Magnetic Field Induced Effects in Quark Matter*

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Abstract

Quark–gluon matter produced in relativistic heavy–ion collisions(RHIC and LHC) is subject to a super–strong magnetic field(MF) $\sim 10^{18} - 10^{20}\ G$. Quark matter(QM) response to MF allows to get a new insight on its properties. We give a cursory glance on MF induced effects.

Relativistic heavy–ion collisions(RHIC and LHC) generate gigantic magnetic field (MF) $eB \sim \Lambda^2_{QCD}$. The response of QM to such a field has been intensively studied during the last several years. In this brief presentation we merely give a list of the corresponding problems. In no way this material can be considered as a review paper. We apologize for the absence of references. A list of 10^2 references would be excessive for this format.

We begin by comparing MF-s encountered in Nature and in the Laboratory.

The Hierarcy of MF-s (in Gauss)

Medical MPI scan	10^4
ATLAS at LHC	$4 \cdot 10^4$
Lab.(preserving the equipment)	10^6
Lab.(explosion)	$28 \cdot 10^6$
Schwinger(for electron)	$4.4 \cdot 10^{13}$
Surface of magnetars	$10^{14} - 10^{15}$
RHIC and LHC	$10^{18} - 10^{20}$
Early Universe	10^{24}

From the above table we see that quark–gluon matter produced in heavy–ion collisions is embedded in the strongest possible MF. This field lasts for only $\tau \sim 0.2\ fm$ unless the conductivity of the produced matter is high enough(see below). It turns out that effects caused by MF at $\tau \leq 0.2\ fm$ almost evaded a thorough investigation. It might be due to the fact that at this stage of the "fireball" evolution and its nature and dynamics are rather complicated. In fact, two drastic oversimplifications have been done in most studies: 1) MF was assumed to be constant both in time and in space direction, and 2) an infinite system in thermodynamic equilibrium has been usually considered. Probably the rare exception in the Chiral Magnetic Effect(CME) which implies the creation of flashing topological charges in hot and dense medium. However, microscopic derivation of topological effects is lacking, while the lattice calculations of CME were performed for low temperature and in thermodynamic equilibrium(see Section 4 below).

Now we start the list of MF induced effects.

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1 MF decay rate

MF duration $\tau \sim 0.2\ fm$ corresponds to the time of maximal overlap of the colliding nuclei. Magnetic response of the produced matter can make this time an order of magnitude larger. On the dimensional grounds the decay time of MF is $\tau' \simeq \sigma L^2$, where σ is the electrical conductivity, L is the characteristic length scale of the spatial variation of MF. If for a rough estimate we take $\sigma/T \simeq 0.5,\ T \simeq 200\ MeV,\ L \simeq 2\ fm$, we obtain $\sigma' \simeq 2\ fm$. In presence of magnetic monopoles(possibly seen on the lattice) this time will be shorter.

2 Phase space arguments

MF stronger than Schwinger critical field ($B_c = \frac{eB}{m_e^2} = 4.4 \cdot 10^{13}\ G$ for electron) results in enlarging of the phase space available for the electron in β-decay and in the corresponding increase of the decay rate. This is due to the Landau orbits phase space. The same effect for quark emerging from a decay requires MF smaller by a factor of m_e^2/m_q^2. To our knowledge this effect has not been investigated.

Another phenomenon concerns the population of Landau levels in dense QM. The dispersion relation for quark in MF reads

$$\omega_{n,\sigma}(k_z) = [k_z^2 + m_f^2 + q_f B(2n + 1 + \sigma)]^{1/2}, \tag{1}$$

where $\mathbf{B}||\mathbf{z}$, f is the flavour index, q_f is the absolute value of quark electric charge, $\sigma = \pm 1$. Consider a dense QM at low temperature, i.e., in the regime $\mu \gg T$, where μ is the chemical potential. Condensed matter wisdom tells that the key physical processes(like transport) are determined by the vicinity of the Fermi surface. If in (1) we take $\omega_{n\sigma} = \mu_F = \sqrt{k_F^2 + m_f^2}$, then only Landau levels up to

$$n_{max} = \frac{\mu_F^2 - m_f^2}{2q_f B} \tag{2}$$

survive. For example, for $\mu_F = 3m_\pi,\ q_f B = 5m_\pi^2,\ m_f = 0$, one gets $n_{max} = 1$. We note that the dominance of the Lower Landau Lavel(LLL) in strong MF has a general nature not inferred from (2). Another related general feature of strong MF in the transverse shrinkage of the system and dimensional reduction $3d \to 1d$.

3 QCD phase diagram in MF

Before going to concrete results on QCD phase diagram, a remark has to be done. With rare exceptions, QCD phase diagram has been studied under a tacit assumption that the system is infinite and in a state of thermodynamic equilibrium. Such an approach might have been appropriate for neutron stars, but reliable lattice calculations are only possible for $\mu = 0$. For $\mu > 0$ one has to resort to models, like NJL, and no clear cut conclusions are available.

The first point is the influence of MF on light quark condensate $\langle\bar{\psi}\psi\rangle$. Until recently it seemed firmly established that $\langle\bar{\psi}\psi\rangle$ is increasing with B. Correspondingly the critical temperature $T_\chi(B)$ also grows. This kind of response got the name of "magnetic catalysis". The new lattice calculations revealed more complicated picture. Magnetic catalysis was confirmed at low temperature, while around T_χ the B-dependence of $\langle\bar{\psi}\psi\rangle$ is not monotonous resulting in the decrease of T_χ. This might be due to indirect interaction between gluons and MF. As already mentioned, at $\mu > 0$ only the results of model calculations are available. There is an indication that due to MF the first–order transition line, which starts at the critical point, goes up.

At $\mu = 0$ the phase transition is an analytic crossover. The chiral T_χ and deconfinement T_L temperatures are splitted, most studies show that $T_L > T_\chi$. The conclusions of different

authors on the MF dependence of T_L are contradictory. The latest lattice calculation indicates a reduction of T_L in MF.

Next we shall consider several more specific problems.

4 Chiral magnetic effect(CME)

It certainly provoked a record wave of discussions among all MF induced effects. It also brought to light the fact that heavy–ion collisions generate a super–intense MF. It still needs a sound experimental confirmation and work in this direction is in progress. In the most concise form CME is represented by the formula

$$\mathbf{j} = N_c \sum_f \frac{q_f^2 \mu_5}{2\pi^2} \mathbf{B}, \tag{3}$$

where $\mathbf{j}$ is the electric current, μ_5 is the chiral chemical potential which induces a difference in number between right–handed and left–handed particles. On the theoretical side equation (3), or similar ones, were obtained starting from different basic ideas: topological charge, axial anomaly, Chern-Simons action, strong θ–angle, etc. It also turned out that equations like (3) were discovered much earlier. As we already mentioned, lattice calculations can hardly be considered as a direct evidence of CME since simulations of chiral fermions at high temperature is out of reach for present lattice calculations.

5 Conductivity in MF

In Section 1 it was shown that the decay of MF depends on the value of the electrical conductivity(EC). Latice calculations at $\mu = 0$ and T around the phase transition temperature or somewhat higher give $\sigma/T \simeq 0.3 - 0.4$ which corresponds to MF decay time of a few fm. Another lattice group calculated EC in the same $(T,\ \mu)$ region with MF. They obtained much less value for EC and very weak dependence on MF. The last fact didn't get a physical explanation. It has a natural explanation in a different regime described in Section 2, namely high density and low temperature. Here the EC can be decomposed into two contributions: the Drude and the quantum ones. Drude part is calculated using Kubo formula and MF dependence enters via the combination $(eB/\mu)^2\tau^2$, where τ is the momentum relaxation time. As a result, MF dependence becomes significant only at $eB \geq 5m_\pi^2$. The quantum part depends on the MF via $1/l_B$, $l_B = (eB)^{-1/2}$, so that it has a relatively weak square root dependence on MF. We note that quantum contribution may be negative. In high density regime quantum EC is dominated by fluctuating (precursor) Cooper pairs. The same mechanism is responsible for another spectacular effect which we consider in the next section.

6 Giant Nernst Effect

Consider $\mathbf{B}||\mathbf{z}$ and the temperature gradient $\nabla_x T$. Then counterpart of Hall effect is the Nernst–Ettingshausen one. It amounts to the induction of the electric field E_y and is characterized by a coefficient

$$\nu = \frac{E_y}{(-\nabla_x T)B} \tag{4}$$

It was shown by Varlamov and co-authors that fluctuating pairs lead to a giant effect. The corresponding work for QM is in progress.

The effects listed in sections 1–6 do not cover the whole subject. In particular, left in the cold are:

1. Magnetic tuning of BCS-BEC crossover
2. Quarkonium dissociation via ionization in MF.
3. Enhancement of flaw anisotropies due to MF.
4. QM viscosity in MF.
5. ...

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