Some divergencies in brane world models

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Abstract

In this paper a model describing four-dimensional spinor electrodynamics localized on a domain wall is discussed. It is shown that in the effective four-dimensional theory there is a divergence in the amplitude and cross-section of the standard process of quantum electrodynamics – light by light scattering, which contradicts the experimental data and predictions of the Standard Model. This divergence is caused by the contributions of higher fermion modes and can not be removed by means of the standard renormalization procedure.

In [1] an attempt was made to construct a model with infinite extra dimension, describing spinor electrodynamics localized on a domain wall. It was found that due to the existence of non-localized fermion modes the renormalized amplitude of the standard process of quantum electrodynamics, – the light by light scattering, is divergent. This divergence can not be removed by means of the standard renormalization procedure and it simply reflects the fact that in the effective four-dimensional theory an infinite number of fermions with the same couplings to the massless vector field contribute to the amplitude. In this short note we present the main results of [1].

The five-dimensional action of the model presented in [1] has the following form:

$$S=\int\left[i\bar\Psi_1\Gamma^M\left(\partial_M-ieA_M\right)\Psi_1-h\Phi\bar\Psi_1\Psi_1+i\bar\Psi_2\Gamma^M\left(\partial_M-ieA_M\right)\Psi_2+h\Phi\bar\Psi_2\Psi_2-\right.$$
$$\left.-M\left(\bar\Psi_1\Psi_2+\bar\Psi_2\Psi_1\right)-\frac{1}{4}\xi^2\left(\Phi_v^2-\Phi^2\right)F^{MN}F_{MN}\right]d^4xdz, \qquad (1)$$

where Ψ_1 and Ψ_2 are different five-dimensional fermion fields, $h>0$, indices $M,N=0,..,4$, $\Gamma^\mu=\gamma^\mu$, $\Gamma^4=i\gamma^5$, $F_{MN}=\partial_M A_N-\partial_N A_M$, the parameter ξ is chosen so that the dimension of A_M is [mass]. We also suppose that the background solution is such that $A_M\equiv 0$, $\Phi(x,z)=\Phi_0(z)$ represents the domain wall

$$\Phi_0=\frac{m}{\sqrt{\lambda}}\tanh\left(\frac{m}{\sqrt{2}}z\right), \qquad (2)$$

$\Phi\to\pm\Phi_v$ at $z\to\pm\infty$ and $\Phi_v^2-\Phi^2>0$. From the very beginning we also choose $\frac{h}{\sqrt{\lambda}}=\frac{1}{\sqrt{2}}$ (in this case there is only one localized fermion mode, see [1]) and impose the gauge $A_4\equiv 0$.

First let us consider the vector field. The mechanism of localization of this field through the overall coupling to the scalar field forming a domain wall is based on the ideas discussed in [2, 3, 4]: an overall coupling of the gauge field to the scalar field to make the wave function of the gauge field normalizable [2] and taking this scalar field to be the one which forms the domain wall proper [4].

The equations of motion coming from the free part of the action of the gauge field (1) take the form

$$-\partial^\mu F_{\mu\nu}+A''_\nu-\sqrt{2}m\tanh\left(\frac{m}{\sqrt{2}}z\right)A'_\nu=0, \qquad \partial^\mu A'_\mu=0. \qquad (3)$$

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It is easy to see that there exists the lowest massless mode

$$A_\mu(x,z) = C a_\mu(x), \tag{4}$$

where C is a constant, satisfying Maxwell's equations

$$\partial^\mu f_{\mu\nu} = 0. \tag{5}$$

The effective action of the zero mode has the form

$$S^A_{eff} = -\int \frac{\xi^2 m^2 C^2}{\lambda \cosh^2\left(\frac{m}{\sqrt{2}}z\right)} dz \int \frac{1}{4} f^{\mu\nu} f_{\mu\nu} d^4x. \tag{6}$$

With the properly normalized wave function of this mode we get $A^0_\mu(x,z) = \frac{\sqrt{\lambda}}{2^{\frac{3}{4}}\xi\sqrt{m}} a_\mu(x)$ and

$$S^A_{eff} = -\int \frac{1}{4} f^{\mu\nu} f_{\mu\nu} d^4x. \tag{7}$$

Note that the constant wave function of the zero mode ensures the universality of charge, which is important for constructing realistic models, see detailed discussion of this topic in [5].

All other modes of the vector field form the continuous spectrum, which starts at $\mu = \frac{m}{\sqrt{2}}$. Thus, we get the lowest mode with the zero four-dimensional mass and the nonzero mass gap between this mode and the continuous spectrum. But these (non-localized) modes will be irrelevant for our analysis and we will drop them in the effective four-dimensional action.

The fermion sector of the model describes the generalization of the well-known Rubakov-Shaposhnikov mechanism [6] resulting in a non-zero mass of a lowest localized fermion. To our knowledge, for the first time this mechanism was proposed in [7]. Later it was discussed and utilized in different models [8, 9, 10, 11]. Thus, the fermion part of the action (1) corresponds to the model of [7], but in the absence of gravity, and coincides with the action of a simple model discussed in the beginning of [10].

Let us consider the equations of motion coming from the fermion part of the action (neglecting the vector field at the moment). They take the form

$$i\gamma^\mu \partial_\mu \Psi_1 - \gamma^5 \Psi_1' - h\Phi_0 \Psi_1 - M\Psi_2 = 0, \tag{8}$$

$$i\gamma^\mu \partial_\mu \Psi_2 - \gamma^5 \Psi_2' + h\Phi_0 \Psi_2 - M\Psi_1 = 0, \tag{9}$$

where $'$ denotes ∂_4. Let us take the following form of the solution to these equations:

$$\Psi_1(x,z) = f(z)\psi_L(x), \qquad \gamma^5\psi_L = \psi_L, \tag{10}$$

$$\Psi_2(x,z) = f(z)\psi_R(x), \qquad \gamma^5\psi_R = -\psi_R. \tag{11}$$

Substituting (10), (11) into (8), (9) we easily get:

$$\Psi_1(x,z) = C_1 e^{-h\int_0^z \Phi_0(z')dz'} \psi_L(x), \qquad i\gamma^\mu\partial_\mu\psi_L - M\psi_R = 0, \tag{12}$$

$$\Psi_2(x,z) = C_1 e^{-h\int_0^z \Phi_0(z')dz'} \psi_R(x), \qquad i\gamma^\mu\partial_\mu\psi_R - M\psi_L = 0, \tag{13}$$

where C_1 is a normalization constant. The four-dimensional equations in (12) and (13) can be combined into one equation by taking

$$\psi_L(x) + \psi_R(x) = \psi(x)$$

and we get the Dirac equation for a massive fermion

$$i\gamma^\mu\partial_\mu\psi - M\psi = 0. \tag{14}$$

The corresponding four-dimensional effective action for this mode can be easily obtained and has the form

$$S = C_1^2 \int e^{-2h\int_0^z \Phi_0(z')dz'} dz \int \left(i\bar{\psi}\gamma^\mu\partial_\mu\psi - M\bar{\psi}\psi\right) d^4x. \tag{15}$$

If C_1 is chosen in a proper way, we get a canonically-normalized four-dimensional effective action having the standard form

$$S_{eff} = \int \left(i\bar{\psi}\gamma^\mu\partial_\mu\psi - M\bar{\psi}\psi\right) d^4x. \tag{16}$$

Thus, we get the localized mode, which describes a massive four-dimensional fermion.

Of course, there are other fermion modes in the model under consideration. For the case $\frac{h}{\sqrt{\lambda}} = \frac{1}{\sqrt{2}}$ they also form the continuous spectrum, which starts at $\sqrt{\frac{m^2}{2} + M^2}$. Contrary to the Kaluza-Klein modes of the vector field, these modes will be important for our analysis. We will not focus here on the technical aspects of the mode decomposition procedure and on the derivation of the four-dimensional effective action, one can find detailed calculations in [1]. We just present the part of the effective four-dimensional action of the model, describing interactions of the massless vector field (the photon) with the lowest localized fermion and the fermion modes from the continuous spectrum. It has the form

$$S_{eff} = \int d^4x \Bigg[i\bar{\psi}\gamma^\nu\left(\partial_\nu - ie_4 a_\nu\right)\psi - M\bar{\psi}\psi - \frac{1}{4}f^{\rho\nu}f_{\rho\nu} + \tag{17}$$
$$+ \int\limits_{\sqrt{\frac{m^2}{2}+M^2}}^{\infty} d\mu \sum_{i=1}^{2}\sum_{j=1}^{2}\left(i\bar{\psi}_i^{(\mu,j)}\gamma^\nu\left(\partial_\nu - ie_4 a_\nu\right)\psi_i^{(\mu,j)} - \mu\bar{\psi}_i^{(\mu,j)}\psi_i^{(\mu,j)}\right)\Bigg]$$

where $e_4 = \frac{\sqrt{\lambda}}{2^{\frac{3}{4}}\xi\sqrt{m}}\, e$, $\psi = \psi_L + \psi_R$ and a_ν are the lowest modes of the fermion and vector boson fields (these fields can be identified with the electron and the photon respectively). One can see that each Kaluza-Klein level, except the lowest one, contains four four-dimensional fermions with the equal masses μ_n and with the same coupling to the zero mode of the gauge field. This degeneracy is of the same nature as the doubling in the UED models, see, for example, [12].

Now we are ready to consider the consequences of the model at hand. To simplify the analysis, let us pass from the continuous spectra to discrete spectra replacing $\int d\mu \to \sum\limits_n \Delta\mu$. We can perform the calculation of particular processes in such a discretized theory and then take the limit $\Delta\mu \to 0$. For simplicity, we also introduce a cut-off scale $\tilde{M}$ such that $\tilde{M}$ is the heaviest mode which can contribute to a process. Thus, we get

$$\int\limits_{\sqrt{\frac{m^2}{2}+M^2}}^{\tilde{M}} d\mu \;\to\; \sum_{n=1}^{N+1}\Delta\mu, \qquad \Delta\mu = \frac{\tilde{M} - \sqrt{\frac{m^2}{2}+M^2}}{N}, \tag{18}$$

$$\mu \;\to\; \mu_n = \sqrt{\frac{m^2}{2} + M^2} + (n-1)\Delta\mu, \tag{19}$$

$$\sqrt{\Delta\mu}\,\psi_i^{(\mu_n,j)} \;\to\; \psi_i^{n,j}. \tag{20}$$

The latter transformation is necessary to acquire the canonical normalization of four-dimensional fields. The discretized action, following from (17), takes the form

$$S_{eff} = \int d^4x \Bigg[i\bar{\psi}\gamma^\nu\left(\partial_\nu - ie_4 a_\nu\right)\psi - M\bar{\psi}\psi - \frac{1}{4}f^{\rho\nu}f_{\rho\nu} + \tag{21}$$
$$+ \sum_{n=1}^{N+1}\sum_{i=1}^{2}\sum_{j=1}^{2}\left(i\bar{\psi}_i^{n,j}\gamma^\nu\left(\partial_\nu - ie_4 a_\nu\right)\psi_i^{n,j} - \mu_n\bar{\psi}_i^{n,j}\psi_i^{n,j}\right)\Bigg].$$

We will be interested in $\gamma\gamma \to \gamma\gamma$ scattering, where γ stands for a photon. According to action (21), the corresponding amplitude in the lowest order in the coupling constant is schematically represented in Figure 1. We will consider the case when the energy of the initial

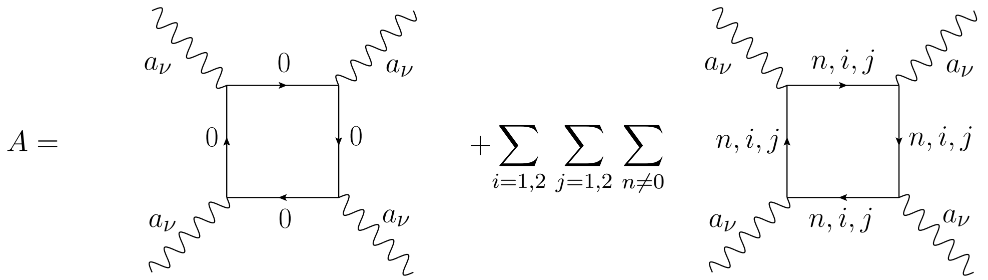

Figure 1: Schematic representation of the elastic $\gamma\gamma$ scattering amplitude. The number marking the fermion line corresponds to the number of the massive fermion mode which is represented by this line.

photon in the c.m. frame is much smaller than $\frac{m}{\sqrt{2}}$, in this case the creation of massive vector bosons is kinematically forbidden. For $\omega \ll \mu_0 = M$, where ω is the energy of the photon in the c.m. frame, the renormalized contribution of each mode to the amplitude in the leading order in $\frac{\omega}{\mu_n}$ is given by (see [13])

$$A_n = \frac{\omega^4}{\mu_n^4} F(\theta), \tag{22}$$

where the function $F(\theta)$ depends on the scattering angle θ and the polarizations of the photons, $n = 0$ corresponds to the zero localized mode with $\mu_0 = M$. The explicit form of $F(\theta)$ will be irrelevant for our analysis, but it can be found, for example, in [13]. Thus, we get

$$A = A_0 + 4\sum_{n=1}^{N+1} A_n = \left(\frac{\omega^4}{\mu_0^4} + 4\sum_{n=1}^{N+1}\frac{\omega^4}{\mu_n^4}\right) F(\theta). \tag{23}$$

It is easy to show that this amplitude diverges in the limit $\Delta\mu \to 0$. Indeed,

$$\begin{aligned} A &= \left(\frac{\omega^4}{\mu_0^4} + 4\sum_{n=1}^{N+1}\frac{\omega^4}{\mu_n^4}\right) F(\theta) > 4\sum_{n=1}^{N+1}\frac{\omega^4}{\mu_{N+1}^4}F(\theta) = 4(N+1)\frac{\omega^4}{\mu_{N+1}^4}F(\theta) = \\ &= 4\frac{\omega^4}{\tilde{M}^4}\left(1 + \frac{\tilde{M}-\mu_1}{\Delta\mu}\right) F(\theta) \underset{\Delta\mu\to 0}{\longrightarrow} \infty, \end{aligned} \tag{24}$$

leading to an infinite cross-section, even with the finite cut-off scale $\tilde{M}$. This infinity is not the same as the infinities, which arise in QED and which can be removed by applying the standard renormalization procedure, the same growth of the amplitude would appear if we begun to add extra "electrons" into the standard QED. Technically this happens because the coupling constant of each fermion to the photon remains finite in the limit $N \to \infty$. It is reasonable to suppose that analogous pathologies can arise in other models with infinite extra dimensions, where a gauge field is supposed to be localized on a brane, and one should take this effect into account when considering brane world models with infinite extra dimensions.

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