Life inside black holes

V. I. Dokuchaev[a*]

[a] *Institute for Nuclear Research of the Russian Academy of Sciences*
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Abstract

We describe the test planet and photon orbits of the third kind inside a rotating and electrically charged Kerr-Newman black hole, which are stable, periodic and neither come out of the black hole nor terminate at the singularity. The third kind orbits exist in between the central singularity and the Cauchy horizon. Interiors of supermassive black holes may be inhabited by advanced civilizations living on the planets with third-kind orbits.

Orbits of the third kind were described in [1, 2, 3, 4] under the assumption of the Kerr-Newman metric validity inside a black hole event horizon. The motion of a test particle (e. g., a planet) with mass μ and electric charge ϵ in the background gravitational field of a Kerr-Newman black hole (BH) with mass M, angular momentum $J = Ma$ and electric charge e is completely defined by three integrals of motion: the total particle energy E, the azimuthal component of the angular momentum L and the Carter constant Q, related to the total angular momentum of the particle and non-equatorial motion.

S. Chandrasekhar [5] designated only two general types of test particle orbits in the black hole gravitational field: orbits of the *first kind*, which are completely confined outside the black hole event horizon, and orbits of the *second kind*, which penetrate inside the black hole. Orbits of the *third kind* are absent in the Schwarzschild case ($a = 0$, $e = 0$). In the Reissner-Nordström case ($e \neq 0$, $a = 0$) there are orbits of the third kind only for charged planets. Respectively, in the Kerr case ($a \neq 0$, $e = 0$) there are only non-equatorial of the third kind for planets and photons. At last, in the most general Kerr-Newman case ($a \neq 0$, $e \neq 0$) there both equatorial and non-equatorial orbits of the kind kinds for planets and photons.

Equations of motion for test particles (e. g. planets) in the Kerr-Newman metric in the Boyer-Lindquist coordinates (t, r, θ, φ) are [6, 7]:

$$\rho^2 \frac{dr}{d\lambda} = \pm\sqrt{V_r}, \quad \rho^2 \frac{d\theta}{d\lambda} = \pm\sqrt{V_\theta}, \tag{1}$$

$$\rho^2 \frac{d\varphi}{d\lambda} = L \sin^{-2}\theta + a(\Delta^{-1}P - E), \tag{2}$$

$$\rho^2 \frac{dt}{d\lambda} = a(L - aE\sin^2\theta) + (r^2+a^2)\Delta^{-1}P, \tag{3}$$

where $\lambda = \tau/\mu$, τ — is the proper time of a particle and

$$V_r = P^2 - \Delta[\mu^2 r^2 + (L - aE)^2 + Q], \tag{4}$$

$$V_\theta = Q - \cos^2\theta[a^2(\mu^2 - E^2) + L^2\sin^{-2}\theta], \tag{5}$$

$$P = E(r^2 + a^2) + \epsilon e r - aL, \quad \rho^2 = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2r + a^2 + e^2. \tag{6}$$

We use the normalized dimensionless variables and parameters: $t \Rightarrow t/M$, $r \Rightarrow r/M$, $a \Rightarrow a/M$, $e \Rightarrow e/M$, $\epsilon \Rightarrow \epsilon/\mu$, $E \Rightarrow E/\mu$, $L \Rightarrow L/(M\mu)$, $Q \Rightarrow Q/(M^2\mu^2)$. The effective potentials V_r

[*] **e-mail**: dokuchaev@inr.ac.ru

and V_θ in (4) and (5) determine the motion of particles in the radial r-direction and latitudinal θ-direction, respectively [7].

The generic non-equatorial orbits of the third kind in the case of rotating black hole ($a \neq 0$) are *periodic* with respect to the separate coordinates r, θ and φ but *non periodic* in time t. Namely, the r-periodicity means that the orbital radial coordinate r oscillates between the minimal (perigee) and maximal (apogee) values, $r_p < r < r_a$. The values of r_p and r_a are defined by zeroes (the bounce points) of the radial potential, $V_r(r_{p,a}) = 0$ in (4). Respectively, the θ-periodicity means that a latitudinal coordinate θ oscillates in the interval $\pi/2 - \theta_{\mathbf{max}} < \theta < \pi/2 + \theta_{\mathbf{max}}$, where the maximum angle of latitudinal elevation $\theta_{\mathbf{max}}$ relative to the equatorial plane ($\theta = \pi/2$) is defined by zeros (the bounce points) of the latitudinal potential $V_\theta(\theta_{\mathbf{max}}) = 0$ in (5). At last, the φ-periodicity means that the azimuth coordinate φ oscillates between some φ_0 and $\varphi_0 + 2\pi$.

In the absence of strict time periodicity of non-equatorial particle orbits around the rotating black hole, it is useful define the *temporal quasi-periods* for oscillation time of separate coordinates, $T_{n,r}$, $T_{m,\theta}$ and $T_{k,\varphi}$), which will depend additionally on the initial orbit conditions and the number of successive periodic circles $(n, m, k) = 1, 2, 3 \dots$ (starting from some initial time t_0), with respect to corresponding coordinates r, θ and φ. These three temporal quasi-periods are incommensurable in general, i. e, all ratios $T_{n,r}/T_{m,\theta}/T_{k,\varphi}$ are non the rational numbers. For this reason the orbits of particles are non closed in space. The strict periodicity in time t is realized only for the equatorial orbits ($Q = 0$) or in the non-rotating case ($a = 0$).

For circular orbits of test particles with $r = const$, equations of motion (4) and (5) provide the conditions:

$$V_r(r) = 0, \quad V_r'(r) \equiv \frac{dV_r}{dr} = 0. \tag{7}$$

The circular orbits would be stable if $V_r'' < 0$, i. e. at the maximum of the effective potential. In the case of a rotating black hole (with $a \neq 0$), a particle in the orbit with $r = const$ may additionally move in the latitudinal θ-direction, if $Q \neq 0$. These non-equatorial orbits are called *spherical orbits* [8]. Purely circular orbits correspond to the particular case of spherical orbits with the parameter $Q = 0$, which are completely confined in the black hole equatorial plane.

In the general Kerr-Newman case there are four possible solutions (some of them may be unstable) of Eqs. (7) for the azimuthal momentum L_i and the total energy E_i of test particles with a charge ϵ on the spherical orbits with $r = const$. Analytical expressions for L_i and E_i are rather cumbersome and presented in [9]. For neutral massive particles ($\epsilon = 0$) from Eqs. (7) we find two pairs of solutions for E and $b = L/E$ for spherical orbits [10]:

$$\begin{aligned} E_{1,2}^2 &= \frac{\mp 2D_2 + \beta_1 r^2 + a^2[2(r-e^2)\Delta - r^2(r-1)^2]Q}{r^4[(r^2-3r+2e^2)^2 - 4a^2(r-e^2)]}, \\ b_{1,2} &= \frac{\pm D_2 r - a^2(r-e^2)\{\beta_2 r + [a^2 - r(r-e^2)]Q\}}{a(r-e^2)\{r[(\Delta-a^2)^2 - a^2(r-e^2)] + a^2(1-r)Q\}}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \beta_1 &= (r^2-3r+2e^2)(r^2-2r+e^2)^2 - a^2(r-e^2)[r(3r-5)+2e^2], \\ \beta_2 &= e^4 - a^2(r-e^2) + 2e^2 r(r-2) - r^2(3r-4), \\ D_2^2 &= [a(r-e^2)\Delta]^2[(r-e^2)r^4 - r^2(r^2-3r+2e^2)Q + a^2Q^2]. \end{aligned} \tag{9}$$

It can be shown that stable spherical orbits are realized for the first pair of solution (E_1, b_1) with $0 < Q < Q_{\max}$, where $Q_{\max}$ is a root of the marginal stability equation $V_r'' = 0$. All spherical orbits with $Q < 0$ are unstable (see also [8]). The stable spherical photon orbit corresponds to the ultrarelativistic limit for massive particle energy on the spherical orbit, $E \to \infty$, which is equivalent to the case $\mu = 0$ [10]. Note that corresponding spherical (or circular) $r = const$ photon orbits outside the black hole horizon are all unstable [7, 8].

The Fig. 1 presents examples of the third kind non-equatorial orbits of a test planet and a photon inside the Kerr black hole, calculated by numerical integration of equation of motion (1) – (3) in the Boyer-Lindquist frame. Respectively, Fig. 2 presents numerically calculated examples of the third kind non-equatorial orbits for test planet and photon inside the Kerr-Newman black hole, viewed in the Locally Non-Rotating Frame (LNRF) [7].

We hypothesize [9, 10] that the advanced civilizations of third type (according to N. S. Kardashev scale [11]) may live safely inside the supermassive black holes in the galactic nuclei being invisible from the outside. Yet, some difficulties (or advantages?) of a life inside black hole are worth mentioning, such as a possible causality violation [6, 12] and mass inflation in close vicinity of the Cauchy horizon [13]. The existence of third kind orbits inside the event horizon may be verified or falsified in principle (without a traveling inside black holes) by observations of white holes counterparts.

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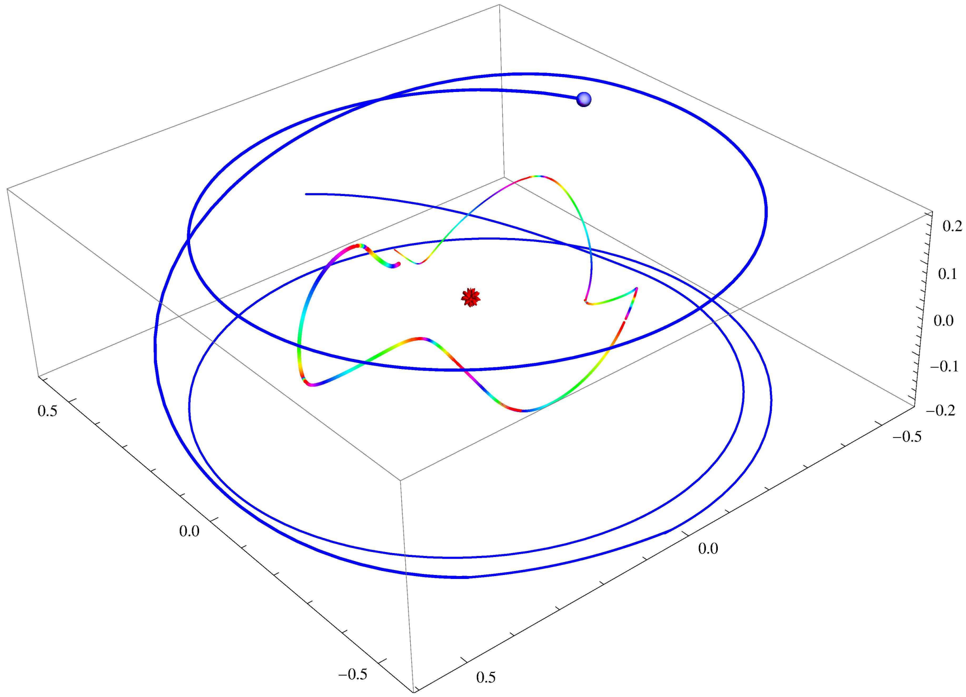

Figure 1: A non-equatorial stable periodic orbit of a planet (the external curve) and a photon orbit (the internal curve, with $b = L/E = 1.5$, $q = Q/E^2 = 0.09$) inside the Kerr black hole ($a = 0.9982$, $e = 0$) in the in the Boyer-Lindquist frame. The thickness of trajectories are growing in the direction of motion. In particular, the first temporal quasi-periods (duration of the first coordinate oscillations from the starting time $t_0 = 0$) for the shown photon orbit $(T_{1,r}, T_{1,\theta}, T_{1,\varphi}) = (0.49, 0.33, 2.95)$, the perigee and apogee photon radii $(r_p, r_a) = (0.24, 0.35)$ and the maximal elevation angle with respect to equatorial plane $\theta_{\max} = 14.6°$.

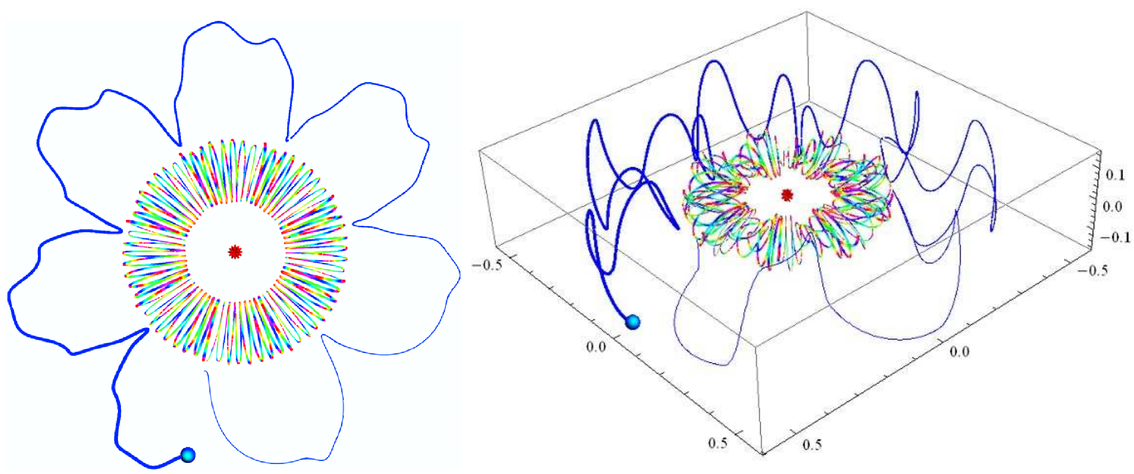

Figure 2: A non-equatorial stable periodic orbit of planet (the external curves) with $E = 0.568$, $L = 1.13$, $Q = 0.13$, $(r_p, r_a) = (0.32, 0.59)$ and photon orbit (the internal curves) with $b = L/E = 1.38$, $q = Q/E^2 = 0.03$, $(r_p, r_a) = (0.13, 0.29)$ inside a black hole ($a = 0.9982$, $e = 0.05$) in the Locally Non-Rotating Frame (LNRF) [7], viewed from the north pole (left panel) and from the outside (right panel). The first temporal quasi-periods are for photon $(T_{1,r}, T_{1,\theta}, T_{1,\varphi}) = (0.51, 0.45, 32.8)$ and for planet $(T_{1,r}, T_{1,\theta}, T_{1,\varphi}) = (4.06, 4.03, 25.2)$, respectively. Few next successive radial quasi-periods for planet are $T_{n,r} = 3.87, 3.65, 3.63, 3.88$ for $n = 2, 3, 4, 5$.