Long-distance contributions to $B \to K\ell^+\ell^-$

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Abstract

The results of computation of long-distance contributions due to charm quark loops to the process $B \to K\mu^+\mu^-$ are presented.

1 Introduction

Presently much attention is drawn to the precision comparison of the SM predictions with the data obtained by experiments at the LHC. The ultimate goal of these studies would be to find deviations from the SM and thereby some hints for New Physics. There are numerous scenarios for physics beyond SM. In general, it seems that the gauge sector of the SM is rather robust and aesthetically attractive since one has just fermionic quanta while the interaction is of geometrical origin and is dictated by the gauge principle. If some new modes are discovered then their interaction will still be based most probably on the gauge invariance principle. The primary task of LHC experiments is to study the nature of mass generation that is realized in the SM through Higgs mechanism. Introduction of this mechanism is dictated mainly by the requirement of renormalizability of the theory which is a strict principle from old fifties. The presence of the fundamental Higgs mode allows for ad hoc Yukawa couplings to fermions. And this part of SM seems to be the most interesting in present and currently under thorough study at LHC with the special purpose detector for flavor physics LHCb. The flavor sector of SM is the most mysterious part of the model as there are no guidelines in its construction but rather some parametrization of data with the requirement of minimality of every sort. However this parametrization is quite efficient and successful. The existence of three generations allows for explanation of CP violation within CKM scheme. Nevertheless the flavor sector is certainly a place to look for physics beyond SM (New Physics).

The experiment LHCb is designed to investigate flavor physics in decays of beautiful mesons. The number of events already registered is such that some traditional "rare" B-decay modes are not so rare anymore. The experimental material is impressive and allows for a precision study of the flavor sector and, in particular, the process $B \to K\ell^+\ell^-$ [1]. However, there is a problem on the theory side to claim that the results are consistent or not with the SM since for precision check of the theory one has to account for QCD effects. Indeed, the SM is formulated in terms of quark-gluon degrees of freedom $\{q, g\}$ while the experiment in performed in terms of hadrons $B, D, K, \pi...$ and the comparison between two pictures requires big effort when one is talking about real precision. For instance, in the case of $B \to K\ell^+\ell^-$ processes the underlying flavor-changing transition is $b \to s\ell^+\ell^-$ and is sensitive to new physics as loops of virtual new particles can contribute to the rates. The main obstacle in the analysis is the absence of the interface between quarks and hadrons that requires the use of QCD in strong coupling regime which is still beyond the present theory capabilities and not treatable at the moment. The solution of the future is probably the numerical simulation on the lattice. Nevertheless the theory analysis is developed especially its parts that are treatable within present techniques.

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2 Effective theory - separation of scales

It seems like factorization is a key word in theoretical analysis of SM at present. In the case of flavor changing transitions it means a scale separation through the OPE and the use of effective theories for describing the low energy dynamics. Indeed, the electroweak scale of the SM is the vacuum expectation value of the Higgs field $v = 250$ GeV or, in practice, masses of gauge bosons M_W, M_Z and that of the top-quark m_t as it happens to be of the same order of magnitude. Since for $\Delta B = 1$ processes the scale is set by m_b with $\mu \sim E \sim m_b \ll v$ one can construct an expansion in m_b/v. This procedure is known as ("integrating out heavy particles" and is well understood in the framework of effective theories and OPE. The pQCQ works and allows for control of the large terms $\alpha_s^n \ln^k(M_W/m_b)$ within renormalization group summation. The result gives an effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

with G_F being the Fermi constant, V_{tb} - elements of CKM matrix. The coefficients $C_i(\alpha_s, \mu)$ are short-distance pQCD quantities and $O_i(\mu)$ are composite local operators. This factorization is of pure PT origin and under control through PT series in $\alpha_s(\mu)$ for the coefficient functions $C_i(\alpha_s(\mu))$. The numerical value of $\alpha_s(\mu)$ is well known (see, e.g. [2, 3]). Thus at the leading order in G_F the SM amplitudes reduce to the leading order expression in an effective theory with the Hamiltonian H_{eff}

$$A(B \to K^{(*)}\ell^+\ell^-) = -\langle K^{(*)}\ell^+\ell^- \mid H_{eff} \mid B\rangle + \mathcal{O}\left(\frac{m_b^2}{M_W^2, m_t^2}\right)$$

Theoretical analysis the relevant processes reduces to computation of hadronic matrix elements of the local operators O_i. There are tree-level operators with charm fields

$$O_1 = (\bar{s}_L \gamma_\rho c_L)(\bar{c}_L \gamma^\rho b_L), \quad O_2 = \left(\bar{s}_L^j \gamma_\rho c_L^i\right)\left(\bar{c}_L^i \gamma^\rho b_L^j\right)$$

but also new ones that are sensitive to new physics as their their coefficient generated through the loops

$$O_9 = \frac{\alpha_{em}}{4\pi}(\bar{s}_L \gamma_\rho b_L)(\bar{l}\gamma^\rho l), \quad O_{10} = \frac{\alpha_{em}}{4\pi}(\bar{s}_L \gamma_\rho b_L)(\bar{l}\gamma^\rho \gamma_5 l)$$

and

$$O_{7\gamma} = -\frac{e}{16\pi^2} \bar{s}\sigma_{\mu\nu}(m_s L + m_b R) b F^{\mu\nu}$$

"V-A" projectors are $L = (1-\gamma_5)/2$, $R = 1 - L$. The dominant contributions are due to operators $O_{9,10}$ and $O_{7\gamma}$. Contributions of loop operators to $B \to K(K^*)\ell^+\ell^-$ are local and reduce to form factors $f_{BK}^+(q^2)$, $f_{BK}^T(q^2)$, ...

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{\pi} V_{tb} V_{ts}^* p^\mu \times$$

$$\left[\bar{\ell}\gamma_\mu \ell \left(C_9 f_{BK}^+(q^2) + \frac{2(m_b + m_s)}{m_B + m_K} C_7^{eff} f_{BK}^T(q^2)\right) + \bar{\ell}\gamma_\mu \gamma_5 \ell C_{10} f_{BK}^+(q^2)\right]$$

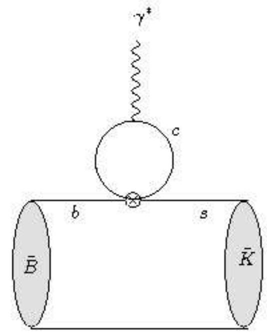
For example, the tensor $B \to K$ form factor is defined as

$$\langle K(p)|\bar{s}\sigma_{\mu\rho} q^\rho b|B(p+q)\rangle = \left[q^2(2p_\mu + q_\mu) - (m_B^2 - m_K^2) q_\mu\right] \frac{i f_{BK}^T(q^2)}{m_B + m_K}.$$

Computation of the form factors in QCD requires nonPT methods. However higher precision calls for accounting for even more complicated matrix elements.

3 Charm loops contribution

The four-quark charm operators O_1 and O_2 lead to charm-loop interactions of the form



as the top quark has but they cannot be integrated out as the scale m_c is small. The amplitude is not local anymore. Indeed, the contribution of tree-level operators to the amplitude $A(B \to K\ell^+\ell^-)$ reads

$$A^{O_{1,2}} = -(4\pi\alpha_{em}Q_c)\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{\bar{\ell}\gamma^\mu\ell}{q^2}\mathcal{H}_\mu^{(B\to K)}(p,q)$$

with $Q_c = 2/3$ and

$$\mathcal{H}_\mu^{(B\to K)}(p,q) = i\int d^4x e^{iqx} \times \langle K(p)|T\bar{c}\gamma_\mu c(x)\Big[C_1O_1(0) + C_2O_2(0)\Big]|B(p+q)\rangle$$

The key quantity of the analysis is the T-product

$$T_\mu = TO_1(0)J_\mu(x) = T\bar{c}\gamma_\mu c(x)O_i(0)$$

which leads to a nonlocal amplitude that can be expanded on the light-cone in the form

$$TO_1(0)J_\mu(x) = \bar{s}\Gamma b \otimes C(x) + \bar{s}Gb \otimes C_G(x) + ...$$

At the leading order in α_s and x, the coefficient C is given by the two-point correlator

$$C \to i\int d^4x e^{iq\cdot x}\bar{c}\gamma_\mu c(x)J_\mu(0).$$

It can be reliably computed in QCD at $q^2 \ll 4m_c^2$ that leads to the factorization approximation.

4 Factorization of charm loops

At the leading order the amplitude

$$\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)|_{fact} = \left(\frac{C_1}{3} + C_2\right)\langle K^{(*)}(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle$$

where both O_1 and O_2 contribute and the new effective operator

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho})\frac{9}{32\pi^2}\, g(m_c^2,q^2)\bar{s}_L\gamma^\rho b_L$$

reduces to the $b \to s$ local current can be expressed through $B \to K^{(*)}$ form factors. The charm-loop coefficient function is given by the well-known expression

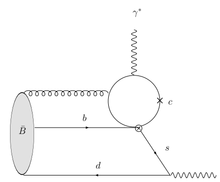
$$g(m_c^2,q^2) = -\frac{8}{9}\ln\left(\frac{m_c}{m_b}\right) + \frac{8}{27} + \frac{4}{9}y - \frac{4}{9}(2+y)\sqrt{y-1}\,\arctan\left(\frac{1}{\sqrt{y-1}}\right)$$

where $y = 4m_c^2/q^2 > 1$ and $\mu = m_b$. It is easier to represent this function through its dispersion relation in the variable q^2 with the spectrum

$$\frac{1}{\pi}\,\mathrm{Im}_s g(m_c^2, s) = \frac{4}{9}\sqrt{1 - \frac{4m_c^2}{s}}(1 + \frac{2m_c^2}{s})\Theta(s - 4m_c^2)$$

The factorizable amplitude is often called "perturbative" or "short-distance" charm-loop effect. There are both PT and nonPT corrections to factorization. For instance, even NLO PT corrections violate factorization.

In this talk I discuss the nonPT soft gluon corrections to factorization approximation [4] that are represented by the graph



where the $B \to K^{(*)}$ matrix element contains a soft-gluon emission from the charm loop. The c-quark loop with the emitted gluon generates the nonlocal effective operator $\widetilde{O}_\mu$.

5 OPE on the light-cone and sum rules

One casts the soft-gluon emission part to a form

$$\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)|_{nonfact} = 2C_1\langle K^{(*)}(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle$$

where $\widetilde{\mathcal{O}}_\mu(q)$ is a convolution of the coefficient function with the nonlocal operator

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L$$

In fact this matrix element resembles a nonforward parton distribution with different initial and final hadrons. The coefficient function is given by its spectral density

$$\frac{1}{\pi}\mathrm{Im}\, I_{\mu\rho\alpha\beta}(q,\omega) = \frac{m_c^2\Theta(\tilde{q}^2 - 4m_c^2)}{4\pi^2\tilde{q}^2\sqrt{\tilde{q}^2(\tilde{q}^2 - 4m_c^2)}}\times$$

$$\int_0^1 du\Big\{\bar{u}\tilde{q}_\mu\tilde{q}_\alpha g_{\rho\beta} + u\tilde{q}_\rho\tilde{q}_\alpha g_{\mu\beta} - \Big[u + \frac{(\bar{u}-u)\tilde{q}^2}{4m_c^2}\Big]\tilde{q}^2 g_{\mu\alpha}g_{\rho\beta}\Big\}$$

with $\tilde{q} = q - u\omega n_-$, so that $\tilde{q}^2 \simeq q^2 - 2u\omega m_b$. Now the amplitude reads

$$\mathcal{H}_\mu^{(B\to K)}(p,q) = \left(\frac{C_1}{3} + C_2\right)\langle K(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle$$

$$+2C_1\langle K(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle = \big[(p\cdot q)q_\mu - q^2p_\mu\big]\mathcal{H}^{(B\to K)}(q^2)$$

and the scalar part is

$$\mathcal{H}^{(B\to K)}(q^2) = \left(\frac{C_1}{3} + C_2\right)A(q^2) + 2C_1\tilde{A}(q^2)$$

with $\tilde{A}$ being a soft-gluon amplitude. Note that $C_1 = 1.12$, $C_2 = -0.27$ that enhances $\tilde{A}$. To compute $\tilde{A}$ determining the soft-gluon emission we employ Light-Cone SR with the B-meson Distribution Amplitudes (DA).

Consider a correlation function

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i\int d^4y e^{ip\cdot y}\langle 0|T\{j^K_\nu(y)\widetilde{\mathcal{O}}_\mu(q)\}|B(p+q)\rangle\,,$$

with $j^K_\nu = \bar{d}\gamma_\nu\gamma_5 s$ and extract a residue

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{if_K p_\nu}{m_K^2 - p^2}[(p\cdot q)q_\mu - q^2 p_\mu]\tilde{A}(q^2) + cont$$

where f_K is the kaon decay constant and *cont* accumulates higher mass states with the kaon quantum numbers located above the threshold s_h.

A general note about Sum Rules vs lattice:

- model-independent, first-principle method. QCD sum rules rely on asymptotic expansions of Green's functions – OPE at large momenta – while on the lattice the entire function can be found numerically
- QCD Sum Rules techniques provide a consistent way of treating PT structure of matrix elements (scale dependence) which is needed to retain RG invariance of physical observables

We use HQET for describing B-meson with the DA decomposed as

$$\langle 0|\bar{d}_\alpha(y)\delta[\omega - \frac{(in_+\mathcal{D})}{2}]G_{\sigma\tau}(0)b_\beta(0)|\bar{B}(v)\rangle = \frac{f_B m_B}{2}\int d\lambda e^{-i\lambda y v}$$

$$\left[(1+\not{v})\left\{(v_\sigma\gamma_\tau - v_\tau\gamma_\sigma)\left[\Psi_A - \Psi_V\right] - i\sigma_{\sigma\tau}\Psi_V - \frac{y_\sigma v_\tau - y_\tau v_\sigma}{v\cdot y}X_A + \frac{y_\sigma\gamma_\tau - y_\tau\gamma_\sigma}{v\cdot y}Y_A\right\}\gamma_5\right]_{\beta\alpha}$$

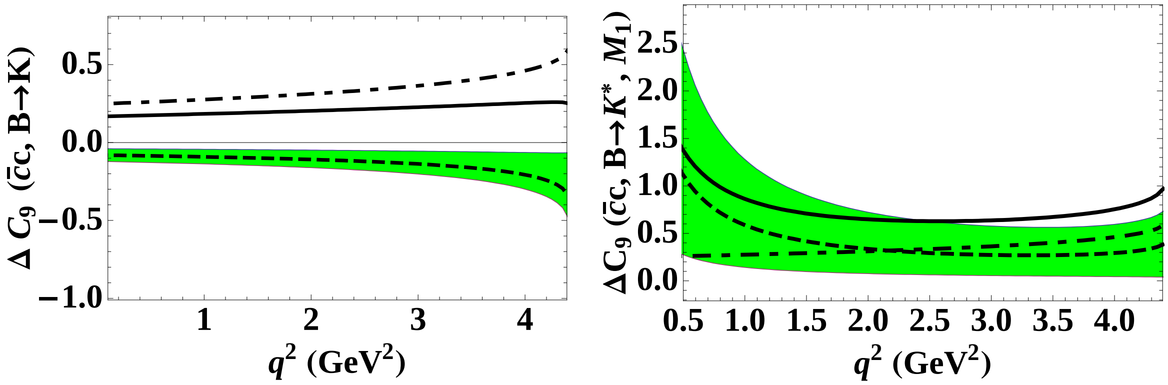
through quantities Ψ_A, Ψ_V, X_A, Y_A and where where f_B and m_B are the B-meson decay constant and the mass. We use the model DA for the B-meson of the form

$$\Psi_A(\lambda,\omega) = \Psi_V(\lambda,\omega) = \frac{\lambda_E^2}{6\omega_0^4}\omega^2 e^{-(\lambda+\omega)/\omega_0},\quad X_A(\lambda,\omega) = \frac{\lambda_E^2}{6\omega_0^4}\omega(2\lambda-\omega)e^{-(\lambda+\omega)/\omega_0}$$

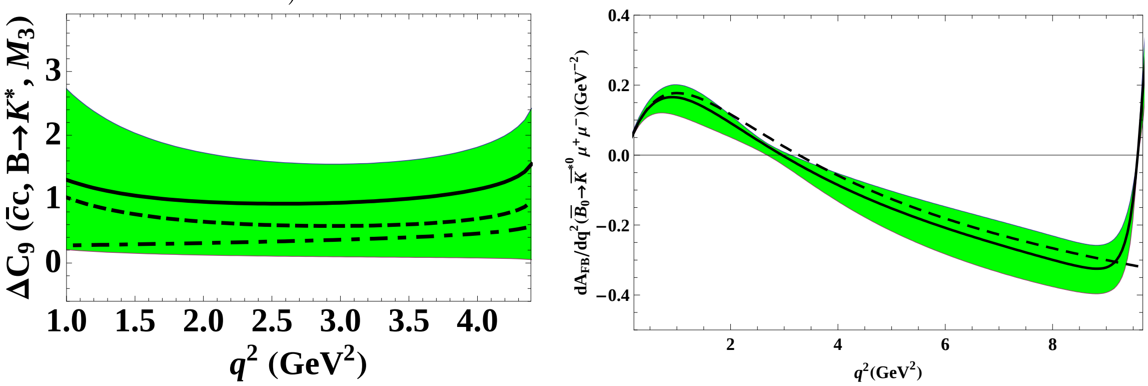
$$Y_A(\lambda,\omega) = -\frac{\lambda_E^2}{24\omega_0^4}\omega(7\omega_0 - 13\lambda + 3\omega)e^{-(\lambda+\omega)/\omega_0}$$

with ω_0 and λ_E being the parameters of the two-particle and three-particle DA's of the B-meson. A note about the B-meson DA's is in order. This is definitely a model which is not yet sufficiently accurate. There are $\sim 1/m_b$ corrections to the HQET correlation function, the gluon radiative corrections and some other that have not been taken into account. The normalization parameters of B-meson DA's, such as the inverse moment, have large uncertainties. The uncertainties of sum rules eventually leads to relatively large theoretical errors of the calculation. Nevertheless, since we are investigating a small effect, at least at low q^2, the achieved accuracy is reasonable.

It is convenient to give results in terms of correction to the coefficient C_9 which is sensitive to new physics. Its value without loop corrections is $C_9 = 4.2$. We write $\Delta C_9 = (C_1 + 3C_2)g(m_c,q) + 2C_1\tilde{g}$ with $\tilde{g} = \tilde{A}/f^+_{BK}$ in case of the decay into K-meson. The results are presented in the following figures.



Left figure: charm-loop effect in $B \to K\ell^+\ell^-$ as a correction to the coefficient C_9 (solid), nonfactorizable soft-gluon contribution (dashed) factorizable contribution (dash-dotted). Right figure: correction to C_9 for amplitude $\mathcal{M}_1$ for the K^* decay (total – solid, soft-gluon – dashed, factorizable – dash-dotted).



Left figure: correction to C_9 for the amplitude $\mathcal{M}_3$ for $\bar{B}_0 \to \bar{K}^* l^+ l^-$ (total – solid, soft-gluon – dashed, factorizable – dash-dotted). Right figure: forward-backward asymmetry A_{FB} for $\bar{B}_0 \to \bar{K}^* \mu^+\mu^-$ decay with charm-loop effect (solid), without this effect (dashed). The zero of A_{FB} is at $q_0^2 = 2.9 \pm 0.3$.

6 Summary

Long-distance soft-gluon emission violating factorization taken into account for c-quark loop contribution. LCSR with B-meson DA is used to calculate matrix elements of emerging operators. Soft-gluon contribution is enhanced by the Wilson coefficient for $B \to K^*\ell^+\ell^-$ and numerically important. The soft-gluon contribution is suppressed by $\sim 1/(4m_c^2 - q^2)$ and the approximation is valid at $q^2 \ll 4m_c^2$. Near the $\bar{c}c$-threshold, multiple soft-gluon emission operators have to be included and one eventually looses control over OPE.

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