Particle production in modified gravity

E. V. Arbuzova[a,b]*, A. D. Dolgov[a,c,d,e]†

[a] *Novosibirsk State University*
Pirogova ul., 2, 630090 Novosibirsk, Russia
[b] *Department of Higher Mathematics, University "Dubna"*
Universitetskaya ul., 19, 141980 Dubna, Russia
[c] *Institute of Theoretical and Experimental Physics*
Bol. Cheremushkinsaya ul., 25, 113259 Moscow, Russia
[d] *Dipartimento di Fisica, Università degli Studi di Ferrara*
Polo Scientifico e Tecnologico – Edificio C, via Saragat 1, I-44124 Ferrara, Italy
[e] *Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara*
Polo Scientifico e Tecnologico – Edificio C, via Saragat 1, I-44124 Ferrara, Italy

Abstract

Gravity modifications at small curvatures which could induce observed accelerated cosmological expansion are discussed. It is shown that astronomical systems with rising energy/mass density evolve to a singular state in the future in finite time. The universe evolution during the radiation-dominated epoch is investigated in the R^2–extended gravity theory. Particle production in cosmology by the oscillating curvature and the back reaction of particle production on the evolution of R are calculated in one-loop approximation. Particle production in infrared-modified gravitational theories is also studied at the contemporary universe epoch. It is shown that in an interesting range of the model parameters the oscillating curvature could be a source of energetic cosmic rays. Possible implications of the models for cosmological creation of non-thermal dark matter are discussed.

A large set of independent, different types astronomical data strongly indicates that the universe today expands with acceleration. The data include the observation of the large scale structure of the universe, the measurements of the angular fluctuations of the cosmic microwave background radiation, the determination of the universe age (for a review see [1]), and especially the discovery of the dimming of distant Supernovae [2]. It was established and unambiguously proved that the universe expansion is accelerated but the driving force behind this accelerated expansion is still unknown.

Among possible explanations, very popular is the assumption of a new (unknown) form of cosmological energy density with large negative pressure, $P < -\varrho/3$, the so-called dark energy (for a review see e.g. [3]). The latter can be either a small vacuum energy, which is identical to cosmological constant, or the energy density associated with an unknown, pressumably scalar field, which slowly varies in the course of the cosmological evolution. The problem of vacuum energy and possible ways to its solution are described in [4] .

Soon after discovery of the accelerated expansion, theories with competing mechanism for producing cosmological acceleration have been proposed. These theories are based on the gravity modifications at large scales by introducing an additional term, $F(R)$, into the usual action of General Relativity (GR):

$$S = \frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}[R + F(R)] + S_m = \frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} f(R) + S_m \, , \tag{1}$$

* **e-mail**: arbuzova@uni-dubna.ru
† **e-mail**: dolgov@fe.infn.it

where $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass and S_m is the matter action.

The pioneering works in this direction was done in ref. [5], which was closely followed by ref. [6]. In these works the singular in R action:

$$F(R) = -\mu^4/R\,, \tag{2}$$

has been suggested to describe the observed cosmological acceleration. The constant parameter μ with dimension of mass squared was chosen as $\mu^2 \sim R_c \sim 1/t_U^2$. However, the small coefficient, μ^4, in front of the highest derivative in the corresponding equation of motion leads to a strong instability in presence of matter [7] with the characteristic time:

$$\tau = \frac{\sqrt{6}\mu^2}{T^{3/2}} \sim 10^{-26}\text{sec}\left(\frac{\varrho_m}{\text{g/cm}^3}\right)^{-3/2}, \tag{3}$$

where ϱ_m is the mass density of the body and $\mu^{-1} \sim t_u \approx 3 \cdot 10^{17}$ *sec.*

To avoid the problem of such instability further modification of the modified gravity has been suggested. We will consider here some class of the models discussed in refs. [8] with the different actions:

$$F_{\text{HS}}(R) = -\frac{R_{\text{vac}}}{2}\frac{c\left(\frac{R}{R_{\text{vac}}}\right)^{2n}}{1+c\left(\frac{R}{R_{\text{vac}}}\right)^{2n}}, \tag{4}$$

$$F_{\text{AB}}(R) = \frac{\epsilon}{2}\log\left[\frac{\cosh\left(\frac{R}{\epsilon}-b\right)}{\cosh b}\right] - \frac{R}{2}, \tag{5}$$

$$F_{\text{S}}(R) = \lambda R_0\left[\left(1+\frac{R^2}{R_0^2}\right)^{-n} - 1\right]. \tag{6}$$

Some other forms of gravity modification are reviewed in ref. [9]. All these functions have been carefully constructed to satisfy conditions considered below (we essentially follow the analysis made in ref. [10]) and, despite different forms, result in quite similar consequences.

The condition of accelerated expansion in absence of matter is the existence of real positive root, $R = R_1 > 0$, of the equation

$$Rf'(R) - 2f(R) = 0\,, \tag{7}$$

where R_1 is (approximately) constant.

The following necessary conditions to avoid pathologies are to be satisfied:

1. Future stability of cosmological solutions:

$$f'(R_1)/f''(R_1) > R_1\,. \tag{8}$$

2. Classical and quantum stability (gravitational attraction and absence of ghosts):

$$f'(R) > 0\,. \tag{9}$$

3. Absence of matter [7] instability:

$$f''(R) > 0\,. \tag{10}$$

4. Existence of the stable Newtonian limit:

$$|f(R) - R| \ll R\,, |f'(R) - 1| \ll 1\,, Rf''(R) \ll 1\,. \tag{11}$$

Note that the effective scalaron mass squared is $M^2(R) = 1/(3f''(R))$ and the third condition (10) means that the scalaron is not a tachyon.

Despite considerable improvement, the models proposed in [8] possess another troublesome feature, namely in a cosmological situation they should evolve from a singular state with an infinite R in the past [11]. In other words, if we travel backward in time from a normal cosmological state, we come to a singular state with infinite curvature while the energy density remains finite.

In cosmology energy density drops down with time and singularity doesn't appear in the future. However, systems with rising mass/energy density will evolve to a singularity, $R \to \infty$, in a finite time [12, 13]. Such future singularity is unavoidable, regardless of the initial conditions, and infinite value of R would be reached in time which is much shorter than the cosmological one.

Following ref. [13] let us consider version (6) of $F(R)$ function in the case of large R. We analyze the evolution of R in massive objects with time varying mass density, $\varrho_m \gg \varrho_c$. The cosmological energy density at the present time is $\varrho_c \approx 10^{-29}\,\mathrm{g/cm}^3$, while matter density of, say, a dust cloud in a galaxy could be about $\varrho_m \sim 10^{-24}\mathrm{g/cm}^3$. Since the magnitude of the curvature scalar of a nonrelativistic system is proportional to its mass density, we find $R \gg R_0$. In this limit we can approximately take:

$$F(R) \approx -\lambda R_0 \left[1 - \left(\frac{R_0}{R}\right)^{2n} \right] . \tag{12}$$

Gravitational field of such object is supposed to be weak, so the background metric is approximately flat and covariant derivatives can be replaced by the usual flat space ones. The corresponding equation of motion takes the form:

$$(\partial_t^2 - \Delta)R - (2n+2)\frac{\dot{R}^2 - (\nabla R)^2}{R} + \frac{R^2}{3n(2n+1)}\left[\frac{R^{2n}}{R_0^{2n}} - (n+1)\right] - \frac{R^{2n+2}}{6n(2n+1)\lambda R_0^{2n+1}}(R+T) = 0\,. \tag{13}$$

The equation is very much simplified if we choose another unknown function: $w \equiv F' = -2n\lambda\,(R_0/R)^{2n+1}$ which satisfies:

$$(\partial_t^2 - \Delta)w + U'(w) = 0\,. \tag{14}$$

Here potential $U(w)$ is equal to:

$$U(w) = \frac{1}{3}\left(T - 2\lambda R_0\right)w + \frac{R_0}{3}\left[\frac{q^\nu}{2n\nu}w^{2n\nu} + \left(q^\nu + \frac{2\lambda}{q^{2n\nu}}\right)\frac{w^{1+2n\nu}}{1+2n\nu}\right], \tag{15}$$

where $\nu = 1/(2n+1)$ and $q = 2n\lambda$.

Notice that the infinite R singularity corresponds to $w = 0$.

If only the dominant terms are retained and if the space derivatives are neglected, equation (14) simplifies to:

$$\ddot{w} + T/3 - \frac{q^\nu(-R_0)}{3w^\nu} = 0\,. \tag{16}$$

Potential U would depend upon time, if the mass density of the object changes with time. We parametrize it as:

$$T = T(t) = T_0(1 + \kappa\tau)\,, \tag{17}$$

where τ is dimensionless time introduced below.

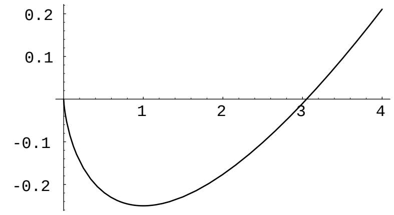

Figure 1: Potential $U(z) = z(1+\kappa\tau) - z^{1-\nu}/(1-\nu),\ \nu = \frac{1}{5}, \tau = 0.$

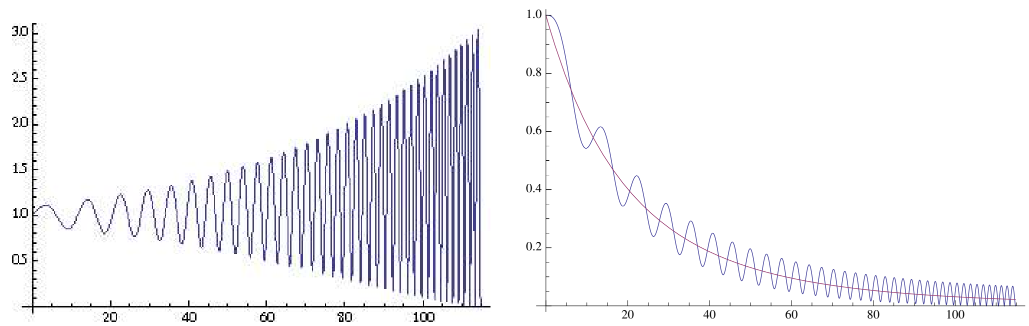

Figure 2: Ratio $z(\tau)/z_{min}(\tau)$ (left) and functions $z(\tau)$ and $z_{min}(\tau)$ (right) for $n = 2$, $\kappa = 0.01$, $\varrho_m/\varrho_c = 10^5$. The initial conditions are $z(0) = 1$ and $z'(0) = 0$.

With dimensionless quantities $t = \gamma\tau$ and $w = \beta z$, where

$$\gamma^2 = \frac{3q}{(-R_0)}\left(-\frac{R_0}{T_0}\right)^{2(n+1)}, \ \beta = \gamma^2 T_0/3 = q\left(-\frac{R_0}{T_0}\right)^{2n+1} \tag{18}$$

the equation further simplifies:

$$z'' - z^{-\nu} + (1+\kappa\tau) = 0\,. \tag{19}$$

Minimum of potential $U(z)$ (Fig. 1) sits at $z_{min} = (1+\kappa\tau)^{-1/\nu}$. When the mass density rises, the minimum moves towards zero and becomes more and more shallow. If at the process of "lifting" of the potential $z(\tau)$ happens to be at $U > 0$ it would overjump potential which is equal to zero at $z = 0$. In other words, $z(\tau)$ would reach zero, which corresponds to infinite R, and so the singularity can be reached in finite time (see Fig. 2).

The aforementioned problems can be cured by adding to the action quadratic in curvature term $R^2/(6m^2)$ [14], which prevents from the singular behavior both in the past and in the future.

In the homogeneous case and in the limit of large ratio R/R_0 the addition of R^2 term leads to the following modification of the equation of motion:

$$\left[1 - \frac{R^{2n+2}}{6\lambda n(2n+1)R_0^{2n+1}m^2}\right]\ddot{R} - (2n+2)\,\frac{\dot{R}^2}{R} - \frac{R^{2n+2}(R+T)}{6\lambda n(2n+1)R_0^{2n+1}} = 0\,. \tag{20}$$

With dimensionless curvature and time

$$y = -\frac{R}{T_0}\,, \quad \tau_1 = t\left[-\frac{T_0^{2n+2}}{6\lambda n(2n+1)R_0^{2n+1}}\right]^{1/2} \tag{21}$$

the equation for R is transformed into:

$$\left(1 + gy^{2n+2}\right)y'' - 2(n+1)\,\frac{(y')^2}{y} + y^{2n+2}\left[y - (1+\kappa_1\tau_1)\right] = 0\,, \tag{22}$$

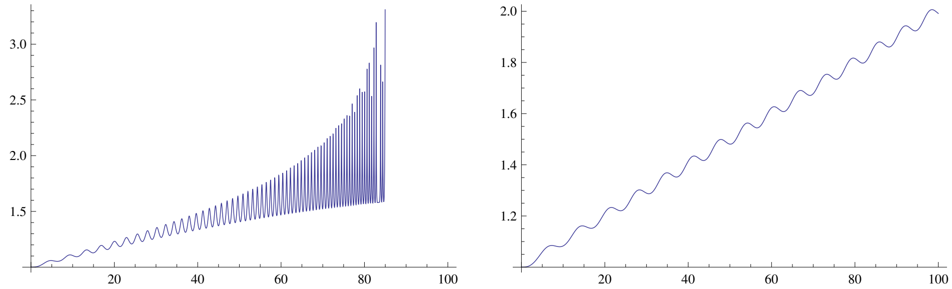

Figure 3: Numerical solutions of eq. (22) for $n = 3$, $\kappa_1 = 0.01$, $y(\tau_{in}) = 1 + \kappa_1 \tau_{in}$, $y'(\tau_{in}) = 0$. Left panel: $g = 0$. Right panel: $g = 1$.

where prime now means derivative with respect to τ_1.

We introduced here the new parameter, g, which can prevent from the approach to infinity and is equal to:

$$g = -\frac{T_0^{2n+2}}{6\lambda n(2n+1)m^2 R_0^{2n+1}} > 0\,. \tag{23}$$

For very large m, or small g, when the second term in the coefficient of the second derivatives in eqs. (20) and (22) can be neglected, numerical solution demonstrates that R would reach infinity in finite time in accordance with the results presented above (see Fig. 3, left panel). Nonzero g would terminate the unbounded rise of R. To avoid too large deviation of R from the usual gravity coefficient g should be larger than or of the order of unity. In the right panel of Fig. 3 it is clearly seen, that for $g = 1$ the amplitude of oscillations remains constant whereas the average value of R increases with time.

As follows from eq. (22), the frequency of small oscillations of y around $y_0 = 1 + \kappa_1 \tau_1$ in dimensionless time τ_1 is

$$\omega_\tau^2 = \frac{1}{g}\,\frac{g y_0^{2n+2}}{1 + g y_0^{2n+2}} \leq \frac{1}{g}\,. \tag{24}$$

It means that in physical time the frequency would be

$$\omega \sim \frac{1}{t_U}\left(\frac{T_0}{R_0}\right)^{n+1} \frac{y_0^{n+1}}{\sqrt{1 + g y_0^{2n+2}}} \leq m\,. \tag{25}$$

In particular, for $n = 5$ and for a galactic gas cloud with $T_0/R_0 = 10^5$, the oscillation frequency would be 10^{12} Hz $\approx 10^{-3}$ eV. Higher density objects e.g. those with $\varrho = 1$ g/cm^3 would oscillate with much higher frequency, saturating bound (25), i.e. $\omega \sim m$. All kind of particles with masses smaller than m might be created by such oscillating field.

As more detailed analysis shows [15] the pattern of oscillations in presence of matter is more complicated. They very much differ from the harmonic ones and consist of high narrow spikes with characteristic frequency of the order of m, separated by wide low amplitude periods, as shown on Fig. 4.

An important effect which is not taken into account in equation (20) and which may also inhibit unbounded rise of R is the particle production by oscillating curvature R. The technique for calculations of particles production applicable to the case of modified gravity (4-6) was worked out in ref. [16] for the case of R^2 gravity in cosmological situation (in the early universe) where the classical results [17] for particle production were reproduced.

As it was done in [16], let us consider the cosmological evolution of the universe in a theory with only an additional R^2 term in the action, neglecting other terms which have been

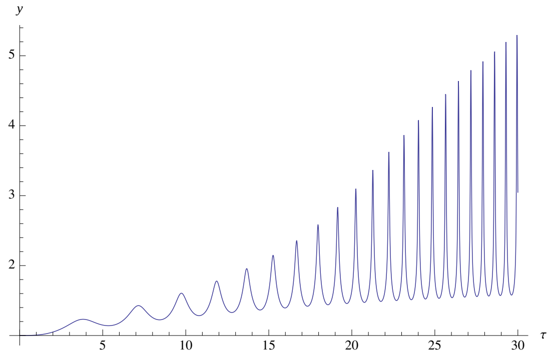

Figure 4: Numerical solution of eq. (22) for $n = 2$, $g = 0.001$, $\kappa_1 = 0.04$, $y(\tau_{in}) = 1 + \kappa_1 \tau_{in}$, $y'(\tau_{in}) = 0$.

introduced to generate the accelerated expansion in the contemporary universe. The impact of such terms is negligible in the limit of sufficiently large curvature, $|R| \gg |R_0|$, where R_0 is the cosmological curvature at the present time.

In other words, we study below the cosmological evolution of the early and not so early universe in the model with the following action:

$$S = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{R^2}{6m^2} \right) + S_m \tag{26}$$

with the account of the back-reaction of particle production.

The modified Einstein equations for this theory read

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{3m^2} \left(R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} + g_{\mu\nu} \mathcal{D}^2 - \mathcal{D}_\mu \mathcal{D}_\nu \right) R = \frac{8\pi}{m_{Pl}^2} T_{\mu\nu} \,, \tag{27}$$

where $\mathcal{D}^2 \equiv g^{\mu\nu} \mathcal{D}_\mu \mathcal{D}_\nu$ is the covariant D'Alembert operator.

The curvature scalar R is expressed through the Hubble parameter $H = \dot{a}/a$ as

$$R = -6\dot{H} - 12H^2 \,. \tag{28}$$

Therefore, the time-time component of eq. (27) reads

$$\ddot{H} + 3H\dot{H} - \frac{\dot{H}^2}{2H} + \frac{m^2 H}{2} = \frac{4\pi m^2}{3m_{Pl}^2 H} \varrho \,, \tag{29}$$

where over-dots denote derivative with respect to physical time t.

Taking trace of eq. (27) yields

$$\ddot{R} + 3H\dot{R} + m^2(R + T) = 0 \,. \tag{30}$$

This equation is a sort of Klein-Gordon equation for a homogeneous scalar field, the "scalaron", of mass m, with a source term proportional to the trace of the energy-momentum tensor of matter. The General Relativity case may be recovered when $m \to \infty$. In this limit we expect to obtain the usual algebraic relation between the curvature scalar and the trace of the energy-momentum tensor of matter:

$$m_{Pl}^2 R_{GR} = -8\pi T_\mu^\mu \,. \tag{31}$$

However, unlike the usual GR, in higher-order theories curvature and matter are related to each other through a differential equation, but not simply algebraically. Therefore, the theory may approach GR as $m \to \infty$ in a non-trivial way or even not approach it at all.

For a perfect fluid with relativistic equation of state, $P = \varrho/3$, the trace of the energy-momentum tensor of matter T_μ^μ vanishes and R satisfies the homogeneous equation. The GR solution $R = 0$ satisfies this equation, but if one assumes that neither R nor $\dot{R}$ vanish initially, the general solution for R will be an oscillating function with a decreasing amplitude. The decrease of the amplitude is induced by the cosmological expansion (the second term in eq. (30)) and by particle production by the oscillating gravitational field $R(t)$. The latter is not included in this equation and will be taken into account below.

In what follows, we study the cosmological evolution in the R^2-theory assuming rather general initial conditions for R and H and dominance of relativistic matter which is red-shifted according to:

$$\dot{\varrho}_\mathrm{R} + 4H\varrho_\mathrm{R} = 0\,. \tag{32}$$

There is a possibility of gravitational particle production, which may non-trivially affect the solutions of the above equations. In first approximation, however, we neglect such contributions, which will be dealt with later on in the final part of this paper.

It is convenient to rewrite the equations in terms of the dimensionless quantities $\tau = H_0\,t$, $h = H/H_0$, $r = R/H_0^2$, $y = 8\pi\varrho/(3m_{Pl}^2 H_0^2)$, and $\omega = m/H_0$, where H_0 is the value of the Hubble parameter at some initial time t_0. Thus equations (29) and (32) transform into the system:

$$\begin{cases} h'' + 3hh' - \dfrac{h'^2}{2h} + \dfrac{\omega^2}{2}\dfrac{h^2 - y}{h} = 0\,, \\ y' + 4hy = 0\,. \end{cases} \tag{33}$$

Here prime indicates derivative with respect to dimensionless time τ.

First we assume that the deviations from GR are small and expand $h = 1/(2\tau) + h_1$ and $y = 1/(4\tau^2) + y_1$, assuming that $h_1/h \ll 1$ and $y_1/y \ll 1$, and linearize the system of equations.

The complete asymptotic solution for h has the form:

$$h(\tau) \simeq \frac{1}{2\tau} + \frac{c_1 \sin(\omega\tau + \varphi)}{\tau^{3/4}}\,, \tag{34}$$

and describes oscillations of the Hubble parameter around the GR value $1/(2\tau)$. Moreover, the amplitude of such oscillations decreases more slowly than $1/\tau$, so for sufficiently large τ the second term would start to dominate, the oscillations will become large, and the condition $h_1 \ll h$ will no longer be satisfied. After this stage is reached, the linear approximation becomes invalid.

However, we can proceed further using a sort of truncated Fourier expansion which allows to take into account the non-linearity of the system in the limit $\omega\tau \gg 1$. As a result we have found that $h_1/h \to$ const. In other words, the amplitude of the oscillating part of h asymptotically behaves as $1/\tau$, i.e. in the same way as the slowly-varying part of h, but the oscillation center is shifted above the GR value 1/2.

Numerical solutions of eqs. (33) with the initial conditions $h_0 = 1 + \delta h_0$, $h_0' = -2$, $y_0 = 1$ presented in Fig. 5 demonstrate good agreement with the analytical results. In the linear regime (left panel of Fig. 5) function $h\tau$ oscillates around the central value $h = 1/2$ with amplitude $h_1 \sim \tau^{-3/4}$. As the deviation from the ideal GR behavior increases, the amplitudes of the oscillating terms of both h and r decrease faster than $\tau^{-3/4}$ (linear regime), and rather close to τ^{-1}. Furthermore, the Hubble parameter does not oscillate around the GR value $h\tau = 1/2$, but around a larger value (right panel of Fig. 5).

Gravitational particle production may non-trivially affect the solutions of the above equations. Below we consider particle production by the external oscillating gravitational field and present the equation of motion for the evolution of R with the account of the back-reaction from particle production. This leads to an exponential damping of the oscillating part of R, while the non-oscillating "Friedmann" part remains practically undisturbed. The particle production

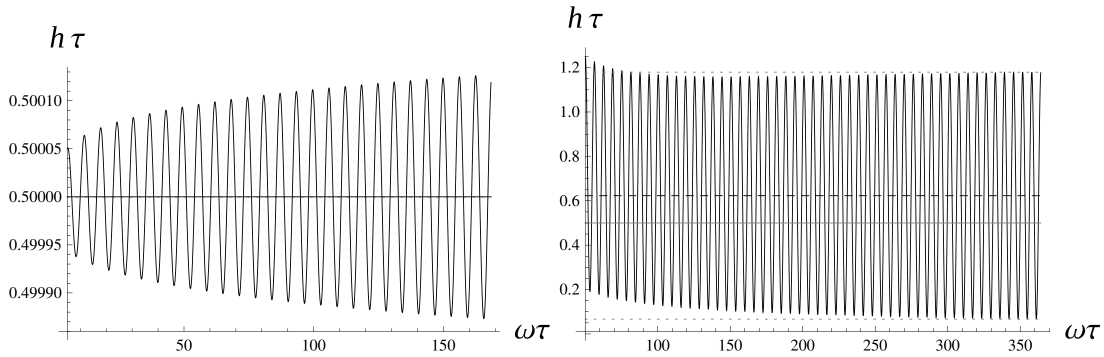

Figure 5: *Left panel* : small deviations from GR with $\delta h_0 = 10^{-4}$, $\omega = 10$. *Right panel*: non-linear regime with $\delta h_0 = 1.5$, $\omega = 100$. The central value of oscillations of Hubble parameter is about 0.6 and is shifted from the GR value 0.5.

influx into the cosmological plasma is estimated in the case of a massless, minimally-coupled to gravity scalar field with the action:

$$S_\phi = \frac{1}{2}\int d^4x\,\sqrt{-g}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi\,. \tag{35}$$

It terms of the conformally rescaled field, $\chi \equiv a(t)\phi$, and conformal time η, such that $a\,d\eta = dt$, we can rewrite the equations of motion as:

$$R'' + 2\frac{a'}{a}R' + m^2a^2R = 8\pi\frac{m^2}{m_{Pl}^2}\frac{1}{a^2}\left[\chi'^2 - (\vec{\nabla}\chi)^2 + \frac{a'^2}{a^2}\chi^2 - \frac{a'}{a}(\chi\chi' + \chi'\chi)\right], \tag{36}$$

$$R = -6a''/a^3\,, \tag{37}$$

$$\chi'' - \Delta\chi + \frac{1}{6}a^2R\,\chi = 0\,. \tag{38}$$

We derive a closed equation for R taking the average value of the χ-dependent quantum operators in the r.h.s. of eq. (36) over vacuum, in presence of an external classical gravitational field R following the procedure described in ref. [18], where such equation was obtained in one-loop approximation.

The dominant contribution of particle production is given by equation:

$$\ddot{R} + 3H\dot{R} + m^2R \simeq \frac{1}{12\pi}\frac{m^2}{m_{Pl}^2}\int_{t_0}^{t} dt'\,\frac{\ddot{R}(t')}{t-t'}\,. \tag{39}$$

This equation is linear in R and naturally non-local in time since the impact of particle production depends upon all the history of the evolution of the system.

Using again the procedure of truncated Fourier expansion including the back-reaction effects in the form of equation (39), we obtaine the decay rate:

$$\Gamma_R = \frac{m^3}{48m_{Pl}^2}\,. \tag{40}$$

This result is in agreement with ref. [17]. Correspondingly the oscillating part of R or H behaves as

$$\cos m_1t \to e^{-\Gamma_R t}\cos m_1t\,, \tag{41}$$

where m_1 is equal to m plus radiative corrections.

We use this result in the calculation of the energy density influx of the produced particles into the primeval plasma and find the rate of variation of the physical energy density of the produced χ-particles:

$$\dot{\varrho}_\chi = \frac{m \langle R^2 \rangle}{1152\pi} \,. \tag{42}$$

Here $\langle R^2 \rangle$ is the square of the amplitude of the oscillations of R .

The total rate of the gravitational energy transformation into elementary particles is obtained by multiplying the above result by the number of the produced particle species, N_{eff}:

$$\dot{\varrho}_{PP} = N_{eff} \dot{\varrho}_\chi \,. \tag{43}$$

The characteristic decay time of the oscillating curvature is

$$\tau_R = \frac{1}{2\Gamma_R} = \frac{24 m_{Pl}^2}{m^3} \simeq 2 \left(\frac{10^5 \text{ GeV}}{m} \right)^3 \text{ seconds} \,. \tag{44}$$

The contribution of the produced particles into the total cosmological energy density reaches its maximum value at approximately this time.

Depending upon the cosmological history and the values of the parameters of the theory, the role of non-thermal particles may vary from negligible up to very significant. It is worth noting that even initially small contribution of the oscillations of R into the total cosmological energy density could rise due to a weak decrease of the oscillation amplitude. Moreover, in R-dominated universe the Hubble parameter could be different from the GR one, $H = \alpha/(2t)$ with $\alpha > 1$ and hence the energy density of relativistic cosmological matter drops faster than $1/t^2$. This also amplifies possible non-thermal contribution into the cosmological energy density.

The influx of energetic protons and antiprotons produced by the oscillations of R could have an impact on BBN, if such protons were not thermalized at BBN era. Their effect would either allow to obtain some bounds on m or even to improve the agreement between the theoretical predictions for BBN and the measurements of primordial light nuclei abundances.

The oscillating curvature might also be a source of dark matter in the form of heavy supersymmetric (SUSY) particles. Since the expected light SUSY particles have not yet been discovered at LHC, to some people supersymmetry somewhat lost its attractiveness. The contribution of the stable lightest SUSY particle into the cosmological energy is proportional to

$$\Omega \sim m_{SUSY}^2 / m_{Pl} \tag{45}$$

and for m_{SUSY} in the range $100 - 1000$ GeV the cosmological fraction of these particles would be of order unity. It is exactly what is necessary for dark matter. However, it excludes thermally produced LSP's if they are much heavier. If LSP's came from the decay of R and their mass is larger than the "mass" of R, i.e. m, but not too much larger, the LSP production could be sufficiently suppressed to make a reasonable contribution to dark matter.

The developed in [16] method for calculations of particle production is applied for the case of $F(R) + R^2$ gravity in the modern astronomical systems which under certain conditions may be observable sources of cosmic rays [15].

Acknowledgements

This work was supported by the Grant of the Government of Russian Federation, No. 11.G34.31.0047.

References

[1] Nakamura K. *et al.* (Particle Data Group), Review of Particle Physics, J. Phys. G. **37** (2010) 075021 and references therein.

[2] Riess A.G. *et al.,* Astron. J. **116** (1998) 1009-1038; Perlmutter S. *et al.,* Nature. **391**(1998) 51; Schmidt B. P. *et al.,* Astrophys. J. **507** (1998) 46; Perlmutter S. *et al.,* Astrophys. J. **517** (1999) 565-586; Riess A.G. *et al.,* Astrophys. J. **607** (2004) 665-687.

[3] Peebles P.J.E., Ratra Bharat, Rev. Mod. Phys. **75** (2003) 559-606; Copeland E.J., Sami M. and Tsujikawa S., Int. J. Mod. Phys. D. **15** (2006) 1753-1936.

[4] Dolgov A.D., arXiv:1206.3725.

[5] Capozziello S., Carloni S., Troisi A., Recent Res. Dev. Astron. Astrophys. **1** (2003) 625.

[6] Carroll S.M., Duvvuri V., Trodden M., Turner M.S., Phys. Rev. D. **70** (2004) 043528.

[7] Dolgov A.D., Kawasaki M., Phys. Lett. B. **573** (2003) 1-4.

[8] Starobinsky A.A., JETP Lett. **86** (2007) 157-163; Hu W., Sawicki I., Phys. Rev. D. **76** (2007) 064004; Appleby A., Battye R., Phys. Lett. B. **654** (2007) 7-12.

[9] Nojiri S., Odintsov S., Phys. Rept. **505** (2011) 59-144.

[10] Appleby S.A., Battye R.A., Starobinsky A.A., JCAP **1006** (2010) 005.

[11] Appleby S.A., Battye R.A., JCAP **0805** (2008) 019.

[12] Frolov A.V., Phys. Rev. Lett. **101** (2008) 061103; Thongkool I., Sami M., Gannouji R., Jhingan S., Phys. Rev. D. **80** (2009) 043523; Thongkool I., Sami M., Choudhury S. Rai, Phys. Rev. D. **80** (2009) 127501.

[13] Arbuzova E.V., Dolgov A.D., Phys. Lett. B **700** (2011) 289-293.

[14] Gurovich V.Ts., Starobinsky A.A., Sov. Phys. JETP. **50** (1979) 844-851; Starobinsky A.A., JETP Lett. **30** (1979) 682-685.

[15] Arbuzova E.V., Dolgov A.D., Reverberi L., work in progress.

[16] Arbuzova E.V., Dolgov A.D., Reverberi L., JCAP **02** (2012) 049.

[17] Zeldovich Ya. B., Starobinsky A. A., JETP Lett. **26** (1977) 252-255; Vilenkin A., Phys. Rev. D **32** (1985) 2511.

[18] Dolgov A.D., Hansen S.H., Nucl.Phys. B **548** (1999) 408-426.