Magnetized electron plasma neutrino luminosity via resonant photon

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Abstract

The process of neutrino pair emission by strongly magnetized plasma is investigated. It is shown that calculation of the plasma emissivity due to a general process via resonant photon $i \to \gamma \to f$ can be reduced to the investigation of the process $\gamma \to f$ only. As an example the plasma emissivity due to the process $\gamma e \to \nu\tilde{\nu}e$ via photon intermediate state is calculated.

1 Introduction

In our research we study the process of neutrino pair emission by strongly magnetized plasma due to neutrino electromagnetic interaction. The process $\gamma e \to \gamma \to \nu\tilde{\nu}e$ via photon intermediate state is of special interest because it has resonant character at a particular energy of virtual photon. Taking into account the possible astrophysical application we calculate the volume density of plasma energy losses due to neutrino emission in this process.

The direct calculations of the plasma emissivity due to process $\gamma e \to \gamma \to \nu\tilde{\nu}e$ are a quite complex, however, it turn out that for study of plasma emissivity there is no need to investigate this process completely. As will be shown below it is enough to calculate the width of the process $\gamma \to \nu\tilde{\nu}$ only.

2 Plasma emissivity due to an arbitrary process $i \to \gamma \to f$

Let us discuss the some process with arbitrary "initial" $|i>$ and "final" $|f>$ states via resonant photon intermediate state in a magnetized plasma (fig.1).

Under the "initial" state we will assume the total set of initial particles of the process as well as the don't leaving plasma final particles. While as the "final" state it is considered the weakly interacting final particles (neutrino, axion, etc), which leave plasma and carry away the some energy.

The S-matrix element of the process $i \to \gamma \to f$ can be written as

$$S_{if} = (A_{i\,\gamma})_\alpha \, G_{\alpha\beta} \, (B_{\gamma f})_\beta. \tag{1}$$

Here $(A_{i\,\gamma})_\alpha$ and $(B_{\gamma f})_\beta$ are the some four-vectors, corresponding to the first part of the process considered (the transition $i \to \gamma$) and the second part (the transition $\gamma \to f$) respectively, $G_{\alpha\beta}$ is the photon propagator, which has the following form

$$G_{\alpha\beta} = i \sum_\lambda \frac{\varepsilon_\alpha^{*(\lambda)} \, \varepsilon_\beta^{(\lambda)}}{q^2 - \Pi^\lambda(q)}, \tag{2}$$

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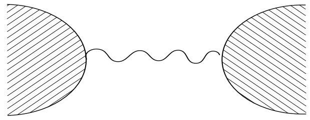

Figure 1: The transition of an arbitrary "initial" state $|i>$ to some "final" state $|f>$ via photon intermediate state.

where $q^\mu = (\omega, \vec{k})$ is the four-momentum of photon, $\varepsilon_\alpha^{(\lambda)}$ are the photon polarization four-vectors, Π^λ are the eigenvalues of photon polarization operator, parameter λ defines the number of eigenvalues and eigenvectors of photon polarization operator.

In a general case three eigenvectors of the photon polarization operator exist in the strongly magnetized plasma:

$$\varepsilon_\alpha^{(1)} = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)} = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}, \tag{3}$$

$$\varepsilon_\alpha^{(3)} = \frac{q^2 (q\varphi\varphi)_\alpha - q_\alpha (q\varphi\varphi q)}{\sqrt{q^2\, q_\parallel^2\, q_\perp^2}},$$

where $\varphi_{\alpha\beta}$ is the dimensionless external field tensor, $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$, $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}$ is the dual tensor, $q_\parallel^2 = (q\tilde{\varphi}\tilde{\varphi}q) = \omega^2 - k_3^2$, $q_\perp^2 = (q\varphi\varphi q) = k_1^2 + k_2^2$, $q^2 = q_\parallel^2 - q_\perp^2$ (it is assumed that a magnetic field is directed along the third axis, $\vec{B} = (0, 0, B)$).

From these modes only second mode with eigenvector $\varepsilon_\alpha^{(2)}$ corresponds to timelike vector with $q^2 > 0$, when the process of photon decay becomes possible. So, we will consider in our investigations the photon of second mode only.

The dispersion law of second mode in the strongly magnetized plasma has the following form:

$$q^2 = \frac{2\alpha e B}{\pi}\,(I(\omega, k_3) - H(z)), \qquad z = \frac{q_\parallel^2}{4m_e^2} < 1, \tag{4}$$

here α is the fine structure constant, m_e is the electron mass, $H(z)$ is the function of $q_\parallel^2$ [3]:

$$H(z) = \frac{1}{\sqrt{z(1-z)}}\, arctg\left(\sqrt{\frac{z}{1-z}}\right) - 1.$$

Function $I(\omega, k_3)$ in (4) can be presented as the integral [4]

$$I(\omega, k_3) = 2\, q_\parallel^2\, m_e^2 \int\limits_{-\infty}^{+\infty} \frac{dp_3}{E}\, \frac{f_{e^-}(E) + f_{e^+}(E)}{4(pq)_\parallel^2 - q_\parallel^4}, \quad f_{e^\pm}(E) = \frac{1}{e^{(E \pm \mu)/T} + 1},$$

where $f_{e^-}(E)$ and $f_{e^+}(E)$ are the distribution functions of plasma electron and positron, $p^\mu = (E, \vec{p})$ is the electron (positron) four-momentum, $(pq) = E\omega - p_3 k_3$, μ and T are the chemical potential and plasma temperature correspondingly.

In general case $\Pi^\lambda(q)$ are the complex functions:

$$\Pi^\lambda(q) = \mathrm{Re}(\Pi^\lambda(q)) + i\,\mathrm{Im}(\Pi^\lambda(q)), \tag{5}$$

where real part of $\Pi^\lambda(q)$ defines the dispersion law:

$$q^2 = \mathrm{Re}(\Pi^\lambda(q)),$$

while the imaginary part is connected with the total width of photon disappearance:

$$\mathrm{Im}(\Pi^\lambda(q)) = -\omega\, \Gamma_{tot}(q). \tag{6}$$

With (2) and (5) the S-matrix element squared can be written in the form:

$$|S_{if}|^2 = \sum_{\lambda,\lambda'} \frac{(A_{i\,\gamma_\lambda}\varepsilon^{*\lambda})(A^*_{i\,\gamma_{\lambda'}}\varepsilon^{\lambda'})(B_{\gamma_\lambda f}\varepsilon^{\lambda})(B^*_{\gamma_{\lambda'} f}\varepsilon^{*\lambda'})}{(q^2 - \mathrm{Re}(\Pi^\lambda(q))^2 + (\mathrm{Im}(\Pi^\lambda(q)))^2}. \tag{7}$$

Taking into account that imaginary part of $\Pi^\lambda(q)$ is more less than real part the resonance factor can be approximated by the delta-function:

$$\frac{1}{(q^2 - \mathrm{Re}(\Pi^\lambda(q)))^2 + (\mathrm{Im}(\Pi^\lambda(q)))^2} = \frac{\pi}{|\mathrm{Im}(\Pi^\lambda(q))|}\,\delta(q^2 - \mathrm{Re}(\Pi^\lambda(q))), \tag{8}$$

where delta-function of q^2 can be reduced to the form

$$\delta(q^2 - \mathrm{Re}(\Pi^\lambda(q))) = \frac{\delta(\omega - \omega_\lambda(k))}{2\omega_\lambda}. \tag{9}$$

Here ω_λ is the root of the equation

$$\omega^2 - \vec{k}^{\,2} - \mathrm{Re}(\Pi^\lambda(q)) = 0.$$

After performing some manipulations and taking into account (8) and (9) the S-matrix element squared can be expressed as

$$|S_{if}|^2 = \frac{V}{\mathcal{T}(2\pi)^3} \int d^3\,k \sum_{\lambda,\lambda'} (S_{i\gamma_\lambda}\, S^*_{i\gamma_{\lambda'}})\,(S_{\gamma_\lambda f}\, S^*_{\gamma_{\lambda'} f})\,\frac{1}{\Gamma_{tot}(q)}, \tag{10}$$

where $\mathcal{T}$ is the total interaction time, V is the normalization volume, $S_{i\gamma_\lambda}$ and $S_{\gamma_\lambda f}$ are the S-matrix elements of the processes $i \to \gamma_\lambda$ and $\gamma_\lambda \to f$ correspondingly.

The total width of photon disappearance $\Gamma_{tot}(q)$ is defined as difference of the widths of photon absorption and creation

$$\Gamma_{tot}(q) = \Gamma_{ab}(q) - \Gamma_{cr}(q). \tag{11}$$

Taking into account the relation between widths of photon absorption and creation [1]

$$\Gamma_{cr}(q) = e^{-\omega/T}\,\Gamma_{ab}(q)$$

the total width of photon disappearance can be represented in the form

$$\begin{aligned} \Gamma_{tot}(q) &= (1 - e^{-\omega/T})\,\Gamma_{ab}(q) = \\ &= (1 - e^{-\omega/T})\,e^{\omega/T}\,\Gamma_{cr}(q) = (e^{\omega/T} - 1)\,\Gamma_{cr}(q). \end{aligned} \tag{12}$$

So for S-matrix element squared of the transition $i \to f$ via photon we obtain

$$|S_{if}|^2 = \frac{V}{\mathcal{T}(2\pi)^3} \int d^3\,k \sum_{\lambda,\lambda'} (S_{i\gamma_\lambda}\, S^*_{i\gamma_{\lambda'}})\,(S_{\gamma_\lambda f}\, S^*_{\gamma_{\lambda'} f})\,\frac{1}{(e^{\omega/T} - 1)\,\Gamma_{cr}(q)}. \tag{13}$$

For calculation the volume density of plasma emissivity due to the process $i \to f$ we need to integrate the S-matrix element squared (13) over both the final and initial states as well

$$\dot{\varepsilon}_{i\to\gamma\to f} = \frac{1}{V}\sum_i \int |S_{if}|^2\,\omega_\lambda\,dn_i\,dn_f, \tag{14}$$

where dn_i and dn_f are the phase-space elements of particles of initial and final states respectively.

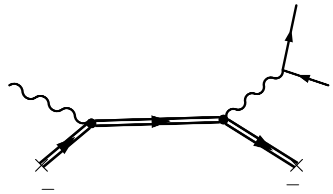

Figure 2: The Feynman diagram of neutrino pair emission via photon intermediate state in magnetized plasma.

Substituting in (14) the S-matrix element squared (13) and omitting the details of calculations, the plasma emissivity can be reduced to the form

$$\dot{\varepsilon}_{i\to\gamma\to f} = \sum_{\lambda}\sum_{i}\int \frac{|S_{i\gamma_\lambda}|^2}{\mathcal{T}}\, dn_i\, \frac{|S_{\gamma_\lambda f}|^2}{\mathcal{T}}\, dn_f\, \frac{d^3k}{(2\pi)^3}\, \frac{\omega}{(e^{\omega/T}-1)\,\Gamma_{cr}(q)}, \tag{15}$$

where

$$\sum_{i}\int \frac{|S_{i\gamma_\lambda}|^2}{\mathcal{T}}\, dn_i = \Gamma_{cr}(q), \quad \int \frac{|S_{\gamma_\lambda f}|^2}{\mathcal{T}}\, dn_f = \Gamma_{\gamma_\lambda\to f}.$$

Finally for the volume density of plasma emissivity due to the process $i \to f$ via photon we obtain the following expression

$$\dot{\varepsilon}_{i\to\gamma\to f} = \sum_{\lambda}\frac{g_\lambda}{V}\int dn_\gamma\, f_{\gamma_\lambda}\, \omega_\lambda\, \Gamma_{\gamma\to f}. \tag{16}$$

Here $dn_\gamma = d^3k\, V/(2\pi)^3$, $f_{\gamma_\lambda}(\omega_\lambda) = (e^{\omega_\lambda/T}-1)^{-1}$ is the photon distribution function, ω_λ is the "resonant" energy of photon, g_λ is the number of photon state with the same dispersion law, $\omega = \omega_\lambda(k)$, $\Gamma_{\gamma_\lambda\to f}$ is the width of the process $\gamma \to f$.

As one can see from (16) the calculation of the plasma emissivity due to the process $i \to \gamma \to f$ via resonant photon can be reduced to the investigation of the process $\gamma \to f$.

3 Plasma emissivity due to the process $\gamma e \to \nu\tilde{\nu}e$

To illustrate possible applications of the result obtained (16) we calculate the plasma emissivity due to neutrino pair emission via resonant photon in the process $\gamma e \to \gamma \to \nu\tilde{\nu}e$, when the neutrino-photon interaction is caused by the neutrino magnetic moment (fig.2). In this case under "initial" state should be considered the initial and final plasma electrons as well as initial photon. As for the "final" state, then it is involves the final neutrino pair.

As it was shown above in (16) for calculation plasma emissivity it is enough to find the width of the process $\gamma \to \nu\tilde{\nu}$. The lagrangian of neutrino-photon interaction caused by the neutrino magnetic moment μ_ν has the form:

$$L = -\frac{i\mu_\nu}{2}\,(\bar{\nu}\,\sigma_{\alpha\beta}\,\nu)\,F^{\alpha\beta}, \tag{17}$$

where $\sigma_{\alpha\beta} = (\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)/2$, ν is the solution of the Dirac equation [5].

The amplitude of the process $\gamma \to \nu\tilde{\nu}$ can be obtained immediately from the lagrangian (17) by means of substitution solutions of the Dirac equation

$$M_{\gamma\to\nu\tilde{\nu}} = \mu_\nu\left(\bar{\nu}(p_1)\nu(p_2)\right)(p_1-p_2)^\alpha\,\varepsilon_\alpha^{(\lambda)}, \tag{18}$$

where $p_1^\mu = (E_1, \vec{p}_1)$ and $p_2^\mu = (E_2, \vec{p}_2)$ are the four-momenta of the final neutrino and antineutrino respectively, $\varepsilon_\alpha^{(\lambda)}$ are the photon polarization four-vectors.

In strongly magnetized plasma only for photon of the second mode a region with $q^2 > 0$ exists, therefore only the photon of second mode can decay into neutrino pair.

The amplitude (18) for photon of the second mode with eigenvector (3) leads to the amplitude squared in the form:

$$|M_{\gamma\to\nu\tilde{\nu}}|^2 = 8\,\mu_\nu^2\,\frac{q^2}{q_\parallel^2}\,(q\tilde{\varphi}p_1)^2. \tag{19}$$

The width of the decay $\gamma \to \nu\tilde{\nu}$ is defined by the standard manner

$$\Gamma_{\gamma\to\nu\tilde{\nu}} = \int \frac{(2\pi)^4\,|M_{\gamma\to\nu\tilde{\nu}}|^2}{(\sqrt{2\omega V}\,\sqrt{2E_1 V}\,\sqrt{2E_2 V}\,)^2}\,V\,\delta^4(q - p_1 - p_2)\,dn_1\,dn_2, \tag{20}$$

where $dn_1 = d^3p_1 V/(2\pi)^3$ and $dn_2 = d^3p_2 V/(2\pi)^3$ are the phase-space of final neutrino and antineutrino correspondingly.

Substituting (19) in (20) and performing integrating over the phase-space of particles for the width of the process $\gamma \to \nu\tilde{\nu}$ we find

$$\Gamma_{\gamma\to\nu\tilde{\nu}} = \frac{\mu_\nu^2}{24\pi}\,\frac{(q^2)^2}{\omega}. \tag{21}$$

The squared of four-vector q^2 in (21) in the strongly magnetized degenerate plasma is defined by the equation (4).

In the limit of low temperature, $T \ll m_e$, at $q_\parallel^2 \ll 4m_e^2$ the dispersion law (4) is essentially simplified:

$$q^2 = \omega_{pl}^2\,\frac{\omega^2 - k_3^2}{\omega^2 - v_F^2\,k_3^2}, \qquad \omega_{pl} = \frac{2\alpha}{\pi}\,eB\,v_F, \tag{22}$$

where ω_{pl} is the plasma frequency, v_F is the Fermi velocity.

The dispersion law (22) has a very simple form in the some limiting cases:

- the case of relativistic plasma, ($v_F \to 1$)

$$q^2 = \omega_{pl}^2, \qquad \omega_{pl}^2 = \frac{2\alpha}{\pi}\,eB, \tag{23}$$

- the case of nonrelativistic plasma, ($v_F \to 0$)

$$q^2 = \omega_{pl}^2\left(1 - \frac{k_3^2}{\omega^2}\right), \qquad \omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}, \tag{24}$$

where n_e is the electron number density.

As one can see from (24) the dispersion law in nonrelativistic plasma is essentially anisotropic as it contains the dependence on third component of photon momentum.

After integration over the phase space of photon intermediate state in accordance with (16) in the case of relativistic plasma for the plasma emissivity in the limit $\omega_{pl} >> T$ we find:

$$\dot{\varepsilon} = \frac{\sqrt{2\pi}\mu_\nu^2}{96\pi^3}\,\omega_{pl}^7\left(\frac{T}{\omega_{pl}}\right)^{3/2} e^{-\omega_{pl}/T}, \tag{25}$$

where T is the plasma temperature.

The plasma emissivity due to neutrino pair emission in the nonrelativistic plasma in the limit $\omega_{pl} \ll T$ has the form:

$$\dot{\varepsilon} = \frac{\mu_\nu^2}{45\pi^3}\,T^3\,\omega_{pl}^4\,\xi(3), \tag{26}$$

where $\xi(3)$ is the Riemann Zeta-function

At the present time in the literature only the numerical estimations of plasma emissivity due to the process $\gamma e \to \nu\tilde{\nu}e$ are presented [6].

The numerical estimations of relativistic plasma emissivity in accordance with the equation (25) reproduces the result [6]. While the numerical estimations in the limit of nonrelativistic plasma differ from one in [6] by a factor 0,64. We believe that this is due to the fact that authors [6] used in the nonrelativistic plasma the same photon dispersion law as in the case of relativistic plasma.

4 Conclusions

We have investigated the general transition process of arbitrary "initial" state to the "final" state via resonant photon in strongly magnetized plasma, $i \to \gamma \to f$. The interest to studies the processes via photon intermediate state is caused by the its resonant character at a particular energy of virtual photon. As the "final" state it is considered neutrino, axion, etc., which due to the weakly interaction with matter leave the plasma and carry away the some energy. The processes with the such weakly interacting particles in final state could be of interest for astrophysical applications, because they could give an additional contribution into the star energy losses and considerable influence on the dynamics of a cooling stars.

The most general expression for the plasma emissivity in the process $i \to \gamma \to f$ via resonant photon is obtained. It was found that the calculation of the plasma emissivity due to the process $i \to \gamma \to f$ via resonant photon can be reduced to the investigation the process $\gamma \to f$.

As an example the plasma emissivity due to the process $\gamma e \to \nu \tilde{\nu} e$ caused by the neutrino electromagnetic interaction is calculated.

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