Constraining deviations from Lorentz invariance in the dark matter sector

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Abstract

We study the possibility to constrain deviations from Lorentz invariance in dark matter (DM) with cosmological observations. The homogeneous expansion of the Universe with Lorentz violating (LV) DM is found to be practically indistinguishable from the standard ΛCDM cosmology. On the other hand, the evolution of the cosmological perturbations significantly differs from the standard case. We discuss, at which level this type of deviations from Lorentz invariance is constrained by the present-day and upcoming cosmological observations.

1 Introduction

Clarifying the nature of dark matter (DM) stands as a major challenge of modern cosmology. One of the basic properties always assumed is that DM satisfies Lorentz invariance (LI). We want to analyse how this assumption can be verified from the study of cosmological perturbations.

Lorentz invariance is one of the best tested symmetries of the Standard Model of particle physics [1]. Thus, it is tempting to postulate that it is a fundamental property of all fields of Nature including gravity and the dark sectors of the Universe, i.e DM and dark energy. However, the unique Lorentz Invariant theory of gravity, GR, suffers from the problem of non-renormalizability precluding its interpretation as a UV complete quantum theory. This is the essence of the notorious problem of quantum gravity.

It is conceivable that the eventual theory of quantum gravity will involve LV in some form. For instance, it was recently suggested by P. Hořava that a UV completion of GR may be possible within perturbative quantum field theory at the cost of abandoning LI at very high energies [2]. However, the renormalizability of gravity in the strict sense along these lines has not yet been demonstrated due to the complexity of the resulting theory. If deviations from LI are present in quantum gravity, it is mandatory to understand which are the consequences for the rest of fields in Nature [3, 4].

A first relevant observation is that even when introduced at very high energies, LV has also consequences at low energies [5]. Precision tests of LI in the Standard Model of particle physics put extremely tight constraints on the LV in the sector of ordinary matter [1, 3]. If similar bounds held for LV in the other sectors, it would have no effect on cosmology. In this paper we adopt the viewpoint that there is a mechanism that enforces LI of the Standard Model with the required precision while allowing for sizable LV in gravitational physics, DM or dark energy. Some consequences of LV for dark energy were unveiled in [12], where it was

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shown that the presence of LV vector field allows to attribute the current acceleration of the Universe to a renormalizable operator not sensitive to UV corrections. The present work is devoted to a systematic investigation of LV effects in the cosmological evolution of DM. To isolate these effects we will assume throughout the paper that the dark energy is represented by the cosmological constant.

We will assume that the velocities of DM are non-relativistic during the observable evolution of the Universe. One might think that this would suppress all LV effects. However, this is not the case. We will see that the coupling with the additional gravitational degrees of freedom modifies the inertial mass of the DM particles, but does not affect their gravitational mass. As a consequence, DM violates the equivalence principle and the dynamics of cosmological perturbations is changed.

2 Gravity theory with Lorentz violation

To describe LV we assume that at every point of space-time there is a time-like vector u_μ that following Ref. [6] we will call "aether". The vector is constrained to have unit norm[1],

$$u_\mu u^\mu = 1 \,. \tag{1}$$

Thus it does not vanish anywhere and sets the preferred time-direction at every point of space-time. This way, the introduction of u_μ allows us to describe LV effects with an action invariant under arbitrary coordinate transformations. We work in the context of the Einstein-aether model [6, 7], and the most general action containing up to two derivatives of u_μ reads

$$S_{æ} \equiv -\frac{M_0^2}{2}\int d^4x\sqrt{-g}\Big[R + K^{\mu\nu}{}_{\sigma\rho}\nabla_\mu u^\sigma \nabla_\nu u^\rho + l(u_\mu u^\mu - 1)\Big] \,, \tag{2}$$

where

$$K^{\mu\nu}{}_{\sigma\rho} \equiv c_1 g^{\mu\nu} g_{\sigma\rho} + c_2\delta^\mu_\sigma \delta^\nu_\rho + c_3 \delta^\mu_\rho \delta^\nu_\sigma + c_4 u^\mu u^\nu g_{\sigma\rho}, \tag{3}$$

and the last term with the Lagrange multiplier l has been added to enforce the unit-norm constraint. We have included in the above action the Einstein-Hilbert term for the metric $g_{\mu\nu}$. The parameter M_0 is related to the Planck mass, cf. (6), while the dimensionless constants c_a, $a = 1, 2, 3, 4$, characterize the strength of the interaction of the aether with gravity. Let us stress that while we are interested in describing LV, the action (2) is explicitly generally covariant. This stems from the assumption that the non-vanishing aether field represents the only source of LV.

A variant of the Einstein-aether model is obtained by restricting the aether to be hypersurface-orthogonal,

$$u_\mu \equiv \frac{\partial_\mu \sigma}{\sqrt{g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma}} \,. \tag{4}$$

In this case the unit-norm constraint is identically satisfied and the Lagrange multiplier term in the action (2) can be omitted. The scalar field σ is assumed to have a time-like gradient at every point of space-time. This defines a preferred time-coordinate, hence σ is called "khronon" (from the Greek word for "time"), and the class of models including the metric and σ "khronometric" models. In this special case of scalar field theory the number of couplings reduces:

$$\lambda \equiv c_2 \,, \quad \beta \equiv c_3 + c_1 \,, \quad \alpha \equiv c_4 + c_1 \,. \tag{5}$$

[1] We use the metric with $(+,-,-,-)$ signature. Latin indices from the middle of the alphabet take the values $i, j = 1, 2, 3$, while Greek letters denote the space-time indices. The latter are manipulated with $g_{\mu\nu}$. Objects in bold face are three-vectors. We use units where the speed of propagation of light is $c = 1$.

This model naturally arises as the low-energy limit of Hořava gravity [9, 8]. In other words, Hořava gravity can provide a UV completion for the khronometric theories which potentially improves their UV behavior as compared to GR.

The main difference between Einstein-aether and khronometric models is the number of degrees of freedom. Einstein-aether describes three types of massless propagating modes: the standard transverse-traceless tensor modes of the metric and the vector and scalar polarizations. For the khronometric case, the transverse vector polarization is absent. In the scalar and tensor sectors the two models are almost equivalent. As in this paper we are mostly interested in the scalar cosmological perturbations, we will often use without loss of generality the terminology of the khronometric model and, in particular, the constants (5). One can show that the parameters c_a (or (5) for the khronometric case) can be chosen such that all modes are stable and have positive energy [7, 10].

The phenomenology of theories with action (2) has been extensively studied [7, 8, 11]. The high precision with which LI is tested within the Standard Model excludes the direct interaction of the aether with baryonic matter, meaning that the latter couples universally to the metric $g_{\mu\nu}$. Still, the aether affects the gravitational field of matter sources. At the Newtonian level the modifications amount to an unobservable renormalization of the gravitational constant that now reads [14, 10],

$$G_N \equiv \frac{1}{8\pi M_0^2}\left(1 - \frac{c_1 + c_4}{2}\right)^{-1} . \tag{6}$$

The deviations from GR for Solar System physics are encoded in two post-Newtonian parameters α_1^{PPN} and α_2^{PPN}. These have been calculated for the generic aether in Ref. [15] and for the khronometric model in Refs. [8, 11]. Observations yield the constraints [16]

$$|\alpha_1^{PPN}| \lesssim 10^{-4} , \qquad |\alpha_2^{PPN}| \lesssim 4 \cdot 10^{-7} . \tag{7}$$

Those constraints are trivially satisfied in GR, where $\alpha_1^{PPN} = \alpha_2^{PPN} = 0$. For LV theories with generic parameters they imply the condition

$$|c_a| \lesssim 10^{-7} .$$

This bound is relaxed for certain relations between the parameters. In the generic aether model one can impose two restrictions on c_a to make both α_1^{PPN} and α_2^{PPN} vanish [7]. Then one is left with a two-parameter family of theories that are indistinguishable from GR at the post-Newtonian level. Remarkably, for the khronometric case the same is achieved by the single condition $\alpha = 2\beta$, which leaves the parameters β and λ arbitrary. Further bounds of order

$$|c_a| \lesssim 10^{-2} ,$$

follow from considerations of Big Bang Nucleosynthesis (BBN) [14] and emission of gravitational waves by binary systems [17, 11].

3 Lorentz violating dark matter: point-particles

To grasp the physical consequences of LV in the DM sector we first study how the coupling to the aether affects the dynamics of gravitationally interacting point particles. In the presence of the aether the relativistic action for a massive point particle can be generalized to

$$S_{pp} \equiv -m \int ds\, F(u_\mu v^\mu) , \tag{8}$$

where

$$ds \equiv \sqrt{g_{\mu\nu} dx^\mu dx^\nu} \tag{9}$$

is the proper length along the trajectory of the particle and

$$v^\mu \equiv \frac{dx^\mu}{ds} \tag{10}$$

is the particle's four-velocity. F is an arbitrary positive function that we normalize to $F(1) = 1$; the GR limit corresponds to $F \equiv 1$. Note that particles described by the action (8) violate the equivalence principle and, actually, do not follow geodesics of any metric.

We will assume that DM is non-relativistic during the relevant stages of cosmological evolution thus one can use the Newtonian limit for qualitative description of the physics in the considered model. This corresponds to expanding the action (8) to quadratic order in the particle three-velocities V^i, quadratic order in the spatial component of the aether, u^i and to linear order in the Newton potential ϕ. The latter appears in the standard Newtonian limit of the metric

$$g_{00} = 1 + 2\phi \;, \quad g_{0i} = 0 \;, \quad g_{ij} = -\delta_{ij}(1 - 2\psi) \;, \tag{11}$$

where for the moment we have neglected the cosmological expansion. We obtain

$$S_{pp} = m \int dt \left[\frac{(V^i)^2}{2} - \phi - Y \frac{(u^i - V^i)^2}{2} \right] , \tag{12}$$

where we have denoted

$$Y \equiv F'(1) \tag{13}$$

and omitted the constant term corresponding to the rest-mass. To understand the effect of LV, we consider first the case when the aether fluctuations are negligible, $u^i = 0$. In this case the last term in (12) renormalizes the particle's inertial mass

$$m \mapsto m(1 - Y) \; .$$

On the other hand, the gravitational mass (the source of ϕ in (12)) remains equal to m, which clearly violates the equivalence principle. To guarantee the positivity of the kinetic energy we impose the restrictions $m > 0$ and $Y < 1$.

Let us now consider the generic situation in the Newtonian limit for a dense medium composed of DM particles. After introducing the mass density one can rewrite the action (12) in the form:

$$S_{pp} = \int d^4x \; \rho \left[\frac{(V^i)^2}{2} - \phi - Y \frac{(u^i - V^i)^2}{2} \right] . \tag{14}$$

One observes that inside the medium the aether perturbations acquire a quadratic potential with the central value set by the velocity of the medium. Not to destabilize the aether the potential must be positive, $Y > 0$. We can anticipate that due to this potential the aether tends to align with the velocity of the medium. When alignment occurs, the last term in (14) disappears, restoring the standard action for the fluid universally coupled to gravity. In other words, the violation of equivalence principle will be screened inside a dense medium, realizing an analog of the chameleon mechanism [19].

The action (14) must be supplemented by the non-relativistic limit of the Einstein-aether action (2). For simplicity, in the rest of this section we will restrict to the case $c_2 = c_3 = c_4 = 0$ and describe the results to leading order in $c_1 \ll 1$, $c_1 \lesssim Y$. In this approximation, the action (2) at post-Newtonian order reads

$$S_{æ} = \frac{M_0^2}{2} \int d^4x \big[4\phi\Delta\psi - 2\psi\Delta\psi + c_1 u^i \Delta u^i \big] \; . \tag{15}$$

Consider a spherical DM halo of size R_h with constant density ρ. According to (14) the aether perturbations inside the halo acquire the effective mass

$$m_{\text{eff}}^2 = \frac{Y\rho}{M_0^2 c_1} \; . \tag{16}$$

Our study shows that there are two different dynamical regimes. First regime takes place for small enough halos,

$$R_h \ll m_{\text{eff}}^{-1} \ , \tag{17}$$

so that the range of the aether interactions exceeds the halo size. Then in (14) one can neglect the u^i-term in the effective potential and obtain that dark matter particles are attracting with the enhanced gravitational force:

$$F = \frac{F_N}{1-Y} \ , \tag{18}$$

where F_N-standard Newton force. As a consequence of an enhanced gravity we have accelerated Jeans instability:

$$\delta \equiv \frac{\delta\rho}{\rho} \propto t^{\gamma} \ , \qquad \gamma = \frac{1}{6}\left[-1+\sqrt{\frac{25-Y}{1-Y}}\right] . \tag{19}$$

Now consider the second case, when the size of the halo is larger than the inverse of the aether mass in its interior,

$$R_h \gg m_{\text{eff}}^{-1} \ . \tag{20}$$

Due to the Yukawa screening, the aether field is frozen at the minimum of its potential, $u^i = V^i$ over most of the halo volume. In this case two lumps of DM are attracting with the standard Newtonian force: $F = F_N$.

One concludes that for large halos the deviations from GR are completely screened. Also in this case we have ordinary growth of the Jeans instability, $\delta \propto t^{2/3}$. Also the size of the Hubble horizon exceeds m_{eff}^{-1} during all the relevant stages, so we can make a conclusion that the effects of Lorentz violating are screened at this scale and expect standard homogeneous cosmology.

4 Lorentz violating dark matter: cosmological consequences

To systematically develop the consequences of the aether – DM interaction beyond the Newtonian limit it is convenient to use a relativistic fluid description of DM. Without going deeply into the detail let us perform only results of our investigations. The reader who is interested in the technical details may find all the relevant information in the [20]. In our model the Universe is filled by the radiation (γ), cold baryons (b),cosmological constant (Λ) and dark matter, interacting with the aether (dm).

As it was discussed above, study of the Newtonian limit hint us that model with LV dark matter should coincide with ΛCDM at the homogeneous level. Taking the FRW metric

$$ds^2 = a^2(\tau)(d\tau^2 - \mathbf{dx}^2) \ , \tag{21}$$

with the conformal time τ, we find the dark matter density behaves as in the standard FRWL-cosmology:

$$\rho_{[dm]} \propto a^{-3} \ , \tag{22}$$

and contribute the Friedmann equation[2],

$$H^2 \equiv \frac{\dot{a}^2}{a^4} = \frac{8\pi G_{cosm}}{3}\big(\rho_{[dm]} + \rho_{[b]} + \rho_{[\gamma]} + \rho_{[\Lambda]}\big) \ . \tag{23}$$

This has the same form as in GR, with the gravitational constant renormalized due to the contribution of the aether (khronon) in the action for gravity (cf. [14]),

$$G_{cosm} = \frac{1}{8\pi M_0^2}\left[1+\frac{\beta+3\lambda}{2}\right]^{-1} . \tag{24}$$

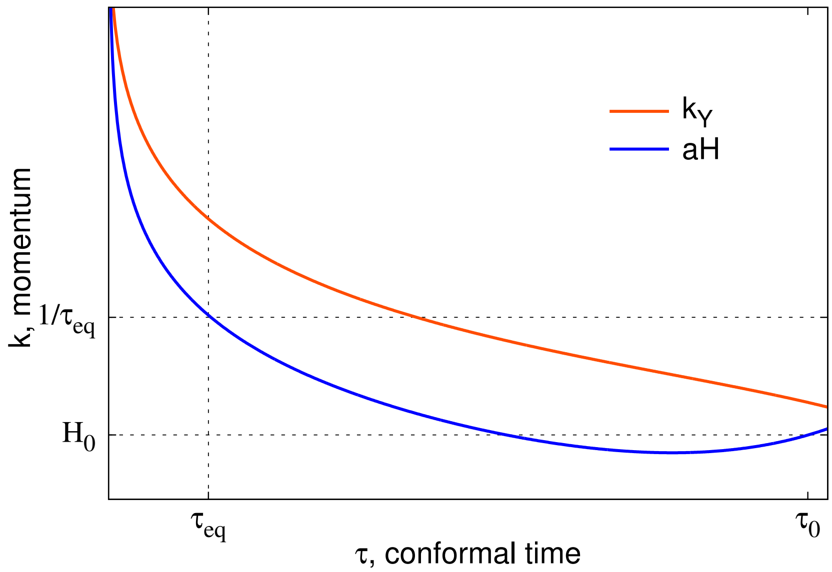

Figure 1: Time dependence of the dynamical scales determining the evolution of the cosmological perturbations. τ_{eq} is the time of radiation–matter equality and τ_0 — the present time. See the main text for other notations.

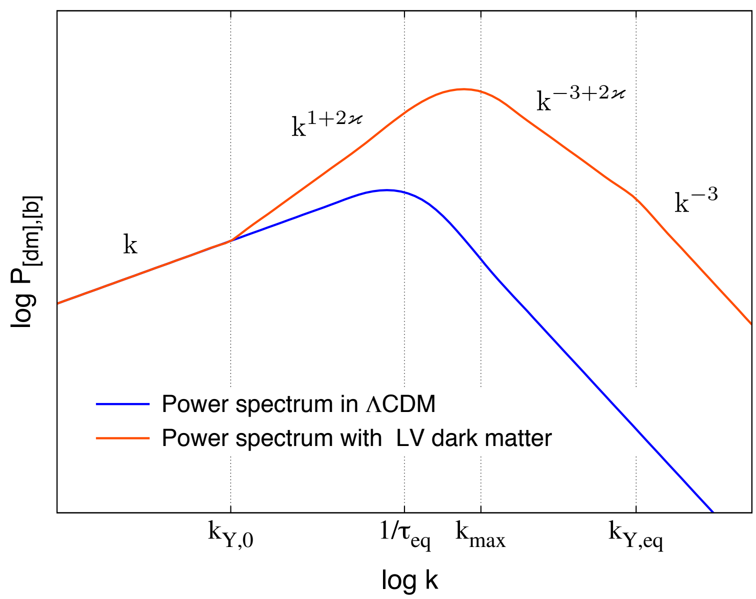

Figure 2: Schematic representation of the matter power spectrum in the LV dark matter model (upper curve) compared to the power spectrum in ΛCDM (lower curve). The same qualitative behaviour is common to the dark matter and baryons. The quantities $k_{Y,0}$, $k_{Y,eq}$, $\varkappa$ are defined in (26), (27), (28). The figure corresponds to the case $k_{Y,0} < 1/\tau_{eq}$.

One clearly see that the expansion history of the Universe with LV dark matter is essentially indistinguishable from that of ΛCDM. The difference between these two theories appears at the level of cosmological perturbations.

[2]The dot denotes henceforth differentiation with respect to the conformal time.

At the language of the cosmological perturbations the dynamics in the Newtonian limit translates into the existence of an additional scale (see Fig.1),

$$k_Y \equiv \left(\frac{Y \bar{\rho}_{[dm]} a^2}{\alpha(1-Y) M_0^2} \right)^{1/2} , \tag{25}$$

which plays the key role in the mode evolution. This new scale is directly related to the density of DM and determines the critical wavenumber below which the effects of LV are screened. Factoring out the explicit time dependence we can write,

$$k_Y = \frac{k_{Y,0}}{\sqrt{a(\tau)}} , \quad k_{Y,0} \equiv H_0 \left[\frac{3Y\Omega_{dm}}{\alpha(1-Y)} \left(1 + \frac{\alpha \mathcal{B}}{2} \right) \right]^{1/2} , \tag{26}$$

where H_0, Ω_{dm} are the present-day Hubble constant and DM density fraction.

When modes are outside new "screening horizon", they exhibit the dynamics, coinciding with that of ΛCDM and effects of LV are hidden. Next, when the scale k_Y is red-shifted to the value k, the effects of LV become unscreened and the modes start to grow faster due to amplification of gravity.

As a consequence of such dynamics, we have scale-depending change in the shape of the matter power-spectrum (see Fig. 2). The change in the slope of power spectrum depends only in the parameter Y, describing LV in DM, while the range of scales where this change occurs is determined both by Y and the khronon parameters α, β, λ. This change takes place for modes in the range

$$k_{Y,0} < k < k_{Y,eq} \equiv \frac{k_{Y,0}}{\sqrt{a(\tau_{eq})}} , \tag{27}$$

and the power-index of enhancement is

$$\varkappa = -\frac{5}{2} + \sqrt{\frac{25}{4} + \frac{6Y}{1-Y} \frac{\Omega_{dm}}{\Omega_{dm} + \Omega_b}}, \tag{28}$$

where Ω_b is the present-day baryons density fraction.

The qualitative analysis of this section is confirmed by the numerical calculations. We consider the effects of different sets of parameters on the spectra of perturbations at the present moment of time. The different choices are listed in Table 1 together with the corresponding values of the screening scales $k_{Y,0}$ and $k_{Y,eq}$ (see Eqs. (26), (27) for definitions). All parameter choices are consistent with the gravitational tests described above. The initial spectrum is taken to be flat with the same normalization in all cases.

	α	β	λ	Y	$k_{Y,0}$ (h Mpc^{-1})	$k_{Y,eq}$ (h Mpc^{-1})
a	$2 \cdot 10^{-2}$	10^{-2}	10^{-2}	0.2	$9.2 \cdot 10^{-4}$	$6.5 \cdot 10^{-2}$
b	$2 \cdot 10^{-4}$	10^{-4}	10^{-4}	0.2	$9.1 \cdot 10^{-3}$	0.65
c	$2 \cdot 10^{-4}$	10^{-4}	10^{-4}	0.02	$2.6 \cdot 10^{-3}$	0.18
d	10^{-7}	0	10^{-7}	0.2	0.41	29

Table 1: The values of the parameters used in numerical simulations.

The comparison between the matter power spectrum in the LV models and in ΛCDM is shown in Fig. 3. The left panel shows the cases when the present screening momentum $k_{Y,0}$ is lower than k_{max} — the position of the power spectrum maximum. We clearly see the change in the slope of the spectrum in the interval $k_{Y,0} < k < k_{Y,eq}$ accompanied by the shift of the position of the maximum. The effect is significant for values of the parameter Y as low as a few per cent, which suggests that these values can be tested observationally. The right panel shows

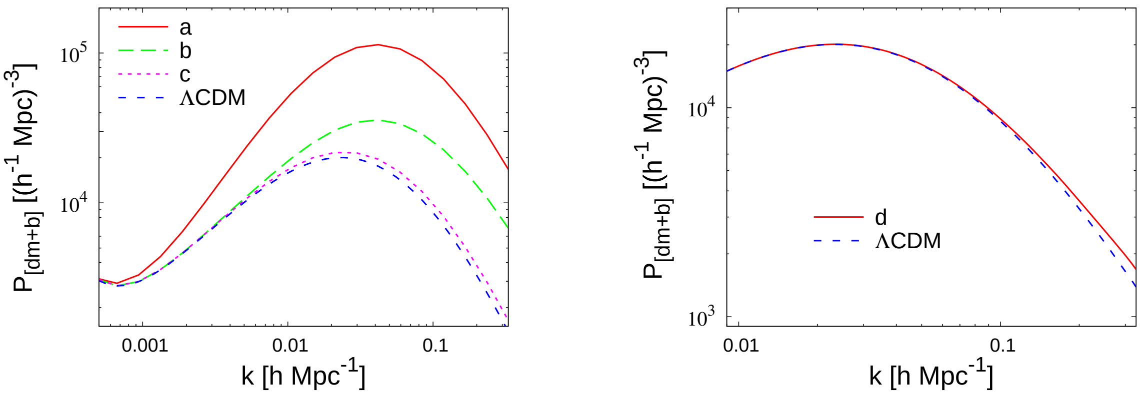

Figure 3: Matter power spectrum for several values of the parameters listed in Table 1. The case of ΛCDM is shown for comparison.

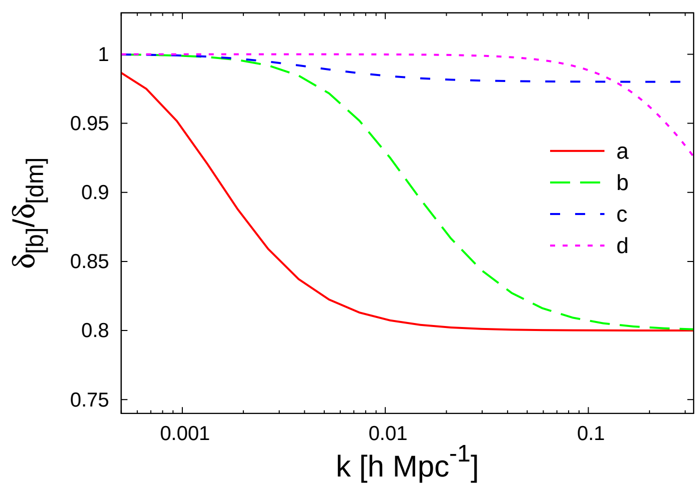

Figure 4: Ratio between the amplitudes of perturbations in the baryonic and dark matter components. The curves correspond to the parameters listed in Table 1.

the situation when $k_{Y,0}$ is larger than k_{max}, corresponding to very small values of the khronon parameters α, β, λ and relatively strong LV in DM, see Table 1. The position of the maximum does not move in this case but the change in the slope is still visible.

Figure 4 shows the ratio between the amplitudes of perturbations in the baryonic and DM components. As expected from the analytic considerations, this ratio drops from 1 at $k < k_{Y,0}$ to $(1 - Y)$ at larger momenta implying a scale dependent bias between baryons and DM.

Finally, Fig. 5 presents the k-dependence of the "relative anisotropic stress" — the difference between the two gravitational potentials ϕ and ψ in the conformal Newton gauge,

$$ds^2 = a(\tau)^2[(1 + 2\phi)d\tau^2 - \delta_{ij}(1 - 2\psi)dx^i dx^j] \,. \tag{29}$$

We observe that at small momenta it has a plateau with the magnitude set by the parameter β. Note, that this effect exists only for non-zero β values. The plateau extends up to $k \approx k_{Y,0}$, beyond which the anisotropic stress drops as the approximate power–law k^{-2}. The persistence of the anisotropic stress up to relatively large momentum, $k_{Y,0} \gg H_0$, is a peculiar signature of the present model that contrasts with the more common situation where the anisotropic stress quickly decays for subhorizon modes.

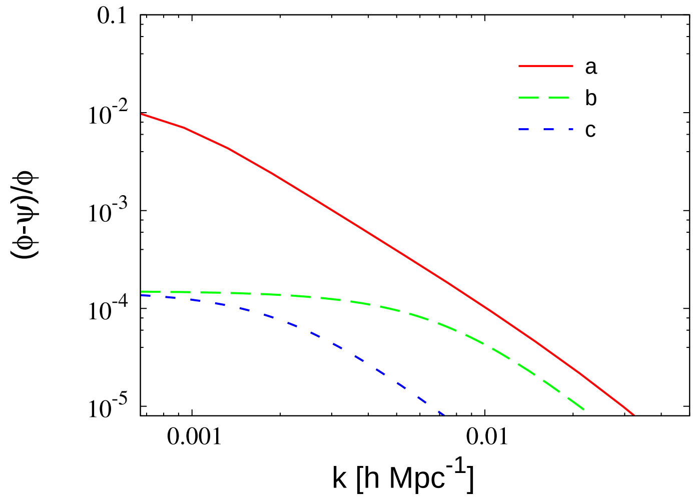

Figure 5: Relative difference between the two scalar gravitational potentials for several choices of parameters listed in Table 1.

5 Summary and discussion

In this short review we have presented the results of testing the Lorentz invariance of dark matter (DM) with cosmological observations. Our description is based on the Einstein-aether/khronometric model, which provides an effective description of Lorentz invariance violation (LV) in gravitational theories. In those models, LV is encoded in a new field (aether) whose expectation value determines a local preferred frame. We generalized DM dynamics to include the LV effects. Those effects amount to different couplings between DM and the aether field. For cosmological perturbations in the linear regime all the LV effects in the DM sector can be summarized in a single parameter Y. This constant is to be added to the parameters of the aether sector (see Eqs. (2), (3) and (5)), which are constrained by local tests of gravity. We considered the Newtonian limit of the model and demonstrated that LV implies modification of the inertial mass for small DM halos thus leading to the violation of the equivalence principle. For large halos this effect is screened by a variant of the chameleon mechanism.

The homogeneous expansion history of the Universe for LV DM was found to be exactly the same as in ΛCDM. However, the evolution of linear cosmological perturbations presents three major effects permitting us to distinguish between the two scenarios. The first effect is the accelerated growth of inhomogeneities for the modes affected by the violation of the equivalence principle. These are the modes that are short enough so that the chameleon mechanism does not switch on. This effect eventually leads to the increase in the slope of the matter power spectrum with respect to ΛCDM in a range of momenta. The enhancement depends only on the parameter Y, whereas the range of momenta is determined also by the aether parameters (cf. Fig. 3). The second effect is the appearance of a new bias between the fluctuations of dark and ordinary baryonic matter. Importantly, the bias exhibits scale-dependence already at the linear level (cf. Fig. 4). Finally, the model predicts non-zero anisotropic stress resulting in the difference between the perturbations of the two gravitational potentials in the conformal Newton gauge. Its present-day power spectrum extends to wavelengths quite inside the current horizon (cf. Fig. 5).

A qualitative comparison between the predictions of the model and those of ΛCDM suggests that the existing data (including local tests of gravity) have the potential to constrain deviations from Lorentz invariance in DM at the level of a few per cent or even better, $Y \lesssim 0.01$. This limit depends on the parameters of the aether sector and may be stronger or weaker for certain regions in the parameter space. Detailed numerical simulations are required to set the precise bounds. This work is currently in progress.

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