Imagery of Symmetry in Current Physics

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Dedicated to the memory
of Albert Tavkhelidze

Instead of Abstract

This text, mainly, summarizes the talk delivered at the Conference "Quarks 2010"; its essence concerns funny duality of symmetry that is broken under phase transition corresponding to the superconductivity and superfluidity, namely, in the semi-phenomelogical description a la Ginzburg-Landau this is wine-bottle rotation symmetry, while in the quantum Bogoliubov theory it is phase symmetry responsible for number of particles (helium atoms, Cooper electron pairs) conservation. This duality is interesting in the context of the logic to intuition (Science to Art) contraposition.
We conclude with a short account on some other aspect of distorted symmetry – the aspect related to varying space-time geometry and, particularly, to dimension reduction.

Albert – man of Idea and Action:

1st episode with New Quantum Number for Quarks introduced by Bogoliubov, Struminsky and Tavkhelidze in 1965. Active Propaganda of this Number in his ICTP 1965 lectures.
Compare with calm attitude of C.N.Yang after the CP-paper (as recalled by F. Dyson) vs. T.D.Lee agitation that forced Chien-Shiung Wu to start the Co^{60} *experiment (according to the story recollected by Ta-You Wu).*

2nd episode: Initiation of the 1971 research with Bogoliubov and V.Vladimirov on compatibility of the self-similarity in the HE Bjorken region with principles of Local QFT.

Other episodes with Albert activity in organizing the Kiev Institute for Theoretical Physics, Moscow Institute for Nuclear Research, Journal of Theoretical and Mathematical Physics and the 1st International conference on Theoretical and Mathematical Physics in 1972.

1 Phase transition and broken symmetry

Connection between Phase transition and symmetry breaking became evident before the QM creation from physics of crystals.

Noteworthy, the famous Landau 1937 paper[1] on the phase transition theory started by pedagogical introduction of symmetries but *only discrete symmetries*; in the paper's conclusion one can find arguments against an idea that superfluid He II could be a liquid crystal. No mention on continuous symmetry !

Meanwhile, in current physics of phase transitions the continuous symmetries are dominating. The break-through from discrete to continuous symmetry was first made by Bogoliubov in his paper[2] on the microscopic theory of superfluidity.

Spontaneous symmetry breaking (SSB) is a well-established term in physical theory; its essence is simple. One has in mind a physical system that can be described by expressions (Lagrangian, Hamiltonian, equations of motion) obeying some symmetry, while a real physical state of the system corresponding to some partial solution of the equations of motion does not obey this symmetry. One meets such a case when the lowest of possible symmetrical states does not provide the system with absolute energy minimum and turns out to be unstable. A particular lowest state is not unique; a full collection of them forms a symmetric set. The real cause of symmetry breaking and transition of the system to some of the lowest non-symmetrical states usually turns out to be an arbitrary small asymmetrical perturbation.

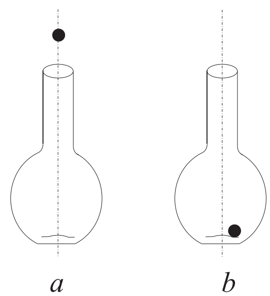


Figure 1: Simple mechanical system illustrating spontaneous symmetry breaking: (a) symmetric initial state; (b) asymmetric final state.

As a simple illustration take a system of an empty vessel(flask) of a convex bottom and a tiny massive ball. Let the vessel, which is a figure of revolution, stand vertically and the ball be located above it, just on the axis (Fig.1a) of symmetry. The system is invariant with respect to rotation around the vertical axis. Let the ball fall down due to the force of gravity. Upon reaching the bottom, the ball will not stand at the center of convex surface and will roll down to some point at the bottom periphery (Fig.1b). Thus, the initial conditions are symmetrical, while the final state is not.

1.1 Symmetries, groups and quantum symmetries

Symmetries and groups, discrete and continuous, are of wide use in theoretical physics.

Continuous groups, Lie group of transformations, are usually formulated within the Hamiltonian or Lagrangian framework. In the second case, from the Lagrangian invariance with the help of the Nöther theorem one obtains conservation laws which are physically important.

In modern physics, along with "natural transformations" and symmetries (like from the Lorentz group) some other, not so obvious and pictorial symmetries, are of utmost importance. Among them, we single out a group of *Quantum Symmetries*. They are quite different from "classical", like spatial (boost, rotation, Lorentz) and internal (isospin, flavor) ones. Quantum Symmetries include phase, gauge, chiral and Super symmetries. The defining feature is that for their formulation and understanding one has to use quantum notions :

* nonobservability of the ψ-function phase;
* spin, chirality ;
* distinction between Bose– and Fermi–statistics.

1.2 Superfluidity

The original explanation of the phenomenon of superfluidity offered by Landau[3] was based on the idea that at low temperatures the properties of liquid ^{4}He are defined by collective excitations (phonons) rather than a quadratic spectrum of individual particle excitations[1]. The need for agreement between the spectrum form and the thermodynamic properties of liquid helium motivated Landau to introduce particular excitations, in addition to phonons, with a quadratic spectrum beginning with a certain energy gap, excitation, which he called rotons[2].

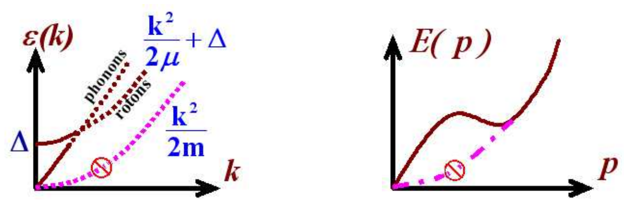


Figure 2: (a) Phonon + roton spectra – Landau 1941 phenomenology; (b) Spectrum of non-ideal Bose-gas in the Bogoliubov 1946 microscopical model.

Bogoliubov's theory of superfluidity is based on a physical assumption that in a weakly nonideal Bose gas there is a condensate akin to an ideal Bose gas. The existence of the Bose condensate leads to a common wave function of the whole system, i.e., collective effect. Therefore, the presence of even a weak interaction transforms single-particle excitations ($\sim k^2/2m^2$) into the spectrum of collective excitations. To calculate this spectrum, Bogoliubov assumed[2] that at low temperatures the Bose condensate contains a macroscopically large[3] (of an order of Avogadro number N_A) number of particles N_0 as a result of which matrix elements of the creation and annihilation operators of the condensate particles are proportional to "large" number $\sim \sqrt{N_0}$, and the main contribution to the system dynamics comes from the processes of particle transition from the condensate to the continuous spectrum and back to the condensate.

The Bogoliubov 1946 theory starts with quantum Hamiltonian for a non-ideal Bose gas

$$H_{\mathrm{B-gas}} = \sum_{\vec{p}} \frac{p^2}{2m} a_p^+ a_p + \frac{1}{2V} \sum v(p_1 - p_2) a_{p_1}^+ a_{p_2}^+ a_{p_2} a_{p_1}$$

with weak repulsion $v(p) > 0$. This Hamiltonian is invariant with respect to phase transformation of creation and annihilation operators

$$a^+ \to e^{i\alpha} a^+ , \quad a \to e^{-i\alpha} a, \tag{1}$$

which is related to the number of particle conservation, as $H_{\mathrm{B-gas}}$ commutes with the number of particle operator $\mathbf{N} = \sum_{\vec{p}} a_p^+ a_p$.

The Bogoliubov physical hypothesis on "macroscopic condensate" $\mathbf{N_{p=0}} = a_0^+ a_0 \sim N_A$ leads to Corollary: condensate operators can be changed for big c-numbers: $a_0^+, a_0 \to \sqrt{N_0}$. At the same time, it allows one to single out a big constant $\Psi_0 \sim \sqrt{N_0}$ from the psi-function $\Psi(x) = \Psi_0 + \psi(x)$ in the position picture.

Expansion in powers of small parameter $1/\sqrt{N_0}$ yields $H_{B-gas} = E_0 + H_{Bog} + ...$ with condensate energy E_0 and H_{Bog} - the Bogoliubov Hamiltonian

$$H_{Bog} = \sum_{p \neq 0} \left(\tfrac{p^2}{2m} + \tfrac{N_0}{V} v(p) \right) b_p^+ b_p + \tfrac{N_0}{2V} \sum_{p \neq 0} v(p) [b_p^+ b_{-p}^+ + b_p b_{-p}] \, . \tag{2}$$

[1]It follows from this assumption that in moving with velocity not exceeding a certain critical one it is impossible to slow down the liquid by transferring energy and momentum from the wall to individual atoms because a linear form of the phonon spectrum does not allow one to obey simultaneously the laws of energy and momentum conservation.

[2]See below Fig. 2(a) in which formulae (2.2) and (2.3) from paper [3] are used.

[3]Bogoliubov's intuitive guess got later a direct data support – see papers [4].

Here, $b_p^+ = a_p^+$, $b_p = a_p$; $(p \neq 0)$ – the "above-condensate" Bose-operators. The second sum describes creation of pairs of Helium atoms with opposite momenta from condensate and their "annihilation" into condensate. Interaction between pairs being small $\sim N_0^{-1/2}$ is omitted. Total *number of these correlated pairs is not fixed.*

The bilinear operator form H_{Bog} can be diagonalized by linear transformation

$$b_p \to \xi_p ; \quad \xi_p = u_p b_p + v_p b_{-p}^+; \quad \xi_p^+ = u_p b_p^+ + v_p b_{-p} \tag{3}$$

with real coefficients $u_p^2 - v_p^2 = 1;\ u_{-p} = u_p;\ v_{-p} = v_p$.
These algebraic relations for operators, in the second quantization language, are realized by unitary transformation

$$b_p \to \xi_p = U_\alpha^{-1} b_p\, U_\alpha = u_p b_p + v_p b_{-p}^+ , \dots ; \quad U_\alpha = e^{\sum_p \alpha(p)\, [b_p^+ b_{-p}^+ - b_p b_{-p}]} . \tag{4}$$

that corresponds to the new ground state

$$\Phi_0 \to \Psi_{Bog} = U_\alpha^{-1} \Phi_0 \sim e^{-\sum_p \alpha(p)\, b_p^+ b_{-p}^+}\, \Phi_0 \tag{5}$$

which contains superposition of correlated pairs of Helium atoms with the total zero momentum.

Bogoliubov spectrum instead of the Landau one. The $b_p \to \xi_p$ transformation (3) correlates pairs of He II atoms with opposite momenta. Transformed Hamiltonian

$$H_{Bog2}(\xi) = \sum_{p \neq 0} E(p)\, \xi_p^+\, \xi_p , \quad E(p) = \sqrt{(T(p))^2 + T(p)\, v(p)}; \qquad T(p) = \tfrac{p^2}{2m} \tag{6}$$

describes new collective excitations [bogolons], as presented on Fig.2b.
Note that the Bogoliubov (u, v) transformation (3) as well as the new ground state Ψ_{Bog} and Hamiltonian (6) are incompatible with initial phase symmetry (1).

1.3 Superconductivity

Phase transition with Symmetry Breaking. In the semi-phenomenological Landau theory of phase transitions[1] one deals with a set of c-functions, the set that forms so-called *order parameter* which vanishes above the critical temperature T_c . A simple example is provided by the case of ferromagnetic. There, one has correlation function $K_{\sigma\sigma}(\mathbf{r})$; its asymptotics

$$K_{\sigma\sigma}(\mathbf{r}) =< \sigma(\mathbf{0})\sigma(\mathbf{r}) > - < \sigma(\mathbf{0}) >< \sigma(\mathbf{r}) > ; \quad \mathbf{K}_{\sigma\sigma}(\mathbf{r} \to \infty) = \begin{cases} 0, & T > T_c \\ M^2(T), & T < T_c \end{cases}$$

provides one with one-component order parameter $M(T)$ explicitly shown on Fig.3.

The Ginzburg-Landau SuperConductivity model exploits two-component order parameter $\Psi(r) = |\Psi(r)| e^{i\Phi(r)}$ for a system of superconducting electrons. It was introduced [5] by an *ad hoc* definition of a free energy functional for superconducting transition; in the G-L theory [1950], the electromagnetic properties of a superconductor are deduced from a functional dependence on $\Psi(\mathbf{r})$ and external magnetic field $\mathbf{B}(\mathbf{r})$:

$$F(\Psi) = F_{n0} + \int d\mathbf{r} \left\{ \tfrac{|\mathbf{B}|^2}{8\pi} + a|\Psi|^2 + \tfrac{1}{2} b|\Psi|^4 + \sum_\alpha \tfrac{1}{2m^*} \left| \left(-i\hbar\nabla_\alpha - \tfrac{q}{c} A_\alpha \right) \Psi(\mathbf{r}) \right|^2 \right\} , \tag{7}$$

where F_{n0} is the free energy in a normal state, $\mathbf{B} = \mathrm{rot}\, \mathbf{A}$. The crucial point is that one of the adjusting parameters *changes the sign* at the Curie point $a \sim T - T_c$, while the other b , as well as the effective charge q and mass m^* of superconducting (SC) electrons, are positive

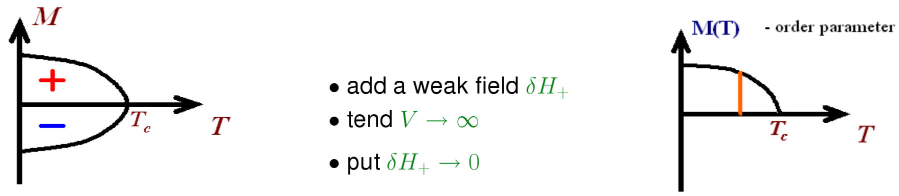


Figure 3: Temperature dependence of the magnetization (left) and order parameter (right) in ferromagnetic.

constants. The last two enter into the SC current $j_\alpha = \frac{q\hbar}{m^*}\,|\Psi|^2\,\nabla_\alpha \Phi$. The structure of potential terms

$$V(\varphi) = a\,\varphi^2 + \tfrac{b}{2}\,\varphi^4\,, \quad \varphi = |\Psi| \tag{8}$$

relates to the geometry of a flask bottom similar to that one presented on Fig.1. The case $T > T_c$; $a > 0$ relates to the concave profile of the bottom. Here, the value $\Psi = 0$ provides the stable minimum for V. For $T < T_c$; $a < 0$ we have an opposite situation with a convex bottom that corresponds to the particular flask presented on Fig.1.
Hence, the symmetry that is broken in the phenomenological (macroscopical) L-G theory is the *rotation symmetry.*

Meanwhile, at the microscopical quantum level (BCS and Bogoliubov models of superconductivity) the broken symmetry is the *phase symmetry* (1), just like in the Bogoliubov superfluidity. Turn to superconductivity.

The BSC SuperConductivity is based on the model quantum Hamiltonian :

$$H_{BCS} = \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}}\, c^+_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{k},\vec{k}'} V_{\vec{k},\vec{k}'}\; c^+_{\vec{k}\uparrow} c^+_{-\vec{k}\downarrow} c_{-\vec{k}'\downarrow} c_{\vec{k}'\uparrow}\,, \quad \varepsilon_{\vec{k}} = \frac{\vec{k}^2}{2m} - \varepsilon_F \tag{9}$$

with $\varepsilon_{\vec{k}}$ being electron energy above the Fermi one ε_F and effective Cooper (antipodes) electron pairs attraction acting in the narrow vicinity of the Fermi surface

$$V(\vec{k},\vec{k}') = -\,V_C\,, \text{ only at } |\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}| < \omega_{ph}\,; \qquad = 0 \quad \text{overwise}\,.$$

The *phase symmetry* is consistent with expression (9). However, it *is absent in the BCS wave function* which forms the new ground state

$$|\psi_{BCS}>= \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} c^+_{\vec{k}\uparrow} c^+_{-\vec{k}\downarrow})|0>= |0>_{BCS}\,, \tag{10}$$

and contains superposition of Cooper pairs.
Omitting a lot of important physical results remind only that the Landau-Ginzburg order parameter can be presented [Gor'kov, 1959] via the average of the BCS operators

$$< c^+_{\vec{k}\uparrow} c^+_{-\vec{k}\downarrow} >_{BCS}= \Psi(\vec{k}) = |\Psi(\vec{k})| exp[i\Phi(\vec{k})] \tag{11}$$

with an important property $|\Psi|^2 \sim n_s$.

Besides, the gap Δ in the resulting energy spectrum $E(\mathbf{k}) = \sqrt{\varepsilon^2(\mathbf{k}) + |\Delta|^2}$ is expressed in terms of an "effective phonon energy" ω_{ph} and the BCS coupling constant

$$\Delta \approx\, exp\left(-\tfrac{1}{\lambda}\right)\,, \quad \lambda = N_0\, V_C\,. \tag{12}$$

The Bogoliubov microscopical theory of superconductivity[6] starts with the Fröhlich electron-phonon interaction:

$$H_{Fr} = \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}} c^+_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{q}} \omega_{\vec{q}}\, b^+_{\vec{q}} b_{\vec{q}} \,+\, g_{Fr} \sum_{\vec{k},\vec{k}',\sigma} \sqrt{\frac{\omega(\vec{q})}{2V}}\, c^+_{\vec{k}\sigma} c_{\vec{k}'\sigma} (b^+_{\vec{q}} + b_{-\vec{q}}) \tag{13}$$

Again, the Hamiltonian obeys the *phase symmetry.* However, the superconducting solution obtained with the help of (u, v) transformation (suitably adjusted to the fermion case)

$$\alpha_{\mathbf{k}\uparrow} = u_k\, c_{\mathbf{k}\uparrow} - v_k\, c^{\dagger}_{-\mathbf{k}\downarrow}\ , \quad \alpha_{\mathbf{k}\downarrow} = u_k\, c_{-\mathbf{k}\downarrow} + v_k\, c^{\dagger}_{\mathbf{k}\uparrow}\ ; \quad u_k^2 + v_k^2 = 1\,, \tag{14}$$

is not compatible with phase symmetry.

We conclude this part of history that started under the XXth mid-Century motto *"Phase transition in Quantum system, as a rule, is accompanied by Spontaneous Symmetry Breaking"* and can be summarized in a few lines

1. The microscopic BCS-Bogoliubov SuperConductivity was shown (Bogoliubov, [7] 1958) to be SuperFluidity of Cooper pairs;

2. The Superfluid and Superconducting transitions in quantum microscopic theory are escorted by Spontaneous Symmetry Breaking of phase ("gauge") symmetry (1) (that is rotation in the complex plane), related to the number of particle conservation;

3. This is in contrast with the macroscopic phenomenology of Ginzburg-Landau type where the breaking symmetry is more pictorial and can be visualized by Fig.1

with Message to XXI:

I. In QFT the Higgs field can be under suspicion of being a formal replica of the Ginzburg-Landau order parameter from the theory of superconductivity.

The Higgs mechanism is an *ad hoc* pragmatic construction with no physics under it.

In the context of transferring the SSB mechanism from quantum statistics to QFT *one should remind an important contribution by Tavkhelidze et al.[8] first mentioned by Bogoliubov at the 1960 Rochester conference. (For details, see Ref.[9]).*

II. Generally, there is no one-to-one correspondence between physical phenomenon with phase transition and its theoretical implementation with symmetry breaking;
Due to this, a general question arises
– What is the Symmetry of a physical system (in particular, symmetry involved in phase transition) ?
– Can it be formulated independently of models ?

Heretical form of the question :

– Does the symmetry exist only in the consciousness of theoreticians ?

"What is the Verity ?" = that is the question by Pilatus to Christ.

The modern analog:

What is the Symmetry ?

"Quid est symmetria ?"

The theoretical physics, is it Art or Science ?

For another aspect of the symmetry duality turn to the Reduction of Dimensions.

2 Reduction of Dimensions

2.1 Coupling running through the DR looking glass

Reduction of Dimensions, the transition in particular form 4D $\to$ 2D, was used in the 90s in HE Regge scattering (Aref'eva, Lipatov). In XXI, it got impetus in quantum gravity opening the way to (super)renormalizability.

We studied[10] the QFT coupling behavior for the $g\,\varphi^4$ model defined in both the 4D, 2D domains; the $\bar{g}(Q^2)$ evolutions being duly conjugated at a reduction scale $Q \sim M$.

Consider

$$L = T - V; \quad V(m,g;\varphi) = \frac{m^2}{2}\,\varphi^2 + \frac{4\pi^{d/2}\,M^{d-4}}{9}\,g\,\varphi^4\,; \qquad g>0$$

in parallel in both the 4D (d=4) and 2D (d=2).

Limit ourselves to the one-loop leading level for $\bar{g}$ corresponding to only diagram, the first correction to the 4-vertex function. Its contribution I stands in the denominator of the running coupling

$$\bar{g}(q^2) = \frac{g_i}{1 - g_i\,I\,(q^2/m^2, m_i^2)}\,. \tag{15}$$

Explicitly, in the UV limit $m^2 \ll q^2$; $\quad I_i^{[4]}(\kappa;\mu) \sim \ln\left(\frac{q^2}{m^2}\right)$ in the 4D region, and $I_i^{[2]}(\kappa;\mu) \sim C + \frac{m^2}{q^2}\ln\frac{q^2}{m^2}$ in the 2D region. Note that the first asymptote is rising, the second – decreasing.

Perform now the smooth Dimension Reduction (=DR) for the momentum picture by modifying metric $dk = d^4k \to d_M k \;= d^4k\,(1 + k^2/M^2)^{-1}$ in the Feynman integrand

$$I\left(\frac{q^2}{m^2}\right) \to \frac{i}{\pi^2}\int \frac{d_M k}{(m^2+k^2)[m^2+(k+q)^2]} = J(\kappa;\mu)$$

with $\kappa = q^2/4\,m^2$, $\mu = M^2/m^2$, $q^2 = \mathbf{q}^2 - q_0^2$.

Explicitly $\quad J_i^{[4]}(\kappa;\mu) \sim \ln\left(\frac{q^2}{m_i^2}\right)$; $\quad$ and $\quad J_i^{[2]}(\kappa;\mu) \sim \ln\left(\frac{4\,M^2}{m_i^2}\right) + \frac{M^2}{q^2}\ln\frac{q^2}{M^2}$
in the 4D region with $m^2 \ll q^2 \ll M^2$ and in the 2D region : $M^2 \ll q^2$; $q^2 \gg M^2$.

Again, the first (intermediate) asymptote is rising, the second – the very final one – is decreasing. Hence, the whole pattern of the coupling evolution changes drastically. The $\bar{g}(q^2)$ diminishes beyond DR scale tending to a finite value

$$\bar{g}_2(\infty) = \frac{g_M}{1 + g_M\,I_2(M^2/m^2)} < g_M\;:$$

The *DR imitates the UV fixed point for the $\bar{g}$ evolution.*

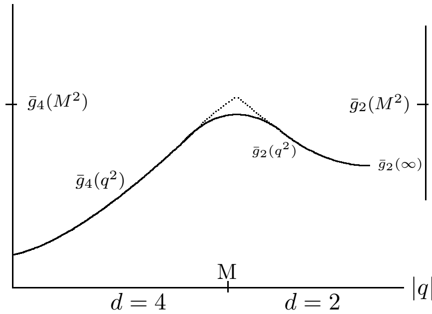
This gives a chance for the "Great Unification via DR Looking-Glass" as shown on the right panel of figure 4.

Resume of the DR hypothesis :

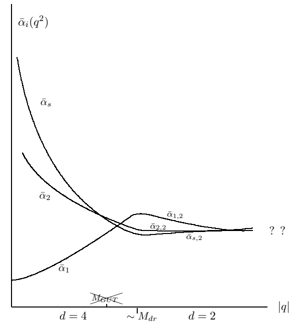
The main result is that DR imitates the UV fixed point for the $\bar{g}$ evolution.

A more general observation is that change of geometry could yield the same final result as an explicit change of dynamics (adding leptoquarks ...).

In order to estimate the possibility of a physical signal "from/through the DR looking-glass", we studied[11] few toy models of space with variable geometry.



The dotted lines corresponds to hard conjunction of (15) at the DR scale.

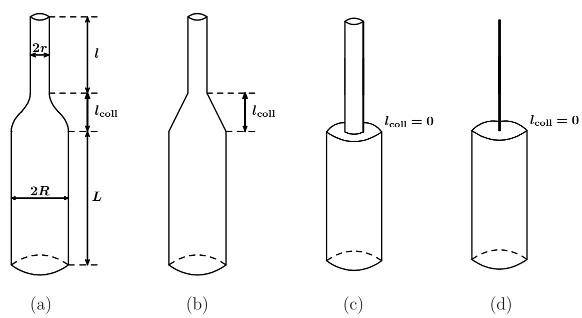


New brave Great Unification by DR instead of leptoquarks.

Figure 4: Running coupling for the $g\varphi^4$ model (left) and possible GUT scenario (right).

2.2 Toy models of the DR Looking-Glass

To get physical intuition and experience, we started with simple problem – Klein-Gordon scalar waves on the variable geometry manifolds, like 2-dim surface of the "bottles":



Resume of the DR hypothesis, 2 Preliminary resume of this study[11] is that there exists a possibility of detecting some signal "through the looking-glass at the DR scale" that would provide us with direct evidence on the existence of variable geometry including dimension reduction.

Acknowledgements

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