Two-component liquid model for quark-gluon plasma

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Abstract

We consider a two-component liquid model, a la Landau, for the quark-gluon plasma. Qualitatively, the model fits well some crucial observations concerning the plasma properties. Dynamically, the model assumes existence of an effective scalar field which is condensed. Existence of such a condensate is supported by lattice data. We indicate a possible crucial test of the model in the lattice simulations.

1 Introduction

Discovery of the strongly interacting quark-gluon plasma at RHIC [1] made a great impact on landscape of theoretical papers devoted to quantum chromodynamics. There emerged a new problem of explaining the exotic properties of the plasma observed. It is the same fundamental and interesting as the confinement problem and in fact the two problems are to be considered in conjunction with each other. Moreover, there is renewed interest in relativistic hydrodynamics, superfluidity and, more generally, in applying the holographic methods to condensed-matter systems [3].

In this paper we consider a possibility that a variation of the famous two-component model of superfluidity applies directly to the quark-gluon plasma [2].

In section 2 we summarize briefly basic properties of the plasma and argue that the model explains naturally these observations. In Section 3 we overview the lattice evidence in favor of existence of a condensate of a scalar field. In section 4 we propose a crucial test of the model through measuring correlator of components of energy-momentum tensor.

2 Qualitative features

It might be useful (for the purpose of model building) to reduce the plasma properties to three points, namely, equation of state, viscosity and the role of quantum effects.

A. Existence of the plasma was conjectured long time ago. Moreover the equation of state of the plasma has been known also since long since it was established via numerical experiments within the lattice formulation of QCD, for references see, e.g., [1]. It turns out that the equation of state is close to that of the ideal gas of quark and gluons:

$$[\epsilon(T)]_{plasma} \;\approx\; [\epsilon(T)]_{ideal\ gas}\big(1\;-\;\delta\;\big)\;, \tag{1}$$

where the correction $\delta \approx 0.15$, $\epsilon(T)$ is the energy density as function of temperature and $[\epsilon(T)]_{ideal\ gas}$ is the energy density for noninteracting quarks and gluons.

[1] For details, discussions and references see, e.g.. reviews [1].

[2] The basic idea is the same as in our unpublished report [2]. Here, we extend the argumentation and address the issue of crucial tests of the model.

Thus, the equation of state indicates that the plasma is close to an ideal gas.

B. The observation (1) produces illusion of simplicity of the properties of the plasma. However, analysis of the data obtained at RHIC led to the conclusion that the plasma possesses the lowest viscosity η among all the substances known so far:

$$\left(\frac{\eta}{s}\right)_{plasma} \approx \frac{1}{4\pi}, \tag{2}$$

where s is the entropy density (introduced to measure the viscosity in dimensionless units). The value of $1/4\pi$ is somewhat symbolical and is quoted for the purpose of memorizing the data. The actual value of η might be larger, say $\eta/s \sim 0.4$ [1] or even lower, see [4]. The value $\eta = 1/4\pi$ represents the conjectured lower limit [5].

Anyhow, the viscosity observed for the plasma is the lowest one among all the known liquids [1]. Thus, measurements of the viscosity indicate that the plasma is close to an ideal liquid (which is defined as having $\eta = 0$). Note that for the ideal gas the viscosity tends to infinity,

$$\left(\frac{\eta}{s}\right)_{ideal\ gas} \rightarrow \infty\ . \tag{3}$$

More precisely, this ratio is inverse proportional to the coupling constant squared $\eta/s \sim 1/\alpha_s^2$.

C. As a kind of variation of the point **B**, one argues [5] that such a low value of viscosity implies that the quantum effects are crucial and the liquid cannot be, rigorously speaking, treated classically. Indeed, basing on the estimates common to kinetics one readily finds that

$$\eta/s \sim k_B^{-1}\tau_{relaxation}(\epsilon/n)\ ,$$

where k_B is the Boltzmann constant, τ is the relaxation time, ϵ is the energy density and n is the density of particles. ¿From the uncertainty principle, the product of energy of a particle, ϵ/n times its free time, *tau* cannot be smaller that the Planck constant. Thus:

$$\frac{\eta}{s} \sim \frac{\tau_{relaxation}}{\tau_{quantum}}\ , \tag{4}$$

where the "quantum time" $\tau_{quantum} \sim \bar{h}/k_BT$. Then the observation (2) implies quantum nature of the QGP.

It is a challenge to theory to explain all the three observations, (1), (2), (4) which are apparently showing in the opposite directions. Indeed, one starts with the idea that the plasma is an ideal gas and ends up with a kind of a proof that the plasma is a quantum liquid.

It is amusing that it is quite straightforward to suggest a model which allows–on a qualitative level– to unify all the would-be contradictory features of the plasma [2]. We have in mind the two-component model of superfluidity as formulated by L.D. Landau.

Indeed, what is 'special' about the viscosity? How is it possible to have the equation of state close to that of the ideal gas and, still, nearly vanishing viscosity? Let us imagine that we are dealing with a two-component substance. One of the components occupies larger phase space, c_1 and is responsible for the equation of state. The other one has smaller phase space, c_2 but very small viscosity. Then the total viscosity can be small since, at least naively, to evaluate the total viscosity one adds inverse power of the partial viscosities:

$$\frac{1}{\eta_{tot}} = \frac{c_1}{\eta_1} + \frac{c_2}{\eta_2}\ , \tag{5}$$

where $c_{1,2}$ are normalized by $c_1 + c_2 = 1$ Indeed, the meaning of the viscosity η is similar to that of resistance and if we have two independent motions then we would apply the rule (5) [3].

[3]Equation (5) can be found in, e.g., in old books on classical solutions [6]. In more modern terms, the example of the superfluidity itself might serve as the best illustration to (5). Indeed, the superfluid fraction can be small while the whole liquid is superfluid. On more detailed level, some care should be exercized since one has distinguish between viscosity with respect to a capillar motion and with respect to rotations, for a recent exposition see, e.g., [13].

Thus, the two-component model accommodates naturally points **A, B** above. Assuming one of the components be superfluid explains, as a bonus, the point **C** as well.

Another point is worth emphasizing. In the non-relativistic case the superfluid component evaporates at finite temperature T_c. The physics behind is readily understood. Indeed, at $T = 0$ the superfluid component is related to the condensate of particles with momentum $\mathbf{p} = 0$. At non-vanishing temperature the particles excited by temperature. Because of the conservation of the number of particles in the non-relativistic case, the superfluid component disappears at finite temperature.

In the relativistic case, that is in the absence of conservation of the particles, the theoretical constraints on the phase space occupied by the superfluid component are weaker. Indeed, even at $T \to \infty$ the non-perturbative component in case of Yang-Mills theories vanishes only logarithmically:

$$\lim_{T\to\infty} c_2(T) \sim g_s^6(T) \sim \frac{1}{(\ln T)^3} , \tag{6}$$

where $g_s^2(T)$ is the coupling of the original 4d theory.

3 Scalar condensate

3.1 General constraints

Dynamically validity of the superfluidity scenario depends strongly on the existence of an (effective) scalar field condensed the thermal vacuum

$$\langle\varphi\rangle_{ground\ state} \neq 0 . \tag{7}$$

The phase of this condensate corresponds then to a new light degree of freedom.

The condition (7) looks very restrictive and, in more detail, assumes a number of constraints:

a. The field φ is a complex field:

$$\varphi^* \neq \varphi .$$

b. Nevertheless the condensate (7) should not violate conservation of any known quantum number, like charge.

c. In case of superfluidity, one is to think rather in terms of a *three-dimensional* field $\varphi\mathbf{r}$ while its time derivative is determined by the chemical potential μ :

$$\partial_t\varphi = \mu . \tag{8}$$

Generalizations of (8) to the case of relativistic plasma can be found in [7]. The chemical potential μ is conjugate to the charge which distinguishes the field φ.

3.2 Thermal scalar

If we consider the conditions a)-c) above in an abstract form, they look very difficult to satisfy. It is then even more amusing that a 3d field with similar properties arises naturally [8] within the string approach to the deconfinement phase transition and is commonly called thermal scalar, for a concise review and further insights see [9] .

One considers temperatures T below and close to the temperature of the phase transition T_c. In the string picture $\beta_H \equiv 1/T_c = 1/\alpha'$ where $(2\pi\alpha' \equiv l_s^{-2}$ is the string tension. At $T = T_c$ the statistical sum over the states diverges. The main observation is that at small $|T - T_c|$ the sum is dominated by contribution of single degree of freedom, that is scalar meson with mass

$$m_\beta^2 \approx \frac{\beta_H(\beta_H - \beta)}{2\pi^2(\alpha')^2} , \tag{9}$$

In other words, at $T = T_C$ the mass is becoming tachyonic.

In more detail, it is convenient to use the polymer approach to field theory of a scalar particle so that the action associated with a trajectory of length L is $S = M \cdot L$ where M is the bare mass. The trajectories are random walks with renormalized mass. The free energy of the thermal scalar can be represented as a sum over random walks and the final expression reduces to:

$$F = = \beta \ln Z = = \beta \int_0^\infty \frac{dL}{L} \frac{\exp(-m_\beta^2 l_s L)}{(l_s L)^{d/2}} \quad , \tag{10}$$

where d is the number of spatial coordinates, in our case $d = 3$. Expression (10) is quite generic to the polymer approach. A specific feature of (10) is that l_s plays the role of the length of the links and is fixed in terms of the string tension.

The crucial point is that the free energy of the thermal scalar is exactly the partition function for a single static string with tension $1/2\pi\alpha'$. Moreover, the single string dominates the free energy of gas of strings.

3.3 Three dimensional scalar at $T > T_c$

What happens to the thermal scalar at $T > T_c$ is an open question. In particular, it could condense. Such a scenario is typical for the percolation picture. The basic features can be understood from Eq. (10). At $m_\beta^2 = 2$ the exponential suppression of very large lengths L disappears. However, the integral over L is still divergent in the ultraviolet, not in the infrared. This means that small clusters with $L \sim l_s$ dominate. The probability of having infinite length is suppressed by a power of L at $L \to \infty$. For a tachyonic mass there emerges an infinite cluster. However, its density is suppressed as a power of $|m_\beta^2$ and small for temperatures above and close to T_c. In field theoretic language appearance of the infinite cluster means condensation of the field, $< \phi > \neq 0$.

Imagine that the thermal scalar is indeed condensed at $T > T_c$. Then, remarkably enough, the conditions we formulated above are satisfied. Indeed,

a) The thermal scalar is a complex field. It is encoded in the fact that the integration in (10) is over closed loops which means a complex field in the polymer language.

b) The thermal scalar is associated with topological quantum number which is a wrapping around the compactified time direction (due to finite temperature).

c) The thermal scalar is a 3d scalar field, as it follows from the representation (10).

Nowadays, it is common to consider dual models of Yang-Mills theories in terms of strings living in extra dimensions with non-trivial geometry. The thermal scalar at temperatures below and close to T_c is generic to such models as well, see [9] and references therein. One would not claim, however, that the most naive version of the condensation of the thermal scalar is realized within this scenario. Rather, the phase transition is a change of geometry in the extra dimensions.

However, the scalar fields at $T > T_c$ are resurrected in another disguise. Namely, one predicts existence of defects of various dimensions, see in particular [10]. At $T > T_c$ the models predict existence of time-oriented strings. Their 3d projection then looks as trajectories and correspond indeed to scalar 3d particles. There are independent lattice data which seem to support the validity of this prediction [11].

To summarize, there is strong evidence that at $T > T_c$ there exists an effective 3d scalar field condensed in the thermal vacuum of QCD. Existence of such a scalar is a necessary condition for the validity of the two-component model.

4 Possible crucial test of the model

The considerations given above demonstrate that the two-component model of the quark-gluon plasma does not contradict existing data. One cannot claim, however, that the model is indeed validated by the data.

A crucial test of the model could performed through lattice measurements of a correlator of components of the energy-momentum tensor $T^{ti}, i = 1, 2, 3$. In more detail, consider the retarded Green's function defined as:

$$G_R^{tj,ti}(k) \equiv i \int d^4 x e^{-ikx} \theta(t) \langle [T^{tj}(x), T^{ti}(0)] \rangle \quad . \tag{11}$$

Moreover, concentrate on the case of vanishing frequency, $k_0 = 0$. There are two independent form factors, corresponding to transverse and longitudinal waves.

$$G_R^{tj,ti}(0, \mathbf{k}) = \frac{k^i k^j}{\mathbf{k}^2} G_R^L(\mathbf{k}) + \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right) G_R^T(\mathbf{k}) \tag{12}$$

Contribution of the superfluid component to the $G_R^{L,T}$ has been discussed in many papers and textbooks. Here, we quote the result of the paper [12] which includes also relativistic corrections:

$$\lim_{\mathbf{k} \to 0} G_R^T(\mathbf{k}) = -(sT + \mu\rho_n), \qquad \lim_{\mathbf{k} \to 0} G_R^L(\mathbf{k}) = -(sT + \mu\rho), \tag{13}$$

where s is the entropy density, T is the temperature, μ is the chemical potential, $\rho \equiv \rho_n + \rho_s$ is the total density, while ρ_n and ρ_s are the densities of the normal and superfluid components, respectively. In an alternative language:

$$\lim_{\mathbf{k} \to 0} G_R^{tj,ti}(0, \mathbf{k}) = \mu \rho_s \frac{k^i k^j}{\mathbf{k}^2} \tag{14}$$

In other words, it is only the superfluid component which results in non-analyticity at small $\mathbf{k}$.

Note that the proposed crucial test of the model (14) refers to static quantities, $k_0 = 0$. Since there is no time (or frequency) dependence, the continuation from the Euclidean to Minkowski space is straightforward and the prediction of the model, $\rho_s \neq 0$, can be tested on the lattice.

5 Conclusions

It is amusing that known qualitative features of the quark-gluon plasma seem to favor a two-component model of superfluidity for the plasma. In terms of field theory, the model implies condensation of an effective 3d scalar field. This consequence of the model seems to be supported by the lattice data as well.

A crucial test of the model could be performed through search of non-analyticity in the correlator of components of the energy-momentum tensor on the lattice. can be tested directly in the lattice simulations of Yang-Mills theories.

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