Localization of scalar fields on self-gravitating thick brane: branons vs. higgs-like bosons

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Abstract

The model of a domain wall ("thick brane") in noncompact five-dimensional space-time is considered with geometries of AdS type generated by self-interacting scalar matter composed of two fields in the presence of gravity. The mixing of scalar and gravitational degrees of freedom leads to nonperturbative corrections to the spectrum in the scalar sector. The possibility of localization of scalar modes on such "thick branes" is investigated.

1 Introduction

The models of the BSM physics based on the hypothesis that our universe is a four-dimensional space-time hypersurface (3-brane) embedded in a fundamental multi-dimensional space are quite popular, see, for example, [1] and references therein. The influence of gravity is especially interesting, which plays an important role in a (de) localization of matter fields on the brane [2] - [8], [9]. As regarding to gravity the question arises under what circumstances the localization of spin-zero matter fields on a brane is still possible when the minimal interaction with gravity is present? This work is partially devoted to answer this question.

In our talk we consider a model of the domain wall formation with finite thickness ("thick" branes) and gravity in five-dimensional noncompact space-time [11]. The formation of "thick" brane with the localization of light particles on it was obtained earlier in [10] with the help of two self-interacting scalar fields, when their vacuum configurations have nontrivial topology. The limit of turned off gravity happens to be smooth for background fields however the spectrum of the scalar fluctuations is changed by nonperturbative corrections due to mixing of the scalar and gravitational degrees of freedom. As it was previously shown [9], the existence of the centrifugal potential leads to absence of localized Goldstone mode related to spontaneous breaking of the translational symmetry. The question of phenomenological importance that arises is what influence this mixing has on the mass of the light scalar produced by fluctuations of the second scalar field which can be identified with Higgs-like boson observed at LHC.

2 Formulation of the model

Consider the five-dimensional space with a pseudo Riemann metric,

$$X^A = (x^\mu, z), \quad x^\mu = (x^0, x^1, x^2, x^3), \quad \eta^{AB} = \mathrm{diag}(+,-,-,-,-) \tag{1}$$

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It is assumed that the size of extra dimension z is large or infinite.

Let's supply this space with gravity providing it with a pseudo Riemann metric tensor g_{AB}, which is reduced to η_{AB} in flat space and for the rectangular coordinate system. We define the dynamics of two scalar fields $\Phi(X)$ and $H(X)$ with a minimal interaction to gravity by the following action functional,

$$S[g,\Phi,H] = \int d^5X\sqrt{|g|}\mathcal{L}(g,\Phi,H),\ \mathcal{L} = -\frac{1}{2}M_*^3R + \frac{1}{2}(\partial_A\Phi\partial^A\Phi + \partial_A H\partial^A H) - V(\Phi,H), \quad (2)$$

where R stands for a scalar curvature, $|g|$ is the determinant of the metric tensor, and M_* denotes a five-dimensional gravitational Planck scale.

The equations of motion are

$$R_{AB} - \frac{1}{2}g_{AB}R = \frac{1}{M_*^3}T_{AB}, \quad D^2\Phi = -\frac{\partial V}{\partial\Phi}, \quad D^2H = -\frac{\partial V}{\partial H}, \quad (3)$$

where D^2 is a covariant D'Alambertian, and the energy-momentum tensor reads,

$$T_{AB} = \partial_A\Phi\partial_B\Phi + \partial_A H\partial_B H - g_{AB}\left(\frac{1}{2}\partial_C\Phi\partial^C\Phi + \partial_C H\partial^C H - V(\Phi,H)\right). \quad (4)$$

In order to build a thick $3+1$-dimensional brane we study such classical vacuum configurations which do not violate spontaneously 4-dimensional Poincare invariance. It's convenient to present a metric in the conformally flat form, $g_{AB} = A^2(z)\,\eta_{AB}$.

For this metric the equations of motion read,

$$\left(\frac{A'}{A^2}\right)' = -\frac{\Phi'^2 + H'^2}{3M_*^3A}, \quad -2A^5V(\Phi,H) = 3M_*^3\Big(A^2A'' + 2A(A')^2\Big), \quad (5)$$

$$\left(A^3\Phi'\right)' = A^5\frac{\partial V}{\partial\Phi}, \quad \left(A^3H'\right)' = A^5\frac{\partial V}{\partial H}. \quad (6)$$

One can prove [11], that only three of these equations are independent. In the limit of zero gravity the equations on classical backgrounds smoothly reproduce the corresponding equations in the model without gravity.

3 Fluctuations around the background metric

The action (2) is invariant under diffeomorphisms. Infinitesimal diffeomorphisms correspond to the Lie derivative along an arbitrary vector field $\tilde{\zeta}^A(X)$, defining the coordinate transformation $X \to \tilde{X} = X + \tilde{\zeta}(X)$.

Let us introduce the fluctuations of the metric $h_{AB}(X)$ and of the scalar fields $\phi(X), h(X)$ around the background solutions, of the equations of motion,

$$g_{AB}(X) = A^2(z)(\eta_{AB} + h_{AB}(X)); \quad \Phi(X) = \Phi(z) + \phi(X), H(X) = H(z) + h(X) \quad (7)$$

Since 4D Poincare symmetry is not broken, we define the corresponding 4D part of the metric metric fluctuations as $h_{\mu\nu}$ and introduce the notation for gravivectors $h_{5\mu} \equiv v_\mu$ and graviscalars $h_{55} \equiv S$. Lets rescale the vector fluctuations $\tilde{\zeta}_\mu = A^2\zeta_\mu$ and the scalar ones $\tilde{\zeta}_5 = A\zeta_5$.

Now we expand the action to quadratic order in fluctuations. The full action after this procedure is a sum,

$$\mathcal{L}_{(2)} = \mathcal{L}_h + \mathcal{L}_\phi + \mathcal{L}_S + \mathcal{L}_V, \quad (8)$$

where

$$\begin{aligned}\sqrt{|g|}\mathcal{L}_h \quad \equiv \quad & -\frac{1}{2}M_*^3A^3\left\{-\frac{1}{4}\,h_{\alpha\beta,\nu}h^{\alpha\beta,\nu} - \frac{1}{2}\,h^{\alpha\beta}_{,\beta}h_{,\alpha} + \frac{1}{2}\,h^{\alpha\nu}_{,\alpha}h^{\beta}_{\nu,\beta} + \frac{1}{4}\,h_{,\alpha}h^{,\alpha}\right. \\ & \left.+\frac{1}{4}h'_{\mu\nu}h'^{\mu\nu} - \frac{1}{4}h'^2\right\}, \end{aligned} \quad (9)$$

$$\sqrt{|g|}\mathcal{L}_{\phi,\chi} \equiv \frac{1}{2}A^3(\phi_{,\mu}\phi^{,\mu} - \phi'^2 + \chi_{,\mu}\chi^{,\mu} - (\chi')^2) - \frac{1}{2}\, A^5\Big(\frac{\partial^2 V}{\partial\Phi^2}\phi^2 + 2\frac{\partial^2 V}{\partial\Phi\partial H}\phi\chi + \frac{\partial^2 V}{\partial H^2}\chi^2\Big) + \frac{1}{2}A^3 h'(\Phi'\phi + H'\chi), \tag{10}$$

$$\sqrt{|g|}\mathcal{L}_S \equiv \frac{1}{4}\Big(- A^5 V S^2 + S\Big(M_*^3 A^3\,(h^{\mu\nu}_{,\mu\nu} - h^{;\mu}_{,\mu}) + M_*^3\,(A^3)'\,h' + 2\big(A^3(\Phi'\phi + H'\chi)\big)' - 4A^3(\Phi'\phi' + H'\chi')\Big)\Big), \tag{11}$$

$$\sqrt{|g|}\mathcal{L}_V \equiv -\frac{1}{8}M_*^3 A^3 v_{\mu\nu}v^{\mu\nu} + \frac{1}{2}v^\mu\Big[- M_*^3 A^3\,(h^{;\nu}_{\mu\nu} - h_{,\mu})' + 2A^3(\Phi'\phi_{,\mu} + H'\chi_{,\mu}) + M_*^3\Big(A^3\Big)' S_{,\mu}\Big], \tag{12}$$

where $v_{\mu\nu} = v_{\mu,\nu} - v_{\nu,\mu}$, $h = h_{\mu\nu}\eta^{\mu\nu}$.

4 Separation of equations for physical degrees of freedom

To determine a physical sector we separate different spin components of the five-dimensional gravitational field. It can be accomplished by description of ten components of 4-dim metric in terms of the traceless-transverse tensor, vector and scalar components and expansions of vector fields v_μ into the transverse and longitudinal parts, [5, 12],

$$h_{\mu\nu} = b_{\mu\nu} + F_{\mu,\nu} + F_{\nu,\mu} + E_{,\mu\nu} + \eta_{\mu\nu}\psi, \qquad v_\mu = v^\perp_\mu + \partial_\mu\eta, \tag{13}$$

where $b_{\mu\nu}$, F_μ and $v^\perp_\mu$ obey the relation $b^{;\mu}_{\mu\nu} = b = 0 = F^{;\mu}_\mu = v^{;\mu}_\mu$. The gravitational fields $b_{\mu\nu}$ are gauge invariant and thereby describe graviton fields in the 4-dim space.

The decomposition (13) entails a partial separation of degrees of freedom in the lagrangian quadratic in fluctuations,

$$\begin{aligned}\sqrt{|g|}\mathcal{L}_{(2)} = {} & \frac{1}{8}M_*^3 A^3\,\Big\{\, b_{\mu\nu,\sigma}b^{\mu\nu,\sigma} - (b')_{\mu\nu}(b')^{\mu\nu} - f_{\mu\nu}f^{\mu\nu}\Big\} \\ & + \frac{3}{4}M_*^3 A^3\,\Big\{-\psi_{,\mu}\psi^{,\mu} + \psi_{,\mu}S^{,\mu} + 2(\psi')^2 + 4\frac{A'}{A}\psi' S\Big\} \\ & + \frac{1}{2}A^3\,\Big\{\phi_{,\mu}\phi^{,\mu} - (\phi')^2 + \chi_{,\mu}\chi^{,\mu} - (\chi')^2 - \, A^2\Big(\frac{\partial^2 V}{\partial\Phi^2}\phi^2 + 2\frac{\partial^2 V}{\partial\Phi\partial H}\phi\chi + \frac{\partial^2 V}{\partial H^2}\chi^2\Big) \\ & - \frac{1}{2}A^2 V(\Phi,H)S^2 + 4\psi'(\Phi'\phi + H'\chi) + S\Big(-\Phi'\phi' - H'\chi' + A^2\Big(\frac{\partial V}{\partial\Phi}\phi + \frac{\partial V}{\partial H}\chi\Big)\Big)\Big\} \\ & + \frac{3}{4}M_*^3 A^3\,\Box(E' - 2\eta)\Big(\frac{A'}{A}S + \psi' + \frac{2}{3M_*^3}(\Phi'\phi + H'\chi)\Big),\end{aligned} \tag{14}$$

where $f_\mu \equiv F'_\mu - v^\perp_\mu$, $\quad f_{\mu\nu} \equiv f_{\mu,\nu} - f_{\nu,\mu}$.

Obviously, in the quadratic approximation graviton, gravivector and graviscalar are decoupled from each other. It is convenient to perform the further analysis in gauge invariant variables. Let us perform the following rotation in (ϕ,χ) sector:

$$\begin{gathered}\phi = \check\phi\cos\theta + \check\chi\sin\theta, \quad \chi = -\check\phi\sin\theta + \check\chi\cos\theta \\ \cos\theta = \frac{\Phi'}{\mathcal{R}}, \quad \sin\theta = \frac{H'}{\mathcal{R}}, \quad \mathcal{R}^2 = (\Phi')^2 + (H')^2\end{gathered} \tag{15}$$

While $\check\chi$ is gauge invariant $\check\phi$ is not. We can exclude redundant gauge invariance introducing three gauge invariant variables:

$$\check\psi = \psi - \frac{2A'}{A\mathcal{R}}\check\phi, \quad \check S = S + \frac{2}{\mathcal{R}}\check\phi' - \frac{2A}{\mathcal{R}^2}\left(\frac{\mathcal{R}}{A}\right)'\check\phi, \quad \check\eta = E' - 2\eta - \frac{2}{\mathcal{R}}\check\phi. \tag{16}$$

Accordingly the scalar part of the lagrangian quadratic in fluctuations takes the form:

$$\begin{aligned}\sqrt{|g|}\mathcal{L}_{(2),scal} \;=\;& \frac{3}{4}M_*^3 A^3 \left\{-\check{\psi}_{,\mu}\check{\psi}^{,\mu} + \check{\psi}_{,\mu}\check{S}^{,\mu} + 2(\check{\psi}')^2 + 4\frac{A'}{A}\check{\psi}'\check{S}\right\} + \frac{1}{2}A^3 \left\{\check{\chi}_{,\mu}\check{\chi}^{,\mu} - (\check{\chi}')^2 - \right.\\ & \left. -\Big[(\theta')^2 + \frac{A^2}{\mathcal{R}^2}\Big(\frac{\partial^2 V}{\partial\Phi^2}(H')^2 - 2\frac{\partial^2 V}{\partial\Phi\partial H}\Phi' H' + \frac{\partial^2 V}{\partial H^2}(\Phi')^2\Big)\Big]\check{\chi}^2\right\} + A^3\mathcal{R}\theta'\check{S}\check{\chi} - \\ & -\frac{1}{4}A^5 V(\Phi, H)\check{S}^2 + \frac{3}{4}M_*^3 A^3\, \Box\check{\eta}\Big(\frac{A'}{A}\check{S} + \check{\psi}'\Big)\Big). \end{aligned} \tag{17}$$

where $\theta' = (\arctan\frac{H'}{\Phi'})' = (H''\Phi' - \Phi'' H')/\mathcal{R}^2$

From the last line it follows that the scalar field $\check{\eta}$ is a gauge invariant Lagrange multiplier and generates a gauge invariant constraint,

$$\frac{A'}{A}\check{S} + \check{\psi}' = 0. \tag{18}$$

Thus after taking this constraint into account only two independent scalar fields remain. To normalize kinetic terms the fields should be redefined $\hat{\chi} = A^{3/2}\check{\chi}$, $\hat{\psi} = \Omega\check{\psi}$, where $\Omega = A^{5/2}\mathcal{R}/2A'$.

$$\begin{aligned}\sqrt{|g|}\mathcal{L}_{(2),scal} =& \frac{1}{2}\left\{\partial_\mu\hat{\psi}\partial^\mu\hat{\psi} - (\partial_z\hat{\psi})^2 - \frac{\Omega''}{\Omega}\hat{\psi}^2\right\} - 2\theta'\hat{\chi}\left(\partial_z - \frac{\Omega'}{\Omega}\right)\hat{\psi} \\ &+\frac{1}{2}\left\{\partial_\mu\hat{\chi}\partial^\mu\hat{\chi} - (\partial_z\hat{\chi})^2 - \frac{(A^{3/2})''}{A^{3/2}} - \left((\theta')^2 + \frac{A^2}{\mathcal{R}^2}\begin{pmatrix}H'\\-\Phi'\end{pmatrix}^\dagger \partial^2 V\begin{pmatrix}H'\\-\Phi'\end{pmatrix}\right)\hat{\chi}^2\right\}\end{aligned} \tag{19}$$

To simplify analytical calculations let us represent the quadratic action for scalar fields in the gaussian normal coordinates x_μ, y,

$$ds^2 = A^2(z)\left(dx_\mu dx^\mu - dz^2\right) = \exp\left(-2\rho(y)\right)dx_\mu dx^\mu - dy^2. \tag{20}$$

Below the prime denotes differentiation with respect to y. To simplify the form of the action let us introduce $\widetilde{\mathcal{R}} = \exp(\rho)\mathcal{R}$ and in addition redefine the fields in order to normalize kinetic term, $\hat{\psi} = \exp(-\rho/2)\tilde{\psi}$,$\hat{\chi} = \exp(-\rho/2)\tilde{\chi}$.

$$\begin{aligned}S_{(2),scal} =& \int d^4x dy\left[\frac{1}{2}\partial_\mu\tilde{\psi}\partial^\mu\tilde{\psi} + \frac{1}{2}\partial_\mu\tilde{\chi}\partial^\mu\tilde{\chi} - 2\exp(-2\rho)\theta'\tilde{\chi}\left(\partial_y + \rho' + \frac{\rho''}{\rho'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}}\right)\tilde{\psi} - \right. \qquad (21)\\ & -\frac{1}{2}\exp(-2\rho)\tilde{\psi}\left\{\left(-\partial_y + \frac{\rho''}{\rho'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}}\right)\left(\partial_y + \frac{\rho''}{\rho'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}}\right) + 2\rho'\partial_y + 3(\rho')^2 + 3\rho'' - 4\rho'\frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}}\right\}\tilde{\psi} - \\ & \left. -\frac{1}{2}\exp(-2\rho)\tilde{\chi}\left\{-\partial_y^2 + (\theta')^2 + \frac{1}{\widetilde{\mathcal{R}}^2}\begin{pmatrix}\tilde{H}'\\-\tilde{\Phi}'\end{pmatrix}^\dagger \partial^2 V\begin{pmatrix}\tilde{H}'\\-\tilde{\Phi}'\end{pmatrix} + 2\rho'\partial_y + 3(\rho')^2 - \rho''\right\}\tilde{\chi}\right].\end{aligned}$$

where the second variation of the field potential reads,

$$\partial^2 V = \begin{pmatrix} -2M^2 + 6\tilde{\Phi}^2 + 2\tilde{H}^2 & 4\tilde{\Phi}\tilde{H} \\ 4\tilde{\Phi}\tilde{H} & -2\Delta_H + 2\tilde{\Phi}^2 + 6\tilde{H}^2 \end{pmatrix}. \tag{22}$$

Let's perform the mass spectrum expansion,

$$\begin{aligned}&\tilde{\psi}(X) = \exp(\rho)\sum_m \Psi^{(m)}(x)\psi_m(y), \quad \tilde{\chi}(X) = \exp(\rho)\sum_m \Psi^{(m)}(x)\chi_m(y), \\ &\partial_\mu\partial^\mu\Psi^{(m)} = -m^2\Psi^{(m)},\end{aligned} \tag{23}$$

where the factor $\exp(\rho)$ is introduced to eliminate first derivatives in the equations. We obtain the following equations,

$$\left(-\partial_y + \frac{\rho''}{\rho'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}} + 2\rho'\right)\left(\partial_y + \frac{\rho''}{\rho'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}} + 2\rho'\right)\psi_m -$$

$$-2\theta'\left(\partial_y - \frac{\rho''}{\rho'} + \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}} - 2\rho' + \frac{\theta''}{\theta'}\right)\chi_m = \exp(2\rho)m^2\psi_m, \tag{24}$$

$$\left(-\partial_y^2 + (\theta')^2 + \frac{1}{\widetilde{\mathcal{R}^2}}\begin{pmatrix}\tilde{H}'\\ -\tilde{\Phi}'\end{pmatrix}^{\dagger}\partial^2 V\begin{pmatrix}\tilde{H}'\\ -\tilde{\Phi}'\end{pmatrix} + 4(\rho')^2 - 2\rho''\right)\chi_m +$$

$$+2\theta'\left(\partial_y + \frac{\rho''}{\rho'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}} + 2\rho'\right)\psi_m = \exp(2\rho)m^2\chi_m. \tag{25}$$

These equations can be treated as equations on a zero-mode considering the mass term as part of a potential. Mass term contribution is essentially negative for all $m^2 > 0$. Then as the exponent $\rho(y)$ is positive and growing at very large y it becomes evident that this term in the potential makes it unbounded below. Thus any eigenfunction of the spectral problem (25) is at best a resonance state though it could be quasilocalized in a finite volume around a local minimum of the potential. In [10] the probability for quantum tunneling of quasilocalized light resonances with masses $m \ll M$ was estimated as $\sim \exp\{-\frac{3}{\kappa}\ln\frac{2M}{m}\}$ which for phenomenologically acceptable values of $\kappa \sim 10^{-15}$ and $M/m \gtrsim 30$ means an enormous suppression. Moreover in the perturbation theory the decay does not occur as the turning point to an unbounded potential energy is situated at $y \sim 1/\kappa$. Therefore one can calculate the localization of resonances following the perturbation schemes.

In the limit $\kappa \longrightarrow 0$ we obtain,

$$\left(-\partial_y + \frac{\rho_1''}{\rho_1'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}}\right)\left(\partial_y + \frac{\rho_1''}{\rho_1'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}}\right)\psi_m - 2\theta'\left(\partial_y - \frac{\rho_1''}{\rho_1'} + \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}} + \frac{\theta''}{\theta'}\right)\chi_m = m^2\psi_m, \tag{26}$$

$$\left(-\partial_y^2 + (\theta')^2 + \frac{1}{\widetilde{\mathcal{R}^2}}\begin{pmatrix}\tilde{H}'\\ -\tilde{\Phi}'\end{pmatrix}^{\dagger}\partial^2 V\begin{pmatrix}\tilde{H}'\\ -\tilde{\Phi}'\end{pmatrix}\right)\chi_m + 2\theta'\left(\partial_y + \frac{\rho_1''}{\rho_1'} - \frac{\widetilde{\mathcal{R}}'}{\widetilde{\mathcal{R}}}\right)\psi_m = m^2\chi_m \tag{27}$$

where ρ_1 is first order of κ.

While equation on χ mostly reproduces the same equation in the model without gravity the equation on ψ changed dramatically due to the mixing of scalar and gravitational degrees of freedom. Note that for smooth symmetric backgrounds in the $\rho' \sim const\cdot y$. This generally leads to the centrifugal potential $\sim 2/y^2$ in the equation on ψ and absence of a branon zero-mode [9].

5 Model with quartic potential

Now let's study the formation of a brane in the theory with a minimal stable potential admitting kink solutions. It possesses a quartic scalar self-interaction and wrong-sign mass terms for both scalar fields. This potential is designed with $U_\tau(1)$-symmetry of dim-4 vertices but with different quadratic couplings. The conveniently normalized effective action has the form,

$$S_{eff}(\tilde{\Phi}, g) = \frac{1}{2}M_*^3\int d^5X\sqrt{|g|}\Big\{-R + 2\lambda + \frac{3\kappa}{M^2}\Big(\partial_A\tilde{\Phi}\partial^A\tilde{\Phi} + \partial_A\tilde{H}\partial^A\tilde{H}$$

$$+ 2M^2\tilde{\Phi}^2 + 2\Delta_H\tilde{H}^2 - (\tilde{\Phi}^2 + \tilde{H}^2)^2 - \tilde{V}_0\Big)\Big\}, \tag{28}$$

where the normalization of the kinetic term of scalar fields is chosen in order to simplify the Eqs. of motion (see below)[1]. For relating it to the weak gravity limit we guess that $\kappa \sim M^3/M_*^3$ is a small parameter, which characterizes the interaction of gravity and matter fields. Let us take $M^2 > \Delta_H$ then the true minima are achieved at $\tilde{\Phi}_{min} = \pm M$, $\tilde{H}_{min} = 0$ and a constant shift of the potential energy must be set $V_0 = M^4$ in order to determine properly the cosmological constant λ.

Now we change the coordinate frame to the warped metric in gaussian normal coordinates, this choice happens to be more tractable for analytic calculations than the conformal one used for (6),

$$\tilde{\Phi}'' = -2M^2\tilde{\Phi} + 4\rho'\tilde{\Phi}' + 2\tilde{\Phi}(\tilde{\Phi}^2 + \tilde{H}^2), \tag{29}$$

$$\tilde{H}'' = -2\Delta_H\tilde{H} + 4\rho'\tilde{H}' + 2\tilde{H}(\tilde{\Phi}^2 + \tilde{H}^2), \tag{30}$$

$$\rho'' = \frac{\kappa}{M^2}\,(\tilde{\Phi}'^2 + \tilde{H}'^2), \tag{31}$$

$$\lambda = -6\rho'^2 + \frac{3\kappa}{2M^2}\left\{(\tilde{\Phi})'^2 + (\tilde{H}')^2 + 2M^2\tilde{\Phi}^2 + 2\Delta_H\tilde{H}^2 - (\tilde{\Phi}^2 + \tilde{H}^2)^2 - M^4\right\}. \tag{32}$$

The above equations contain terms which have different orders in small parameter κ, and accordingly they can be solved by perturbation theory assuming that,

$$\frac{|\rho'(y)|}{M} = O(\kappa) = \frac{|\rho''(y)|}{M^2}.$$

Then in the leading order in κ the equations for the fields $\tilde{\Phi}\,(y)\,, \tilde{H}\,(y)$ do not contain the metric factor, and the metric is completely governed by matter order by order in κ.

Depending on the relation between quadratic couplings M^2 and Δ_H there are the two types of z-inhomogeneous solutions of the equations (32) which have the form of a two-component kink [11]. *For gravity switched off* the first one holds for $\Delta_H \leq M^2/2$,

$$\tilde{\Phi} \to \Phi_0 = \pm M\tanh\left(My\right) + O\left(\kappa\right), \quad \tilde{H}(y) = 0, \tag{33}$$

and therefore the conformal factor to the leading order in κ reads,

$$\rho_1\left(y\right) = \frac{2\kappa}{3}\left\{\ln\cosh\left(My\right) + \frac{1}{4}\tanh^2(My)\right\} + O\left(\kappa^2\right), \tag{34}$$

Using perturbation theory in κ one can obtain next order correction to the background solution

$$\Phi = \tilde{\Phi}(y) = M\tanh\beta My\left(1 - \kappa\frac{2}{9\cosh^2\beta My}\right) + \mathcal{O}(\kappa^2); \quad \beta = 1 - \frac{2}{3}\kappa \tag{35}$$

The second phase arises only when $M^2/2 \leq \Delta_H \leq M^2$, i.e. $2\Delta_H = M^2 + \mu^2,\ \mu^2 < M^2$,

$$\Phi_0(y) = \pm M\tanh\left(\beta My\right), \quad H_0(y) = \pm\frac{\mu}{\cosh\left(\beta My\right)}, \quad \beta = \sqrt{1 - \frac{\mu^2}{M^2}}, \tag{36}$$

wherefrom one can find the conformal factor to the leading order in κ in the following form,

$$\rho_1\left(y\right) = \frac{\kappa}{3}\left\{\left(3 - \beta^2\right)\ln\cosh\left(\beta My\right) + \frac{1}{2}\beta^2\tanh^2(\beta My)\right\} + O\left(\kappa^2\right), \tag{37}$$

[1]It could be inherited from the low-energy effective action of composite scalar fields induced by the one-loop dynamics of five-dimensional pre-fermions [10].

To study this phase one can use perturbation theory both in κ and $\frac{\mu}{M}$,

$$\tilde{H}(\tau) = M \sum_{n,m=0}^{\infty} \kappa^n \left(\frac{\mu}{M}\right)^{2m+1} H_{n,m}(\tau); \quad \tilde{\Phi}(\tau) = M \sum_{n,m=0}^{\infty} \kappa^n \left(\frac{\mu}{M}\right)^{2m} \Phi_{n,m}(\tau);,$$

$$\rho(\tau) = \kappa \sum_{n,m=0}^{\infty} \kappa^n \left(\frac{\mu}{M}\right)^{2m} \rho_{n+1,m}(\tau); \quad \Delta_H = \frac{1}{2} M^2 \sum_{n=0}^{\infty} \kappa^n \Delta_{H,c}^n + \frac{1}{2}\mu^2, \tag{38}$$

We present here first corrections,

$$H_{1,0} = \frac{2}{27\cosh\tau}\left(C_{1,0}^H - 2\log\cosh\tau + 3\tanh^2\tau\right), \quad \Delta_{H,c}^1 = -\frac{44}{27} \tag{39}$$

6 Spectrum of the light scalar fluctuations in the model with quartic interaction

When $H(y) = 0$ the two scalar sectors decouple because $\theta = 0$. The equation on ψ,

$$\left(-\partial_y + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho'\right)\left(\partial_y + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho'\right)\psi = \exp(2\rho) m^2 \psi, \tag{40}$$

can have zero mass solution in the form,

$$\psi_0 = \frac{\Phi'}{\rho'} e^{-2\rho} = \frac{1}{\cosh^2 My} \frac{3}{3\tanh My - \tanh^3 My} + O(\kappa) \tag{41}$$

However this solution happens to be singular. It is caused by existence of the centrifugal barrier in the vicinity of the brane $\sim 2/y^2$ [9]. This differ dramatically from the model without gravity [11] where the massless particle in this channel (branon) corresponds to Goldstone mode related to spontaneous breaking of the translational symmetry. This happens because the corresponding brane fluctuation represents, in fact, a gauge transformation and does not appear in the invariant part of the spectrum. One could say that in the presence of gravity induced by a brane the latter becomes more rigid as only massive fluctuations are possible around it. Of course, the gauge transformation considered leaves invariant only the quadratic action and thereby a track of Goldstone mode may have influence on higher order vertices of interaction between gravity and scalar fields.

There can also be localized states with masses of order M. However they happen to be unstable resonances as it will be evident from the spectral problem formulated in gaussian normal coordinates.

The equation on χ takes the form,

$$\left[-\partial_\tau^2 + \frac{1}{\beta^2 M^2} e^{-2\rho}\left(-2\Delta_H + 2\Phi^2\right) + 4(\rho')^2 - 2\rho''\right]\chi_m = \frac{m^2}{M^2\beta^2} e^{2\rho}\chi_m, \tag{42}$$

where the variable $\tau = \beta M y$ is employed and the derivative is defined against it. The limit of turned off gravity is smooth and the differential operator on the left-hand side of (42) can be factorized,

$$\left[\frac{M^2 - 2\Delta_H}{M^2} + (-\partial_\tau + \tanh\tau)(\partial_\tau + \tanh\tau)\right]\chi_{m,0} = \frac{(m^2)_0}{M^2}\chi_{m,0}, \tag{43}$$

which corresponds to $\Delta_H = \Delta_{H,c} = M^2/2$ for zero scalar mass (phase transition point). It can be shown that it remain massless in the next order of κ. In general, for $M^2 - 2\Delta_H > 0$ one finds one localized state with positive m^2,

$$\chi = \frac{1}{\cosh\tau} + O(\kappa), \quad m^2 = M^2 - 2\Delta_H + O(\kappa), \tag{44}$$

When $\Delta_H > \Delta_{H,c}$ the squared mass becomes negative signalling the instability of the phase with zero H. In the phase with nonzero H mixing terms are nonzero and one has to study spectrum by perturbation theory near critical point. The calculations not presented because of their high complexity show that the leading order of mass for light scalar state is the same as in the model [11] without gravity, namely, $m^2 = 2\mu^2 + O(\mu^4/M^2)$.

7 Conclusions

In this work we have considered a model of domain wall ("thick brane") in the noncompact five-dimensional space-time generated by self-interacting fermions in the presence of gravity. In the model with quartic potential there are two classes of background solutions corresponding to two phases. The phase of phenomenological interest is the phase with nonzero H which can be used for generation of fermion masses [11]. While the massless Goldstone is not present in the physical sector the light scalar particle in this phase which can be associated with observed Higgs-like boson has the same leading order of mass as in the model without gravity.

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