# Gravitational quantum states of antihydrogen atoms as a tool for studies of gravitational properties of antimatter

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## Abstract

We present a method to measure gravitational properties of antimatter, which could potentially provide high precision and compactness compared to alternatives considered methods. It profits from interferometry of long-living quasistationary gravitational quantum states of antihydrogen atoms above a surface. The states are settled within the potential well of the Earth's gravitational potential and the Casimir - van der Waals potential of the surface.

## 1 Introduction

Testing of the Weak Equivalence Principle (WEP) in a wide range of physical phenomena is of fundamental interest. The best test so far confirms WEP for macroscopic bodies to the accuracy of $2 \cdot 10^{-13}$ (using a rotating torsion balance [1]). Ongoing projects aim at the accuracy of 1 part in$10^{18}$ (in an Earth orbiting satellite [2]). It is of interest to investigate gravitational properties of quantum mechanical objects; such experiments have been performed with neutrons [3, 4] and atoms [5, 6, 7, 8]. Experiments with antiatoms (see [9, 10] and references therein) are even more interesting, because some theories striving to unify gravity and quantum mechanics (such as supersymmetric string theories) tend to assume violation of the gravitational equivalence of particles and antiparticles [11]. Experiments testing gravitational properties of antiatoms are planned in the near future (see e.g. ATHENA-ALPHA [12], ATRAP [13] and AEGIS [14]). One of the challenging aspects in experiments of this kind is to control the initial parameters of antiatoms, such as their temperature and position, with sufficient accuracy [15]. Assuming that the most difficult task of producing high phase-space density of ultracold antihydrogen atoms will be solved, we consider here only the final stage of such an experiment: the technique to measure the gravitational interaction of ultracold antihydrogen atoms. In particular, we investigate a possibility to explore gravitational properties of antiatoms in the ultimate quantum limit. We study antihydrogen atoms levitating in the lowest gravitational states above a material surface.The existence of such gravitational states for *neutrons* was proven experimentally [16, 17, 18]. The existence of analogous states for antiatoms seems, at a first glance, impossible because of annihilation of antiatoms in the material walls. However, we have shown that ultracold antihydrogen atoms are efficiently reflected from material surface [19, 20] due to so-called quantum reflection from the Casimir-Polder atom-surface interaction potential. We have shown that antihydrogen atoms, confined by the quantum reflection via Casimir forces from below, and by the gravitational force from above, would form metastable gravitational quantum states [21]. They would bounce on a surface for a finite life-time (of the order of 0.1 s) [19]. This simple system can be considered as a microscopic laboratory for testing the gravitational interaction under extremely well specified (in fact, quantized!) conditions.

High precision in measuring energy of quantum states, thus in measuring the strength of the gravitational interaction, is possible provided relatively long lifetimes of the states compared to the characteristic lifetime of formation of the states; numerical estimations will be given below.

## 2 Properties of $\bar{H}$ gravitational states

The Schrödinger equation, which governs antihydrogen ($\bar{H}$) dynamics in a superposition of the Casimir-Polder potential $V(z)$ (with an asymptotic behavior $-C_4/z^4$ at large atom-surface separation z), and the gravitational potential $Mgz$ is:

$$\left[-\frac{\hbar^2 \partial^2}{2m \partial z^2} + V(z) + Mgz - E\right]\Psi(z) = 0. \tag{1}$$

Here $m$ is the inertial mass, $M$ is the gravitational mass of $\bar{H}$, $g$ is the Earth's gravitational field strength. The above equation meets a boundary condition of total absorption of antiatoms in the bulk of the material surface ($z = 0$) [20]. Eq. 1 defines the following length and energy scales:

$$l_0 = \sqrt[3]{\frac{\hbar^2}{2mMg}}, \tag{2}$$

$$l_{CP} = \sqrt{2mC_4}, \tag{3}$$

$$\varepsilon_0 = \sqrt[3]{\frac{\hbar^2 M^2 g^2}{2m}}, \tag{4}$$

$$\varepsilon_{CP} = \frac{\hbar^2}{4m^2 C_4}. \tag{5}$$

Here $l_0 = 5.871$ $\mu m$ is the characteristic spatial scale of gravitational states, $l_{CP} = 0.027$ $\mu m$ is the characteristic Casimir-Polder interaction spatial scale, $\varepsilon_0 = 6.011 \cdot 10^{-13}$ eV is the characteristic gravitational states energy scale, and $\varepsilon_{CP} = 2.739 \cdot 10^{-8}$ eV is the Casimir-Polder energy scale. As one can see, the gravitational spatial scale is much larger than the Casimir-Polder spatial scale $l_0 \gg l_{CP}$, while the gravitational energy scale is much smaller than the Casimir-Polder energy scale $\varepsilon_0 \ll \varepsilon_{CP}$. It is essential that for the characteristic energies of interest ($E \sim \varepsilon_0$) an effect of the Casimir-Polder potential and antiatom annihilation in the material wall can be described by only one complex parameter, namely the complex scattering length, calculated for a perfect conducting wall in ref. [20]:

$$a_{CP} = -(0.1 + i1.05)l_{CP}, \tag{6}$$

$$a_{CP} = -0.0027 - i0.028 \ \mu m. \tag{7}$$

In this case the condition of the scattering length approximation validity $|a_{CP}|\sqrt{2mE}/\hbar \ll 1$ is met. An effect of quantum reflection consists of the over-barrier reflection of atoms from the rapidly changing (asymptotic) tail of the Casimir-Polder potential, the reflection probability $R$ in the low energy case is given by:

$$R \approx 1 - 4|\operatorname{Im} a_{CP}|\sqrt{2mE}/\hbar \tag{8}$$

Detailed calculations [20] (see Fig.1) show that the reflection probability reaches 95% for an antihydrogen energy of $E = 10^{-12}$ eV , 70% for $E = 10^{-10}$ eV and30% for $E = 10^{-9}$ eV. As one can see, the reflection probability tends to unity in the zero energy limit. For the lowest gravitational states the energies $E_n$ are: $\sqrt{2mE_n}|\operatorname{Im} a_{CP}|/\hbar \sim |\operatorname{Im} a_{CP}|/l_0 \ll 1$. Thus an antihydrogen atom in the lowest gravitational states is effectively reflected from a material surface. In such a case an effect of the Casimir-van der Waals interaction and annihilation in
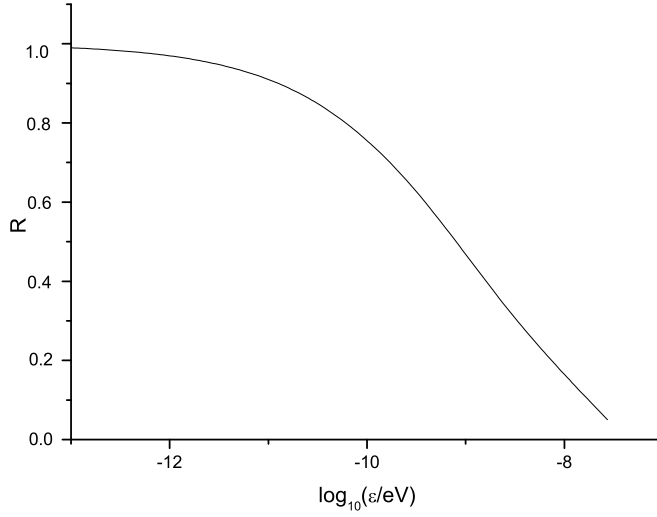
Figure 1: The reflection coefficient from the CP/vdW interaction potential as a function of the antihydrogen incident energy.

the bulk of the wall can be accounted for by a proper boundary condition. The equation for the antihydrogen gravitational states takes a form:

$$
\begin{cases}
\left[-\frac{\hbar^2 \partial^2}{2m \partial z^2} + Mgz - E_n\right]\Psi_n(z) = 0 \\
\frac{\Phi(0)}{\Phi'(0)} = -a_{CP}.
\end{cases}
\tag{9}
$$

The hierarchy of the Casimir-Polder and gravitational scales $l_{CP} \ll l_0$ assumes that the energy levels of antihydrogen in the gravitational field in the presence of the Casimir-Polder potential can be calculated using the following simple modification of the quantum bouncer (i.e. a particle bouncing in the gravitational field above a reflecting mirror) energy levels:

$$
E_n = Mgl_0\lambda_n + Mga_{CP}.
\tag{10}
$$

Here $Mgl_0\lambda_n$ is the quantum bouncer eigen-values, which satisfies the equation

$$
\mathrm{Ai}(-\lambda_n) = 0.
\tag{11}
$$

Here Ai is the Airy function [22]. The negative imaginary part of the scattering length gives the width of the gravitational states:

$$
\Gamma/2 = -Mg\,\mathrm{Im}\,a_{CP}.
\tag{12}
$$

The corresponding lifetime for antihydrogen above a perfect conducting wall is $\tau = 0.1$s.

Using proper materials, such a lifetime could be increased significantly. In particular, calculations based on the Casimir-van der Waals potential for silica, gives a lifetime of $\tau \simeq 0.2$ s. An important message from expression Eq.12 is that the complex shift $\Gamma/2$ is equal for all low-lying quasi-stationary gravitational levels. This result means that the width of all states is equal and the frequencies of transitions between gravitational states are not affected by the Casimir-Polder interaction in the first order of $a_{CP}/l_0$ . In particular, the frequency of transition between the two lowest gravitational states is $\omega_{12} = 254.54$ Hz. It is interesting to estimate

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the number of a highest gravitational state, for which the concept of quasistationary states can be still applied, i.e. the state with a lifetime longer than the corresponding semiclassical period:

$$\frac{\hbar}{\Gamma_n} > T_n, \tag{13}$$

where $T_n = 2\pi\hbar/(dE_n/dn)$ is the semiclassical period of a corresponding gravitational state. Using the semiclassical relation between the width, period and reflection coefficient $R$ we get from Eq.(13):

$$|\ln(R)| < 1. \tag{14}$$

One could find that the lifetime of a gravitational state is longer than its semiclassical period for $n_{max} < 30000$ . The corresponding critical energy is $E_{max} = 1.6 \cdot 10^{-9}$ eV, the classical turning point ("height of a jump") is $H_{max} = 1.3$ cm. Thus an antihydrogen atom "dropped down" from a height of about 1 cm would "bounce" a few times before being totally absorbed. This estimation shows that the concept of gravitational states is applicable even for highly excited states. One can obtain the following expression for the width of an excited state as a function of its energy, within the semiclassical approximation:

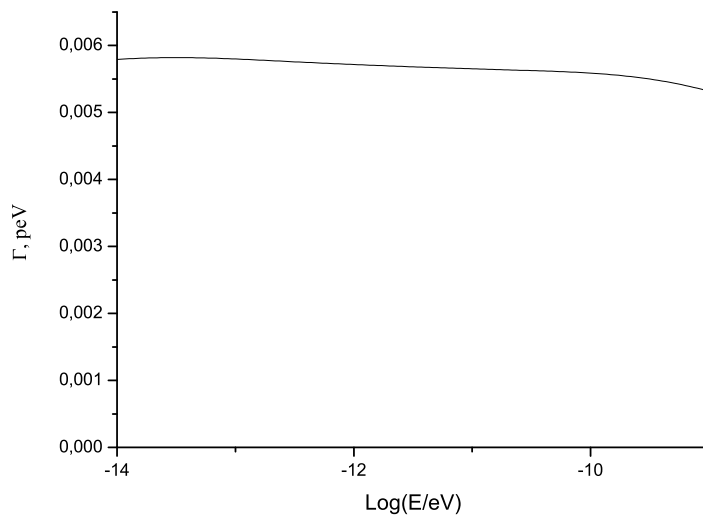$$\Gamma(E) = -\frac{\ln R}{2\sqrt{E}}\varepsilon_0^{3/2}, \tag{15}$$



Figure 2: The width of the antihydrogen gavitational state as a function of the state energy .

The state width dependence on the state energy is shown in Fig.2.

## 3 Interference of antihydrogen gravitational states

The decaying character of gravitational states gives rise to a nonzero probability current through the surface $z = 0$, which determines the annihilation rate of a bouncing antihydrogen atom:

$$j(z,t) = \frac{i\hbar}{2M}\left(\Psi(z,t)\frac{d\Psi^*(z,t)}{dz} - \Psi^*(z,t)\frac{d\Psi(z,t)}{dz}\right). \tag{16}$$

In case of a superposition of a few gravitational states such a current exhibits beatings, corresponding to the gravitational energy level difference. Taking into account the boundary

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condition (5) and an explicit form of the wave-functions (Airy functions) one can get for the annihilation rate $dF_{12}(t)/dt$ of two lowest states a superposition:

$$\frac{dF_{12}(t)}{dt} = -\frac{\Gamma}{\hbar}\exp(-\frac{\Gamma}{\hbar}t)\left(1+\cos(\omega_{12}t)\right). \qquad (17)$$

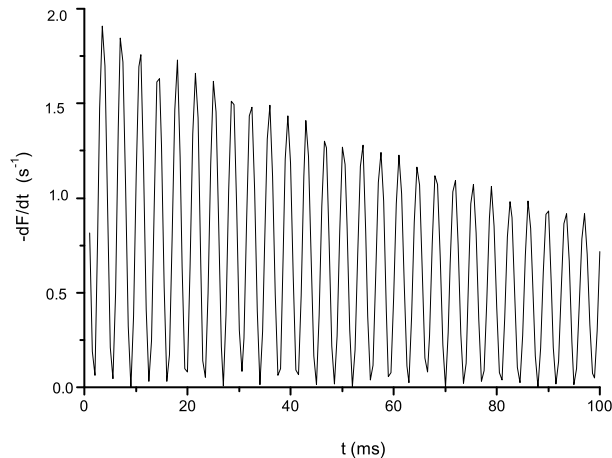Here $\omega_{12} = (\lambda_2 - \lambda_1)\varepsilon_0/\hbar$.



Figure 3: Evolution of the annihilation rate of $\bar{H}$ atom in a superposition of the first and the second gravitational states.

Evolution of annihilation rate for the superposition of two lowest gravitational states is shown in Fig.3. A measurement of the oscillation frequency (due to counting of the annihilation events as a function of time) would allow us to extract the following combination of the gravitational and the inertial masses:

$$\frac{M^2}{m} = \frac{2\hbar\omega_{12}^3}{g^2(\lambda_2 - \lambda_1)^3}. \qquad (18)$$

Under the additional assumption of equality of the known inertial mass of hydrogen atom and that of the antihydrogen, imposed by CPT, we get:

$$M = \sqrt{\frac{2m\hbar\omega_{12}^3}{g^2(\lambda_2 - \lambda_1)^3}}. \qquad (19)$$

Preparing a superposition of several excited states with quantum numbers from the interval $n_0 - \Delta n \leq n \leq$, with $\Delta n \ll n_0$ we can observe semiclassical periodicity in the maxima of annihilation events with the period:

$$T_{cl} = \frac{\hbar}{\varepsilon_0}(12\pi n_0)^{1/3} = 3.35 n_0^{1/3} \text{ ms}. \qquad (20)$$

We show the corresponding evolution of the annihilation rate in Fig.4.
Semiclassical period for $n_0 = 10$ is $T_{cl} = 7.2$ ms.

## 4 Conclusions

We argue for the existence of long-lived quasi-stationary states of $\bar{H}$ above a material surface in the gravitational field of the Earth; the typical lifetime of quantum states is larger than
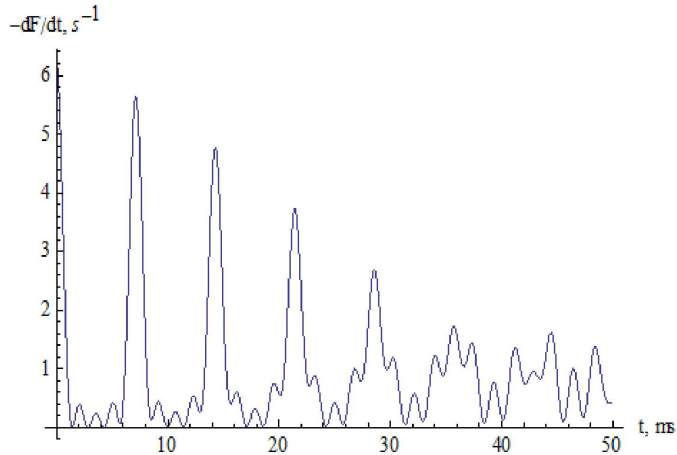
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Figure 4: Evolution of the annihilation rate of $\bar{H}$ atom in a superposition of states with $8 \leq n \leq 12$.

0.1 s. Such a confinement is possible due to the quantum reflection of ultra-cold (anti)atoms from the Casimir-Polder (anti)atom-surface potential. The effective reflection of antiatoms from a material wall is due to smallness of the ratio of the characteristic spatial antiatom-surface interaction scale $l_{CP} = 0.027 \ \mu m$ and the spatial gravitational scale $l_0 = 5.871 \ \mu m$. Choosing a proper material, the reflection coefficient and thus the lifetime of corresponding states could be enhanced, in particular for a silica surface the calculated lifetime is $\tau \simeq 0.2$ s. We show that the spectrum of $\bar{H}$ decaying gravitational levels is quasi-discrete even for highly excited states $n < n_{max} = 30000$ and the corresponding localization is $H_{max} = 1.3$ cm. We show that, by counting the number of annihilation events in a surface the transition frequencies between the gravitational energy levels can be measured. An important observation in this context is that a modification of the above mentioned properties of gravitational states due to the interaction with a surface disappears in the first order of the small ratio $l_{CP}/l_0$ . Such a measurement enables extracting the gravitational mass of and testing WEP in the antimatter sector.

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