Light scalars in semi-leptonic decays of heavy quarkonia

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Abstract

We study the mechanism of production of the light scalar mesons in the $D_s^+ \to \pi^+\pi^-\, e^+\nu$ decays: $D_s^+ \to s\bar{s}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$, and compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays: $D_s^+ \to s\bar{s}\, e^+\nu \to (\eta/\eta')\, e^+\nu$. We show that the $s\bar{s} \to \sigma(600)$ transition is negligibly small in comparison with the $s\bar{s} \to f_0(980)$ one. As for the the $f_0(980)$ meson, the intensity of the $s\bar{s} \to f_0(980)$ transition makes near thirty percent from the intensity of the $s\bar{s} \to \eta_s$ ($\eta_s = s\bar{s}$) transition. So, the $D_s^+ \to \pi^+\pi^-\, e^+\nu$ decay supports the previous conclusions about a dominant role of the four-quark components in the $\sigma(600)$ and $f_0(980)$ mesons.

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At present the nontrivial nature of the well-established light scalar resonances $f_0(980)$ and $a_0(980)$ is denied by very few people. As for the nonet as a whole, even a cursory look at PDG Review [1] gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $\kappa(800)$, $f_0(980)$, and $a_0(980)$, inverted in comparison with the classical P wave $q\bar{q}$ tensor meson nonet, $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, $\phi_2'(1525)$. Really, while the scalar nonet cannot be treated as the P wave $q\bar{q}$ nonet in the naive quark model, it can be easy understood as the $q^2\bar{q}^2$ nonet, where σ has no strange quarks, κ has the s quark, f_0 and a_0 have the $s\bar{s}$ pair. Similar states were found by Jaffe in 1977 in the MIT bag [2].

By now it is established also that the mechanisms of the $a_0(980)$, $f_0(980)$, and $\sigma(600)$ meson production in the ϕ radiative decays [3, 4, 5, 7, 8], in the photon-photon collisions [9, 10], and in the $\pi\pi$ scattering [7, 8] are the four-quark transitions and thus indicate to the four-quark structure of the light scalars [11].

In addition, the absence of the $J/\psi \to \gamma f_0(980)$, $a_0(980)\rho$, $f_0(980)\omega$ decays in contrast to the intensive the $J/\psi \to \gamma f_2(1270)$, $\gamma f_2'(1525)$, $a_2(1320)\rho$, $f_2(1270)\omega$ decays intrigues against the P wave $q\bar{q}$ structure of $a_0(980)$ and $f_0(980)$ also [12].

It is time to explore the light scalar mesons in the decays of of heavy quarkonia [13, 14, 15, 16]. The semi-leptonic decays are of prime interest because they have the clear mechanisms, see, for example, Fig. 1.

As Fig. 1 suggests, the $D_s^+ \to s\bar{s}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$ decay is the perfect probe of the $s\bar{s}$ component in the $\sigma(600)$ and $f_0(980)$ states [13, 14].

Below we study the mechanism of production of the light scalar mesons in the $D_s^+ \to \pi^+\pi^-\, e^+\nu$ decays: $D_s^+ \to s\bar{s}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$, and compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays: $D_s^+ \to s\bar{s}\, e^+\nu \to (\eta/\eta')\, e^+\nu$, in a model of the Nambu-Jona-Lasinio type [17].

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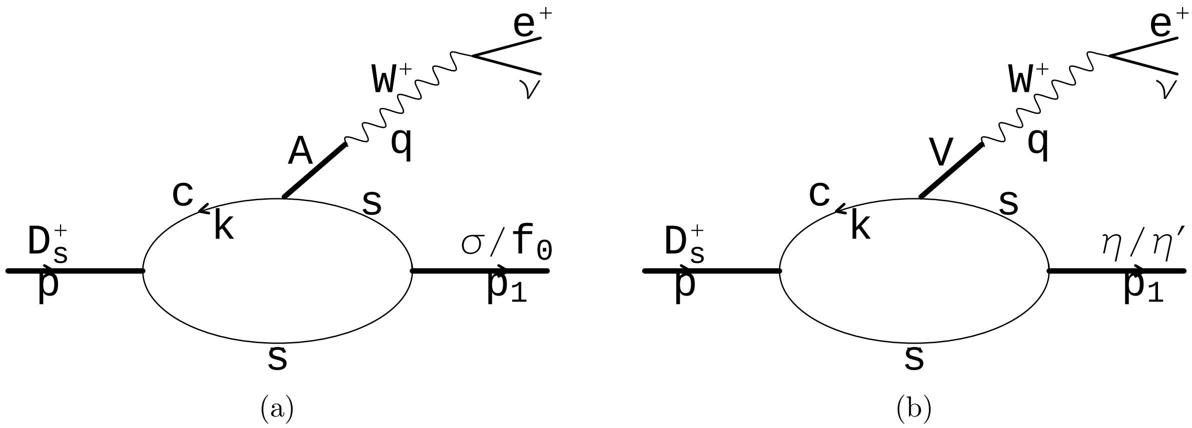

Figure 1: Model of the $D_s^+ \to \sigma/f_0\, e^+\nu$ and $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays

The amplitudes of the $D_s^+ \to P(\text{pseudoscalar})\, e^+\nu$ and $D_s^+ \to S(\text{scalar})\, e^+\nu$ decays have the form

$$M[D_s^+(p) \to P(p_1)W^+(q) \to P(p_1)\, e^+\nu] = \tfrac{G_F}{\sqrt{2}} V_{cs} V_\alpha L^\alpha\,,$$
$$M[D_s^+(p) \to S(p_1)W^+(q) \to S(p_1)\, e^+\nu] = \tfrac{G_F}{\sqrt{2}} V_{cs} A_\alpha L^\alpha\,, \qquad (1)$$

where G_F is the Fermi constant, V_{cs} is the CKM matrix element,

$$V_\alpha = f_+^P(q^2)(p+p_1)_\alpha + f_-^P(q^2)(p-p_1)_\alpha\,,$$
$$A_\alpha = f_+^S(q^2)(p+p_1)_\alpha + f_-^S(q^2)(p-p_1)_\alpha\,,$$
$$L_\alpha = \bar\nu\gamma_\alpha(1+\gamma_5)e\,, \qquad q=(p-p_1)\,. \qquad (2)$$

The influence of the $f_-^P(q^2)$ and $f_-^S(q^2)$ form factors are negligible because of the small mass of the positron.

The decay rates in the stable P and S states are

$$\frac{d\Gamma(D_s^+\to P\, e^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} p_1^3(q^2)|f_+^P(q^2)|^2, \quad \frac{d\Gamma(D_s^+\to S\, e^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} p_1^3(q^2)|f_+^S(q^2)|^2,$$

$$p_1(q^2) = \frac{\sqrt{m^4_{D_s^+} - 2m^2_{D_s^+}(q^2+m_P^2) + (q^2-m_P^2)^2}}{2m_{D_s^+}}\,, \text{ or } \; p_1(q^2) = \frac{\sqrt{m^4_{D_s^+} - 2m^2_{D_s^+}(q^2+m_S^2) + (q^2-m_S^2)^2}}{2m_{D_s^+}}\,. \qquad (3)$$

For the $f_+^P(q^2)$ and $f_+^S(q^2)$ form factors we use the vector dominance model

$$f_+^P(q^2) = f_+^P(0)\frac{m_V^2}{m_V^2-q^2} = f_+^P(0) f_V(q^2)\,, \qquad f_+^S(q^2) = f_+^S(0)\frac{m_A^2}{m_A^2-q^2} = f_+^S(0) f_A(q^2)\,, \qquad (4)$$

where $V = D_s^*(2112)^\pm$, $A = D_{s1}(2460)^\pm$, [1].

Following Fig. 1 we write $f_+^P(0)$ and $f_+^S(0)$ in the form

$$f_+^P(0) = g_{D_s^+ c\bar s} F_P g_{s\bar s P}\,, \qquad f_+^S(0) = g_{D_s^+ c\bar s} F_S g_{s\bar s S}\,, \qquad (5)$$

where $g_{D_s^+ c\bar s}$ is the $D_s^+ \to c\bar s$ coupling constant, $g_{s\bar s P}$ and $g_{s\bar s S}$ are the $s\bar s \to P$ and $s\bar s \to S$ coupling constant.

We know the structure of η and η'

$$\eta = \eta_q \cos\phi - \eta_s \sin\phi\,, \qquad \eta' = \eta_q \sin\phi + \eta_s \cos\phi\,, \qquad (6)$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$. The angle $\phi = \theta_i + \theta_P$, where θ_i is the ideal mixing angle with $\cos\theta_i = \sqrt{1/3}$ and $\sin\theta_i = \sqrt{2/3}$, i.e., $\theta_i = 54.7^\circ$, and θ_P is the angle between the flavor-singlet state η_1 and the flavor-octet state η_8.

So,

$$g_{s\bar{s}\eta} = -g_{s\bar{s}\eta_s}\sin\phi\,, \qquad g_{s\bar{s}\eta'} = g_{s\bar{s}\eta_s}\cos\phi\,. \tag{7}$$

Particle Data Group [1] give the θ_P band $-20^\circ \lesssim \theta_P \lesssim -10^\circ$ that gives us the opportunity to extract information about the $s\bar{s} \to \eta_s$ coupling constant, $g_{s\bar{s}\eta_s}$, from experiment and to compare with the $s\bar{s} \to f_0$ coupling constant, $g_{s\bar{s}f_0}$, extracted from experiment also. We consider the next set of θ_P.

$$\begin{aligned}
\theta_P = -11^\circ\ :&\quad \eta = 0.72\eta_0 - 0.69\eta_s\,, \quad \eta' = 0.69\eta_0 + 0.72\eta_s\\
\theta_P = -14^\circ\ :&\quad \eta = 0.76\eta_0 - 0.65\eta_s\,, \quad \eta' = 0.65\eta_0 + 0.76\eta_s\\
\theta_P = -18^\circ\ :&\quad \eta = 0.8\eta_0 - 0.6\eta_s\,, \quad \eta' = 0.6\eta_0 + 0.8\eta_s\,.
\end{aligned} \tag{8}$$

The amplitude of the the $D_s^+ \to s\bar{s}\,e^+\nu \to [\sigma(600) + f_0(980)]\,e^+\nu \to$ decay is

$$\begin{aligned}
M(D_s^+ \to s\bar{s}\,e^+\nu \ \to \pi^+\pi^-\ e^+\nu) &= \frac{G_F}{\sqrt{2}}V_{cs}\,L^\alpha\,(p+p_1)_\alpha\,g_{D_s^+ c\bar{s}}\,f_A(q^2)\\
\times e^{i\delta_B^{\pi\pi}}\,\frac{1}{\Delta(m)}\Big(&F_\sigma g_{s\bar{s}\sigma}D_{f_0}(m)g_{\sigma\pi^+\pi^-} + F_\sigma g_{s\bar{s}\sigma}\Pi_{\sigma f_0}(m)g_{f_0\pi^+\pi^-}\\
&+ F_{f_0}g_{s\bar{s}f_0}\Pi_{f_0\sigma}(m)g_{\sigma\pi^+\pi^-} + F_{f_0}g_{s\bar{s}f_0}D_\sigma(m)g_{f_0\pi^+\pi^-}\Big)\,,
\end{aligned} \tag{9}$$

where m is the invariant mass of the $\pi\pi$ system, $\Delta(m) = D_{f_0}(m)D_\sigma(m) - \Pi_{f_0\sigma}(m)\Pi_{\sigma f_0}(m)$, $D_\sigma(m)$ and $D_{f_0}(m)$ are the inverted propagators of the σ and f_0 mesons, $\Pi_{\sigma f_0}(m) = \Pi_{f_0\sigma}(m)$ is the off-diagonal element of the polarization operator, which mixes the σ and f_0 mesons. All the details can be found in Refs. [7, 8, 10].

The double differential rate of the $D_s^+ \to s\bar{s}\,e^+\nu \to [\sigma(600) + f_0(980)]\,e^+\nu \to \pi^+\pi^-\,e^+\nu$ decay is

$$\begin{aligned}
\frac{d^2\Gamma(D_s^+ \to \pi^+\pi^-\,e^+\nu)}{dq^2dm} &= \frac{G_F^2\,|V_{cs}|^2}{24\,\pi^3}\,g^2_{D_s^+ c\bar{s}}\,|f_A(q^2)|^2\,p_1^3(q^2,m)\\
\times\frac{1}{8\pi^2}\,m\,\rho_{\pi\pi}(m)\left|\frac{1}{\Delta(m)}\right|^2\Big|&F_\sigma g_{s\bar{s}\sigma}D_{f_0}(m)g_{\sigma\pi^+\pi^-} + F_\sigma g_{s\bar{s}\sigma}\Pi_{\sigma f_0}(m)g_{f_0\pi^+\pi^-}\\
&+ F_{f_0}g_{s\bar{s}f_0}\Pi_{f_0\sigma}(m)g_{\sigma\pi^+\pi^-} + F_{f_0}g_{s\bar{s}f_0}D_\sigma(m)g_{f_0\pi^+\pi^-}\Big|^2\,,
\end{aligned} \tag{10}$$

where $\rho_{\pi\pi}(m) = \sqrt{1 - 4m_\pi^2/m^2}$.

When $\Pi_{\sigma f_0}(m) = \Pi_{f_0\sigma}(m) = 0$ and $g_{s\bar{s}\sigma} = 0$

$$\frac{d^2\Gamma(D_s^+ \to \pi^+\pi^-\,e^+\nu)}{dq^2dm} = \frac{G_F^2\,|V_{cs}|^2}{24\,\pi^3}\,g^2_{D_s^+ c\bar{s}}\,|f_A(q^2)|^2\,p_1^3(q^2,m)\,\frac{2}{\pi}\,\frac{m^2\,\Gamma(f_0 \to \pi^+\pi^-\ m)}{|D_{f_0}(m)|^2}\,. \tag{11}$$

Results of our analysis of the CLEO [13] data are shown in the Table and on Figs. 2 and 3. The parameters of the $\sigma(600)$ and $f_0(980)$ mesons are taken from Fit 1 of Ref. [8] which describes the spectrum on Fig. 2 better than others. When fitting the spectrum shape, we found $f_+^\sigma/f_+^{f_0} = (F_\sigma g_{s\bar{s}\sigma})/(F_{f_0}g_{s\bar{s}f_0}) = 0.039$, that means the practical decoupling of $\sigma(600)$ with $\sigma_s = s\bar{s}$ [18] and agrees well with the decoupling of $\sigma(600)$ with the $K\bar{K}$ system, $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 0.00177$, see Fit 1 in the Table I of Ref. [8]. The decoupling of $\sigma(600)$ with the $K\bar{K}$ system means also the decoupling of $\sigma(600)$ with $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ because σ_q results in $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 1/4$.

So, the CLEO experiment gives strong support in favour of the four-quark ($ud\bar{u}\bar{d}$) structure of the $\sigma(600)$ meson.

In the chirally symmetric model of the Nambu-Jona-Lasinio type the coupling constants of the pseudoscalar and scalar partners with quarks are equal to each other, i.e., $g_{s\bar{s}\eta_s} = g_{s\bar{s}f_{0s}}$, where $f_{0s} = s\bar{s}$. In approximation when the mass of the strange quark much less the mass of the charmed quark ($m_s/m_c \ll 1$) $F_{f_0} = F_{\eta'}$ [19] and we find from the Table (see the last line) that $g^2_{s\bar{s}f_0}/g^2_{s\bar{s}\eta_s} \approx 0.3$. So, the $f_{0s} = s\bar{s}$ part in the $f_0(980)$ wave function is near thirty percent. Taking into account the suppression of the $f_0(980)$ meson coupling with the $\pi\pi$ system, $g^2_{f_0\pi^+\pi^-}/g^2_{f_0K^+K^-} = 0.154$, see Fit 1 in the Table I of Ref. [8], one can conclude that the $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ part in the $f_0(980)$ wave function is suppressed also. So, the CLEO experiment gives strong support in favour of the four-quark ($sd\bar{s}\bar{d}$) structure of the $f_0(980)$ meson, too.

The CLEO collaboration selected only near 44 events of $f_0e^+\nu$ (near 90 efficiency corrected ones). So, urgently requires an experiment with higher statistics.

Of great interest is the experimental search for the decays $D^0 \to a_0^-(980)\,e^+\nu \to \pi^-\eta\,e^+\nu$ and $D^+ \to a_0^0(980)\,e^+\nu \to \pi^0\eta\,e^+\nu$ (or the charge conjugate ones), which will give the information about the $a_q^- = d\bar{u}$ (or $a_q^+ = u\bar{d}$) and $a_q^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ components in the $a_0^-(980)$ and a_0^0 wave functions respectively.

No less interesting is also search for the decays $D^+ \to [\sigma(600) + f_0(980)]\,e^+\nu \to \pi^+\pi^-\,e^+\nu$ (or the charge conjugate ones), which will give the information about the $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ components in the $\sigma(600)$ and $f_0(980)$ wave functions respectively.

Creating a super-b-factories will give opportunity to study the production of light scalar mesons in semileptonic decays of bottomonium, whose mechanisms are more transparent.

Table. Results of the analysis of the CLEO [13] data. All quantities are defined in the text.

$Br(D_s^+ \to f_0e^+\nu \to \pi^+\pi^-e^+\nu) = 0.17\%$			
$(F_\sigma g_{s\bar{s}\sigma})/(F_\sigma g_{s\bar{s}f_0})$	$(F^2_{f_0}g^2_{s\bar{s}f_0})/(F^2_\eta g^2_{s\bar{s}\eta})$	$(F^2_{f_0}g^2_{s\bar{s}f_0})/(F^2_{\eta'}g^2_{s\bar{s}\eta'})$	$(F^2_\eta g^2_{s\bar{s}\eta})/(F^2_{\eta'}g^2_{s\bar{s}\eta'})$
0.039	0.67	0.49	0.73
The $\eta - \eta'$ mixing			
θ_P	-11°	-14°	-18°
$(F^2_{f_0}g^2_{s\bar{s}f_0})/(F^2_\eta g^2_{s\bar{s}\eta_s})$	0.32	0.29	0.24
$(F^2_{f_0}g^2_{s\bar{s}f_0})/(F^2_{\eta'}g^2_{s\bar{s}\eta_s})$	0.27	0.28	0.31

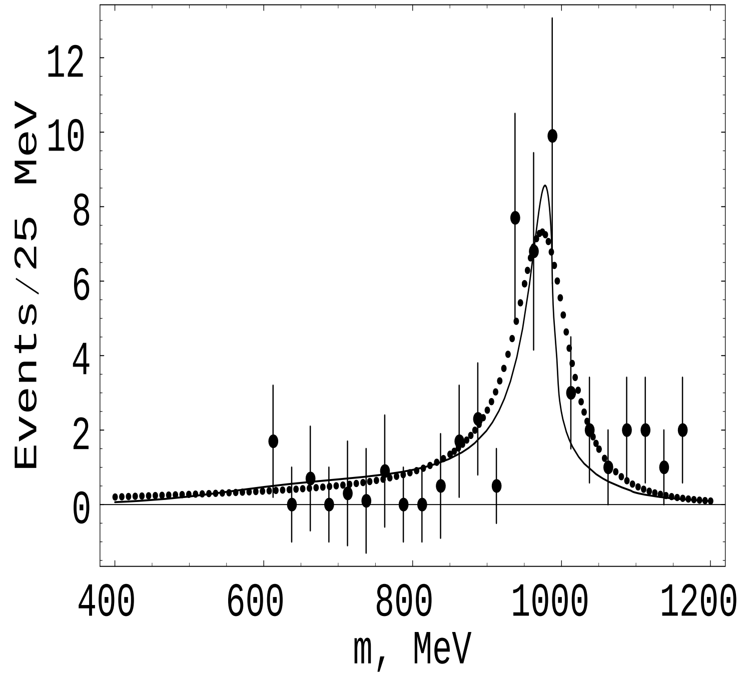

Figure 2: The CLEO data [13] on the invariant $\pi^+\pi^-$ mass (m) distribution for $D_s^+ \to \pi^+\pi^- e^+\nu$ decay with the subtracted backgrounds, which are calculated in Ref. [13]. The dotted line is Fit from Ref. [13], Fig. 9, corresponding to $BR(D_s^+ \to f_0(980)\, e^+\nu)\, BR(f_0(980) \to \pi + \pi^-) = (0.20 \pm 0.03 \pm 0.01)$. Our theoretical curve is the solid line.

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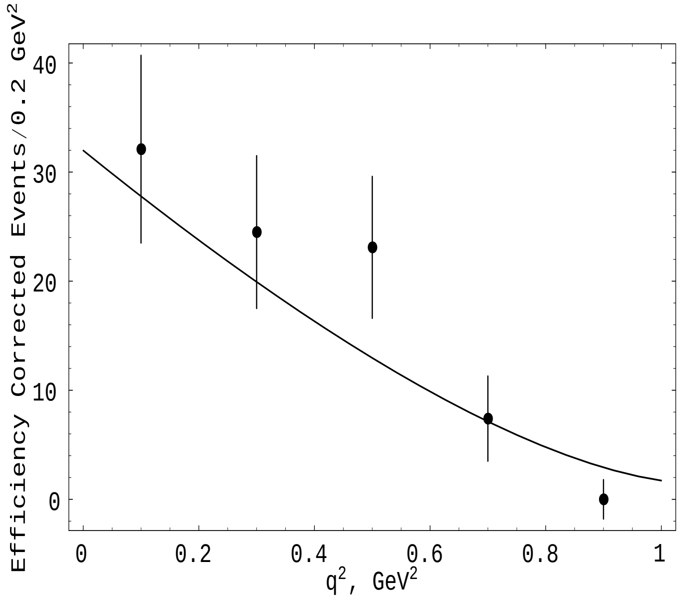

Figure 3: The q^2 distribution for $BR(D_s^+ \to f_0(980)\, e^+\nu)$. The axial-vector dominance model, see Eq. (4), describes the CLEO data [13] quite satisfactorily.

[11] In particular, the was shown that the ideal $q\bar{q}$ model prediction $g^2_{f_0(980)\gamma\gamma} : g^2_{a_0^0(980)\gamma\gamma} = 25 : 9$ is excluded by experiment in contrast to the simalar prediction for the tensor states $f_2(1270)$ and $a_2(1320)$. We mean here the $f_0(980) = (u\bar{u}+d\bar{d})\sqrt{2}$ and $a_0^0(980) = (u\bar{u}-d\bar{d})\sqrt{2}$ case for equality of the masses: $m_{f_0} = m_{a_0^0}$.

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[18] The decoupling of the $\sigma(600)$ with $\sigma_s = s\bar{s}$ is the practically model independent conclusion.

[19] The study beyond this approximation we hope to carry out subsequently.