Chiral dynamics in $\pi\pi$ scattering and light scalars

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Abstract

We present the updated analysis by means of $\pi\pi$ scattering amplitude T_0^0 having regular analytical properties in the s complex plane. We dwell on questions dealing with the small $\sigma - f_0$ mixing, inelasticity description and the kaon loop model for $\phi \to \gamma(\sigma + f_0)$ reaction, and show a number of new fits. In particular, we show that the minimization of the $\sigma - f_0$ mixing results in the four-quark scenario for light scalars: the $\sigma(600)$ coupling with the $K\bar{K}$ channel is suppressed relatively to the coupling with the $\pi\pi$ channel, and the $f_0(980)$ coupling with the $\pi\pi$ channel is suppressed relatively to the coupling with the $K\bar{K}$ channel.

We also suggest a simple background parameterization, practically preserving the resonance features, which is comfortable for experimental data analysis, but allows to describe the results based on chiral expansion and Roy equations only on the real s axis.

1 Introduction

Study of light scalar resonances is one of the central problems of nonperturbative QCD, it is important for understanding the chiral symmetry realization way resulting from the confinement physics.

In QUARKS-2010 [1] we reported for the first time the S-wave $\pi\pi$ scattering amplitude T_0^0 with $I = 0$ with correct analytical properties in the complex s plane. This amplitude was used to analyze the KLOE data on the $\phi \to \pi^0\pi^0\gamma$ decay [2] with the help of the kaon loop model $\phi \to K^+K^- \to (f_0 + \sigma)\gamma \to \pi^0\pi^0\gamma$ [3, 4, 5, 6, 7] simultaneously with the data on the $\pi\pi$ scattering and the $\pi\pi \to K\bar{K}$ process [8]. The chiral shielding of the $\sigma(600)$ meson [9, 10] and the $\sigma - f_0$ mixing were taken into account, the analysis supported the four-quark nature of the $\sigma(600)$ and $f_0(980)$. From that premier results we came to those reported in Ref. [11].

Remind that the primary goal of building such an amplitude is the ability to compare results [8], obtained in our model, with results of Ref. [12], where the $\pi\pi$ scattering amplitude was calculated in the s complex plane, basing on chiral expansion and Roy equations. The σ pole was obtained in Ref. [12] at

$$M_\sigma = 441^{+16}_{-8} - i272^{+9}_{-12.5}\ \text{MeV}\,. \tag{1}$$

Remind that in our model the S matrix of the $\pi\pi$ scattering is the product of the "resonance" and "elastic background" parts:

$$S_0^0 = S_0^{0\,back}\, S_0^{0\,res}\,, \tag{2}$$

and we introduced [1, 11] special $S_0^{0\,back}$ parametrization to obtain the correct T_0^0 analytical properties, while $S_0^{0\,res}$ had correct analytical properties in Refs. [8] already. In Ref. [11] we

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successfully described the Ref. [12] results on the real s axis using the constructed $\pi\pi$ amplitude, while the σ pole was located rather far from the Ref. [12] result. The hypothesis concerning this deviation was suggested. Note that the results [12] were treated along with experimental data.

Here we report the enlarged data analysis [13].

2 Results

The S-wave amplitude T_0^0 of the $\pi\pi$ scattering with I=0 [14, 15, 16, 4] is

$$T_0^0 = \frac{\eta_0^0 e^{2i\delta_0^0} - 1}{2i\rho_{\pi\pi}(m)} = \frac{e^{2i\delta_B^{\pi\pi}} - 1}{2i\rho_{\pi\pi}(m)} + e^{2i\delta_B^{\pi\pi}} \sum_{R,R'} \frac{g_{R\pi\pi} G^{-1}_{RR'} g_{R'\pi\pi}}{16\pi}\,. \tag{3}$$

Here $\eta_0^0 \equiv \eta_0^0(m)$ is the inelasticity, δ_0^0 is the scattering phase, $\delta_B^{\pi\pi}$ is the phase of the elastic background, and the matrix of the inverse propagators [17, 8, 11]

$$G_{RR'}(m) = \begin{pmatrix} D_{f_0}(m) & -\Pi_{f_0\sigma}(m) \\ -\Pi_{f_0\sigma}(m) & D_\sigma(m) \end{pmatrix}.$$

The form of $e^{2i\delta_B^{\pi\pi}}$ may be found in Ref. [11], note that it is practically the same as in Ref. [1].

The results are shown in Tables and Figs.

In Table I we show 5 Fits for different f_0 couplings to the $K\bar{K}$ channel. In Fit 4 the $\sigma(600)$ and $f_0(980)$ are coupled only with the $\pi\pi$ channel and the $K\bar{K}$ channel, while in other Fits $\eta\eta$, $\eta'\eta$ and $\eta'\eta'$ channels are taken into account.

One can see from Table I and Fig. 5 that the small $\sigma - f_0$ mixing ($\delta_0^{0\,res}(m_\sigma)$ close to $90°$ and $\delta_0^{0\,res}(m_{f_0})$ close to $270°$) leads to the four-quark scenario for light scalars: the $\sigma(600)$ coupling with the $K\bar{K}$ channel is suppressed relatively to the coupling with the $\pi\pi$ channel, and the $f_0(980)$ coupling with the $\pi\pi$ channel is suppressed relatively to the coupling with the $K\bar{K}$ channel [18].

In Refs. [8, 11] we used the factor P_K, caused by the elastic $K\bar{K}$ background phase, that allows to correct the kaon loop model, suggested in Ref. [3], under the $K\bar{K}$ threshold[1]:

$$P_K = \begin{cases} e^{i\delta_B^{K\bar{K}}} & m \geq 2m_K\,; \\ \text{analytical continuation of } e^{i\delta_B^{K\bar{K}}} & m < 2m_K\,. \end{cases} \tag{4}$$

Now we investigate how small this correction may be. We upgrade the parametrization of the $\delta^{K\bar{K}}(m)$, used in Refs. [8, 11], now the $\delta_B^{K\bar{K}}$ is parametrized in the following way:

$$e^{2i\delta_B^{K\bar{K}}} = \frac{1 + i2p_K f_K(m^2)}{1 - i2p_K f_K(m^2)}\,, \quad p_K = \frac{1}{2}\sqrt{m^2 - 4m_{K^+}^2}\,,$$

$$f_K(m^2) = -\left(1 - w + w\frac{(m-m_2)^2/\Lambda_2^2}{1 + (m-m_2)^2/\Lambda_2^2}\right)\frac{\arctan(\frac{m^2-m_1^2}{\Lambda_1^2}) - \phi_0}{\Lambda_K}\,. \tag{5}$$

One can see that for Fits 2-5 with $g^2_{f_0K^+K^-}/4\pi \geq 1.5$ GeV^2 the maximum of the $|P_K|^2$ is close to 1 (about 1.2), see Fig. 6b, this means that the correction to the kaon-loop model [3] is small. For lower $g^2_{f_0K^+K^-}/4\pi$ the $|P_K|^2$ increases as a compensation, see Fit 1 and Fig. 6a. This results in a model dependence of the constant determination. A precise measurement of the inelasticity η_0^0 would resolve this problem.

[1] Remind that the P_K also provides pole absence in the analytical continuation of the $\phi \to (f_0+\sigma)\gamma$ amplitude under the $\pi\pi$ threshold, see Ref. [8].

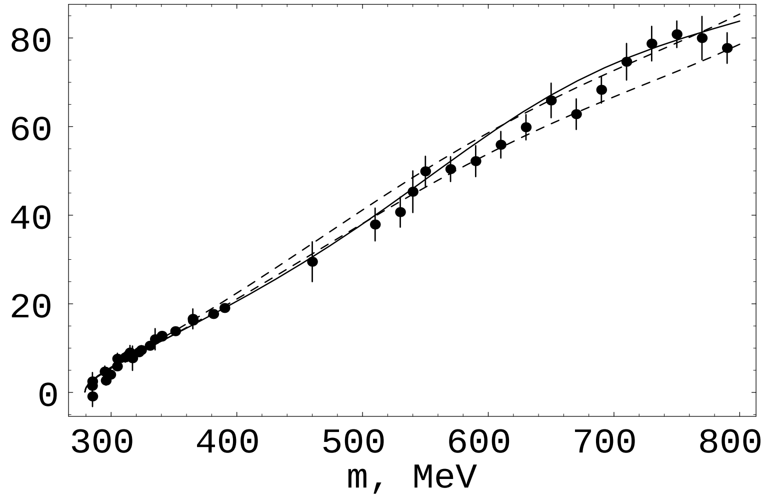


Figure 1: The phase δ_0^0 of the $\pi\pi$ scattering for Fit 3. Solid line is the obtained description, dashed lines mark borders of the corridor [12], points are experimental data.

Table I. Properties of the resonances and main characteristics.

Fit	1	2	3	4	5
m_{f_0}, MeV	978.30	974.78	981.49	979.85	980.40
$g^2_{f_0K^+K^-}/4\pi$, GeV^2	1	1.5	2	2	4.2782
$g^2_{f_0\pi^+\pi^-}/4\pi$, GeV^2	0.154	0.208	0.313	0.215	0.533
$\Gamma_{f_0}(m_{f_0})$, MeV	56.7	76.6	114.8	79.1	195.5
m_σ, MeV	479.40	471.89	470.87	472.87	469.94
$g^2_{\sigma\pi^+\pi^-}/4\pi$, GeV^2	0.564	0.569	0.588	0.584	0.596
$g^2_{\sigma K^+K^-}/4\pi$, GeV^2	0.001	0.048	0.006	0.006	0.002
$\Gamma_\sigma(m_\sigma)$, MeV	362.1	363.2	379.5	376.0	384.7
a_0^0, m_π^{-1}	0.223	0.220	0.224	0.223	0.225
Adler zero in $\pi\pi\to\pi\pi$	(93.5 MeV)^2	(85.6 MeV)^2	(96.8 MeV)^2	(94.6 MeV)^2	(92.3 MeV)^2
$\delta_0^{0\,res}(m_\sigma)$, $^\circ$	91.8	94.1	91.0	90.6	92.3
$\delta_0^{0\,res}(m_{f_0})$, $^\circ$	250.1	250.1	260.1	255.1	258.7
η_0^0(1010 MeV)	0.55	0.52	0.51	0.51	0.51
χ^2_{phase} (44 points)	53.1	48.9	42.0	40.0	55.1
χ^2_{sp} (18 points)	21.2	20.8	21.3	17.0	12.6

Inelasticity is crucial for the analysis, here we describe the peculiar behavior of the data up to 1.2 GeV, indicated by the experimental data, see Fig. 7. This behaviour was discussed quite widely in Ref. [15]. Unfortunately, the current data have large errors, so the precise measurement of the inelasticity η_0^0 near 1 GeV in $\pi\pi\to\pi\pi$ would be very important.

In Ref. [10] it was shown in the SU(2)×SU(2) linear σ model, that the residue of the σ pole in the amplitude of the $\pi\pi$ scattering can not be connected to coupling constant in the Hermitian (or quasi-Hermitian) Hamiltonian for it has a large imaginary part. The T_0^0 residues in the $\sigma(600)$ pole, shown in Table II, illustrate this fact. Note that large imaginary part is both in the residues of the full amplitude T_0^0 and its resonance part $T_0^{0\,Res}$. So, considering the residue of the σ pole in T_0^0 or $T_0^{0\,Res}$ as proportional to the square of its coupling constant to the $\pi\pi$ channel is not a clear guide to understanding the σ meson nature. In addition, this pole can not be interpreted as a physical state for its huge width.

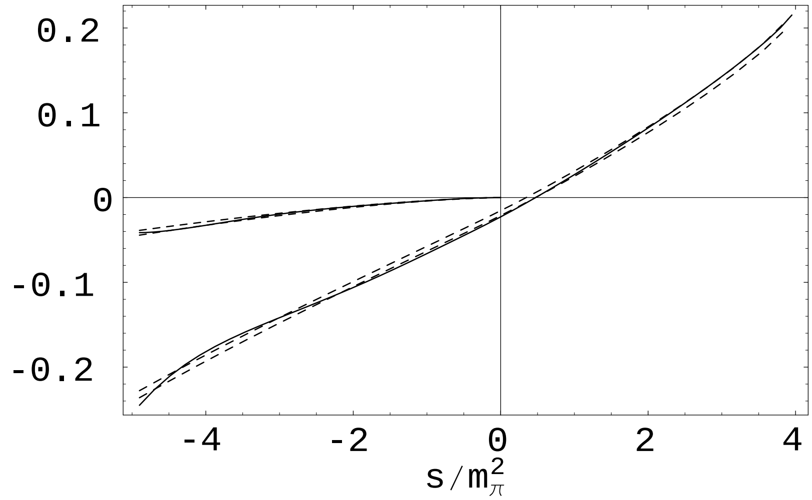


Figure 2: The real and the imaginary parts of the amplitude T_0^0 of the $\pi\pi$ scattering for Fit 3 (s in units of m_π^2). Dashed lines mark borders of the corridors [12].

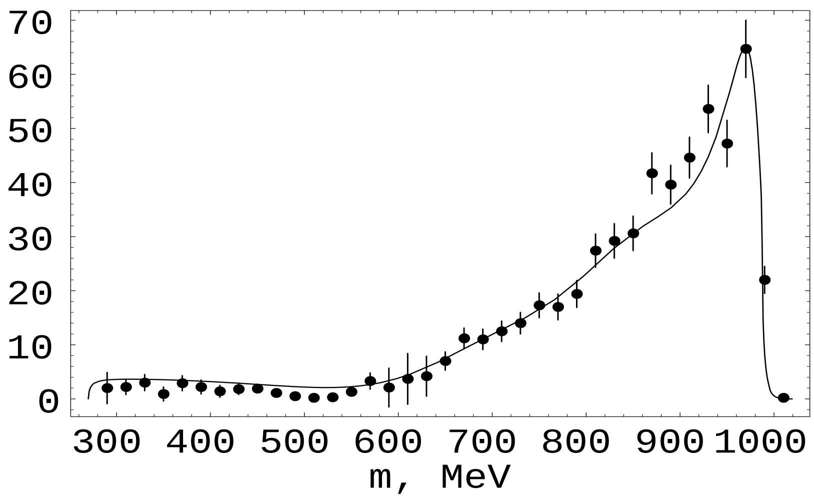


Figure 3: The $\pi^0\pi^0$ spectrum in $\phi \to \pi^0\pi^0\gamma$ decay for Fit 3, solid line is the obtained description.

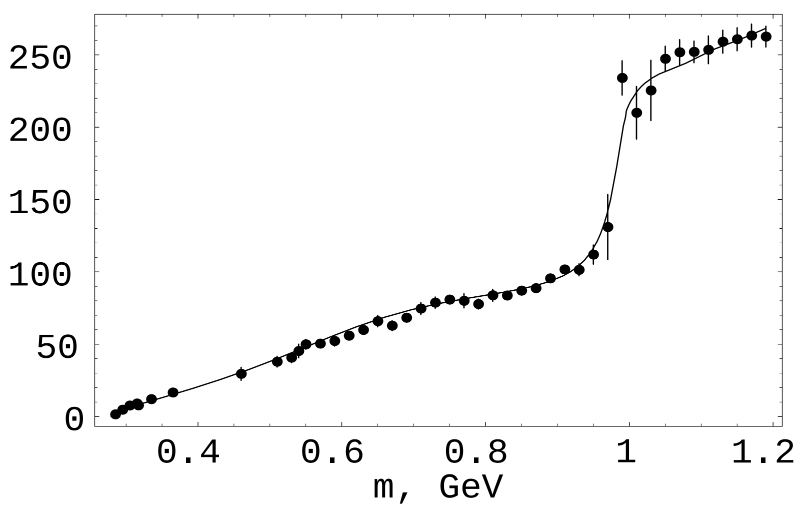
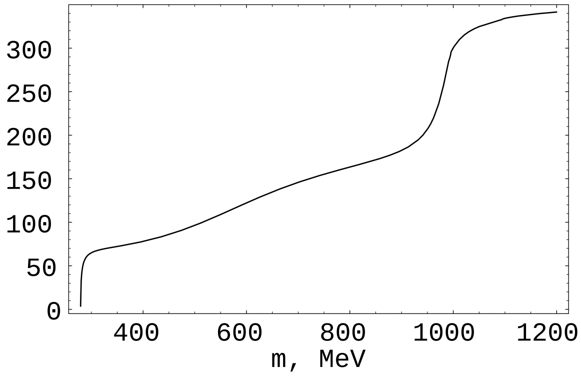
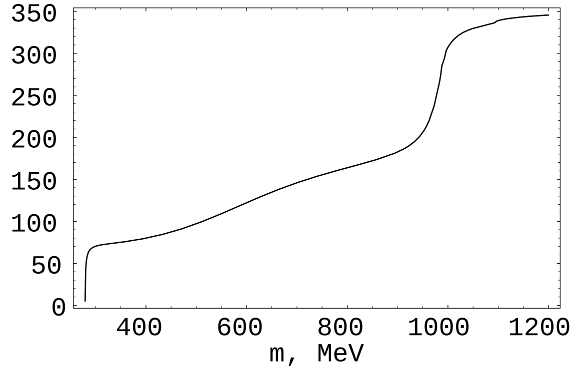


Figure 4: The phase δ_0^0 of the $\pi\pi$ scattering for Fit 3.

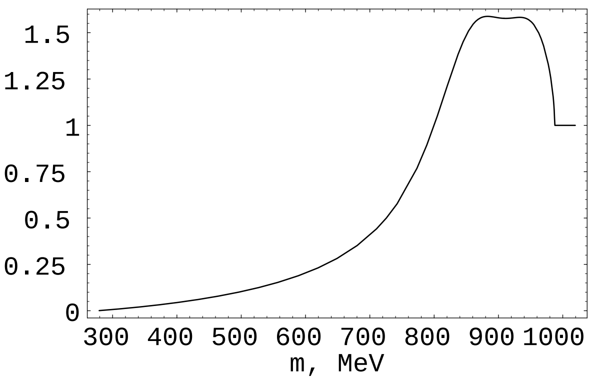


(a)



(b)

Figure 5: The resonance phase of the $\pi\pi$ scattering $\delta_0^{0\,res}$ (degrees): a) Fit 1 ($m_\sigma = 479.4$ MeV, $m_{f_0} = 978.3$ MeV); b) Fit 3 ($m_\sigma = 470.9$ MeV, $m_{f_0} = 981.5$ MeV).



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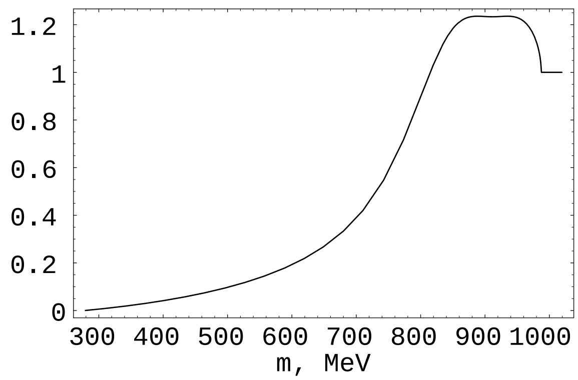
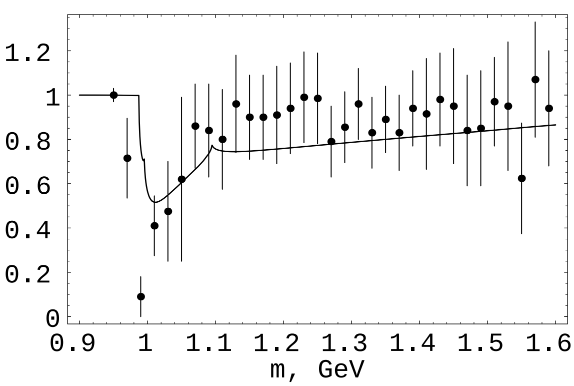
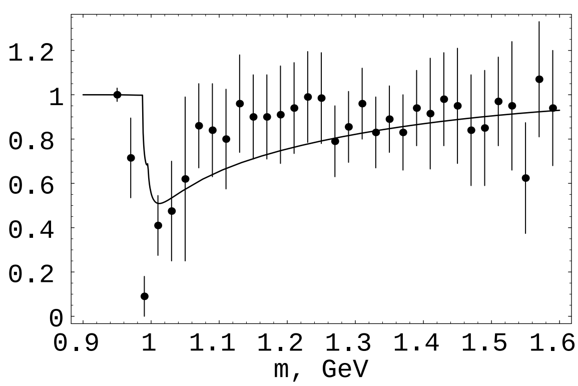


(b)

Figure 6: The $|P_K(m)|^2$ is shown, see Eq. (2): a) Fit 1; b) Fit 3.



(a)



(b)

Figure 7: The inelasticity η_0^0 is shown: a) Fit 3, b) Fit 4.

Table II. The $\sigma(600)$ poles (in MeV), the residues of T_0^0, $\mathrm{Res}\,T_0^0$, and of the resonance part T_0^{0Res}, $\mathrm{Res}\,T_0^{0Res}$, (in 0.01 GeV^2) in this pole on different sheets of the complex s plane depending on sheets of polarization operators $\Pi^{ab}(s)$ are shown.

Sheets of Π^{ab}					Fit 1			Fit 5		
$\Pi^{\pi\pi}$	Π^{KK}	$\Pi^{\eta\eta}$	$\Pi^{\eta\eta'}$	$\Pi^{\eta'\eta'}$	σ pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$	σ pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$
II	I	I	I	I	$565-204\,i$	$-2+14\,i$	$-22-11\,i$	$566-201\,i$	$-1+13\,i$	$-20-12\,i$
II	II	I	I	I	$612-346\,i$	$5+3\,i$	$-18+14\,i$	$569-267\,i$	$3+10\,i$	$-26-1\,i$
II	II	II	I	I	$542-396\,i$	$-1+3\,i$	$-16-3\,i$	$522-379\,i$	$-2+3\,i$	$-14-6\,i$
II	II	II	II	I	$577-522\,i$	$0.2+1\,i$	$-15-1\,i$	$612-626\,i$	$0.3+0.4\,i$	$-15-4\,i$
II	II	II	II	II	$633-534\,i$	$1+1\,i$	$-23+2\,i$	$664-651\,i$	$0.5+0.3\,i$	$-19-4\,i$
Sheets of Π^{ab}					Fit 3			Fit 4		
$\Pi^{\pi\pi}$	Π^{KK}	$\Pi^{\eta\eta}$	$\Pi^{\eta\eta'}$	$\Pi^{\eta'\eta'}$	σ pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$	σ pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$
II	I	I	I	I	$572-206\,i$	$-2+14\,i$	$-21-12\,i$	$579-216\,i$	$-3+15\,i$	$-23-12\,i$
II	II	I	I	I	$572-279\,i$	$3+10\,i$	$-26+2\,i$	$579-273\,i$	$1+12\,i$	$-27-1\,i$
II	II	II	I	I	$526-395\,i$	$-2+2\,i$	$-12-5\,i$	–	–	–
II	II	II	II	I	$623-651\,i$	$0.3+0.3\,i$	$-14-3\,i$	–	–	–
II	II	II	II	II	$683-679\,i$	$1+0.1\,i$	$-19-4\,i$	–	–	–

Table III. The $f_0(980)$ poles (in MeV), the residues of T_0^0, $\mathrm{Res}\,T_0^0$, and of the resonance part T_0^{0Res}, $\mathrm{Res}\,T_0^{0Res}$, (in 0.01 GeV^2) in this pole on different sheets of the complex s plane depending on sheets of polarization operators $\Pi^{ab}(s)$ are shown.

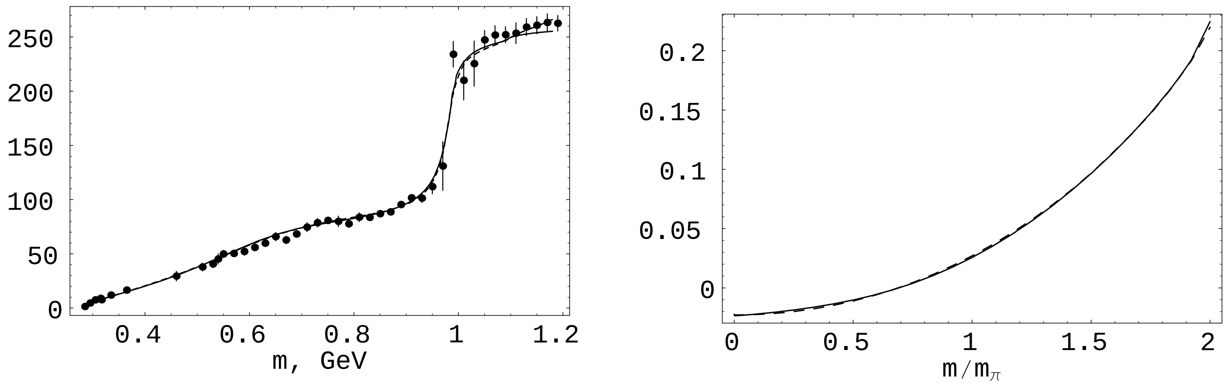
Sheets of Π^{ab}					Fit 1			Fit 5		
$\Pi^{\pi\pi}$	Π^{KK}	$\Pi^{\eta\eta}$	$\Pi^{\eta\eta'}$	$\Pi^{\eta'\eta'}$	f_0 pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$	f_0 pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$
II	I	I	I	I	$986-26\,i$	$6-2\,i$	$-7+2\,i$	$986-21\,i$	$5-1\,i$	$-6+0.1\,i$
II	II	I	I	I	$913-302\,i$	$10+5\,i$	$-19-19\,i$	$1575-553\,i$	$-8-4\,i$	$-21-23\,i$
II	II	II	I	I	$966-450\,i$	$3-1\,i$	$-12-10\,i$	$2101-1065\,i$	$0.1+5\,i$	$-28-10\,i$
II	II	II	II	I	$962-465\,i$	$3-0.3\,i$	$-12-12\,i$	$2173-1158\,i$	$1+5\,i$	$-25-11\,i$
II	II	II	II	II	$954-586\,i$	$1+0.4\,i$	$-3-14\,i$	$2452-1570\,i$	$3+3\,i$	$-22-10\,i$
Sheets of Π^{ab}					Fit 3			Fit 4		
$\Pi^{\pi\pi}$	Π^{KK}	$\Pi^{\eta\eta}$	$\Pi^{\eta\eta'}$	$\Pi^{\eta'\eta'}$	f_0 pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$	f_0 pole	$\mathrm{Res}\,T_0^0$	$\mathrm{Res}\,T_0^{0Res}$
II	I	I	I	I	$986-23\,i$	$6-1\,i$	$-6+1\,i$	$985-20\,i$	$5-1\,i$	$-5+1\,i$
II	II	I	I	I	$1149-485\,i$	$3-6\,i$	$-14-16\,i$	$1187-618\,i$	$0.5-2\,i$	$-11-9\,i$
II	II	II	I	I	$1441-835\,i$	$-3+0.4\,i$	$-20-7\,i$	–	–	–
II	II	II	II	I	$1469-885\,i$	$-2+0.4\,i$	$-16-9\,i$	–	–	–
II	II	II	II	II	$1607-1182\,i$	$-1+1\,i$	$-11-8\,i$	–	–	–

The obtaining of the σ pole in Ref. [12] gave the strong argument in favor of the $\sigma(600)$ existence, but followed efforts aiming the precise determination of the pole are not productive. The Roy equations are one-channel, that is, are approximate and even slight discrepancy in the amplitude in the physical region may lead to large changes in the complex plane. Remind that the Riemannian surface of the $\pi\pi$ scattering amplitude has many sheets (strictly speaking, infinite number of sheets), and even for relatively narrow $f_0(980)$ it is sometimes difficult to determine on what sheet should we find the pole, see, for example, Fit 2 in Ref. [11]. But even if we obtained the pole precisely, it would give us practically no information on the resonance nature. Besides, the residue of the amplitude in the pole is strongly distorted by the background part of the amplitude, that gives essential contribution even for relatively narrow $f_0(980)$, see Tables II and III.

Let us dwell on the results, presented in Table III. Remind that for a stable particle with the mass m_0 there is the pole in the amplitude

$$T = -\frac{g^2/16\pi}{s-m_0^2}$$

at $s=m_0^2$, the residue of the amplitude $ResT$ is connected to the coupling constant (g) of the stable particle with the $\pi\pi$ channel.



(a) (b)

Figure 8: The comparison of Fit 3 and Fit 7 (with the same resonance parameters, but "simple" background): a) The phase δ_0^0, b) the amplitude T_0^0 under the $\pi\pi$ threshold. Solid lines correspond to Fit 7, dashed lines to Fit 3, points - the experimental data.

One can see that the real part of the T_0^0 residue in the $f_0(980)$ pole is positive, so the coupling constant should be practically pure imaginary, what is physically meaningless. Note that the residue of the amplitude resonance part $T_0^{0\,Res}$ is good. That is why the best way for understanding the nature of the light scalars is the investigation of their production mechanisms in physical processes.

3 Simple background

The background function, suggested in Ref. [11] to reach the correct analytical properties of the $\pi\pi$ scattering amplitude and used above, is rather complicated and costly in computation. We suggest much more simple background parameterization, practically preserving the resonance features, which is comfortable for experimental data analysis and allows to describe the results [12] on the real s axis. This background function is an upgrade of the one, used in Ref. [8]:

$$\tan(\delta_B^{\pi\pi}) = -\frac{p_\pi}{m_\pi}\frac{b_0 - b_1\frac{p_\pi^2}{m_\pi^2} + b_2\frac{p_\pi^4}{m_\pi^4} + b_3\frac{p_\pi^6}{m_\pi^6} + \frac{m}{m_\pi}\left(c_0 + c_1\frac{p_\pi^2}{m_\pi^2} + c_2\frac{p_\pi^4}{m_\pi^4} + c_3\frac{p_\pi^6}{m_\pi^6}\right)}{(1+4p_\pi^2/\Lambda_1^{\pi\,2})(1+4p_\pi^2/\Lambda_2^{\pi\,2})}, \tag{6}$$

here $p_\pi = \sqrt{m^2 - 4m_\pi^2}/2$. Note that in comparison with Ref. [8] the function (6) has a left cut.

To illustrate the abilities of the background (6), we perform Fit 6 with the same resonance parameters as for Fit 3. Fit 6 provides practically the same experimental data description as Fit 3. The theoretical curves for phase δ_0^0 are shown in Fig. 8a, they are practically the same. It is obvious that both Fit 3 and Fit 6 provide practically identical mass spectrum in $\phi \to \pi^0\pi^0\gamma$ decay also. The inelasticity is exactly the same. Additionally, Fit 6 and Fit 3 provide indistinguishable curves for T_0^0 at $4m_\pi^2 > s > 0$, see Fig. 8b.

4 Summary

1. The analysis using the $\pi\pi$ scattering amplitude with correct analytical properties has been updated. The requirement of $\sigma - f_0$ mixing smallness leads to the four-quark scenario of the σ and f_0.

2. The determination of the pole positions and their residues give us practically no information on the broad resonances nature.

3. The data are described well with small deviation of the $|P_K|$ from 1 for $m > 850$ MeV.

4. New experiments are important for the study of light scalar mesons.

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