Towards a quantum theory of chiral magnetic effect

V. D. Orlovsky[a]*, V. I. Shevchenko[a]†
[a] *Institute of Theoretical and Experimental Physics*
B.Cheremushkinskaya 25, 117218 Moscow, Russia

Abstract

We discuss three possible ways of addressing quantum physics behind chiral magnetic effect and electric charge fluctuation patterns in heavy ion collisions. The first one makes use of P-parity violation probed by local order parameters, the second considers CME in quantum measurement theory framework and the third way is to study P-odd * P-odd contributions to P-even observables. In the latter approach relevant form-factor is constructed and computed for weak magnetic field in confinement region and for free quarks in strong field. It is shown that the effect is negligible in the former case. We also discuss saturation effect - charge fluctuation asymmetry for free fermions reaches constant value at asymptotically large fields.

1 Introduction

One of the main theoretical challenges of modern quantum chromodynamics (QCD) is to build a detailed theoretical picture of strong interaction physics relevant for heavy ion collisions. Currently running experimental programs have already brought lots of exciting results. Despite tremendous progress in understanding, rich pattern of observed effects is still waiting for being placed into coherent theoretical picture based on QCD.

In the course of studies of hadronic matter at large temperatures and/or densities one can make use of the scale separation allowing to neglect effects of weak and electromagnetic interactions in most cases. A possible interesting exception is pointed out in [1, 2]. When relativistic ions undergo noncentral collision, strong magnetic field is generated in the collision region. The typical magnitude of this field is estimated as $\sqrt{eB} = 10 \div 100$ MeV, i.e. of the order of dynamical QCD scale. Correspondingly, any studies of strongly interacting matter in heavy ion collisions have to take the effects of this abelian magnetic field into account. Of particular interest in this respect is the so called Chiral Magnetic Effect (CME). The physics behind it can be explained in several different but complementary ways [1]-[25]. Let us consider nonzero density of one flavor of free massless quarks in external magnetic field. Suppose there are unequal chemical potentials for left and right handed quarks: $\mu_L \neq \mu_R$. When it can be shown that a nonzero *classical* electric current flows along the magnetic field (see [8] and references therein, see also [26] for another prospective):

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B} \tag{1}$$

where $2\mu_5 = \mu_R - \mu_L$. The physical reason for this chiral charge excess to electric charge current conversion is quark magnetic moment interaction with the magnetic field (which is of different sign for positively and negatively charged quarks) together with the correlation of spin and momentum for chiral fermions. Both sides of (1) have of course the same transformation

***e-mail**: orlovskii@itep.ru
†**e-mail**: shevchen@itep.ru

properties under P- and CP-parity conjugation. Many different aspects of CME have been extensively discussed in the literature and there is no doubt that CME is a robust theoretical effect. However it is not a simple task to apply this clear physical picture to real processes described by nonperturbative QCD. One of the most important questions on this way is about physical origin of chiral chemical potential μ_5, which is absent in fundamental QCD Lagrangian. In original picture [7] appearance of effective $\mu_5 \neq 0$ is a nonperturbative QCD effect, caused by interaction of quarks with topologically nontrivial gluon field configurations *above* the phase transition. The physical explanation goes as follows. As is well known the topological charge in the QCD vacuum fluctuates as described by Veneziano-Witten formula [27, 28]

$$\chi = \int d^4x \langle G\tilde{G}(x) G\tilde{G}(0)\rangle \propto F^2 m_{\eta'}^2 \tag{2}$$

where the nonperturbative parameter in the r.h.s. scales as Λ_{QCD}^4 which means that topological charge fluctuates over Euclidean 4-volumes of typical size determined by nonperturbative QCD scale. It is worth stressing that these fluctuations are *quantum*, i.e. the states of different topological charge are to be summed over for whatever Euclidean 4-volume V and one always has

$$\int_V d^4x \langle G\tilde{G}(x)\rangle = 0 \tag{3}$$

In other words, (3) vanishes because local average $\langle G\tilde{G}(x)\rangle = 0$ and not due to the presence of integration over the volume V. There is no special space-time fluctuation pattern in the problem other than the correlator (2) (and higher ones).

The situation however may change at nonzero temperature/density. Since the Euclidean $\mathbb{O}(4)$ invariance of the vacuum is broken in this case, one can think of different fluctuation patterns in spatial and in temporal directions. Moreover, since in real collision experiments external conditions are time-dependent they can play a dual role of the background and of a measuring device. In other words the meaning of averaging in (3) changes: one has to integrate only over those field excitations which are present at a given Minkowski 3-volume for a given time period and the problem becomes essentially non-stationary in this sense. One can say that the average over fields $\langle .. \rangle$ becomes V-dependent. Such quantity - physically corresponding to a "single event" - can in principle be non-vanishing. Of course it is natural to expect that random character of fluctuations leads to zero result for (3) after averaging over many events.

The CME is often considered as a reasonable explanation of outgoing particles electric charge asymmetry observed at Relativistic Heavy Ion Collider (RHIC) [29] - [40] in $\sqrt{s_{NN}} =$ 200 GeV Au+Au and Cu+Cu collisions. The latter effect can be described as follows. For noncentral collision one can fix the reaction plane by two vectors: beam momentum and impact parameter (without loss of generality this is always chosen as 12 plane in the present paper and no adjustment angle Ψ_{RP} is introduced). Thus angular momentum of the beams (and the corresponding magnetic field) is oriented along the axis 3. The azimuthal angle $\phi \in [0, 2\pi)$ is defined in the plane 23. With this notation, in any particular event one studies charged particles distribution in ϕ using the following conventional parametrization

$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_{1,\pm}\cos\phi + 2v_{2,\pm}\cos 2\phi + 2a_\pm \sin\phi + ... \tag{4}$$

The coefficients $v_{1,\pm}$ and $v_{2,\pm}$ account for the so called directed and elliptic flow. They are believed to be universal for positively and negatively charged particles with good accuracy. The coefficients a_+ and a_- describe charge flow along the third axis, i.e. normal to the reaction plane. This P-parity forbidden correlation between a polar vector (electric current) and the axial one (angular momentum) is considered as a signature of P-parity violation in a given event with $a_\pm \neq 0$. On the other hand, the random nature of the process dictates $\langle a_+\rangle_e = \langle a_-\rangle_e = 0$ (there the averaging over events is taken).

Trying to construct a theory of the phenomenon one has first to choose adequate language. Since at the end the heavy-ion collision is a scattering problem, the ultimate framework would be S-matrix and inelastic scattering amplitudes formalism with two colliding ions as incoming particles. Due to extreme complexity this way seems to be totally hopeless. Instead one uses some effective theories like hydrodynamics to predict distribution of outgoing particles. In the particular problem of charge fluctuations asymmetry the crucial point distinguishing different theoretical models is whether the currents of interest are treated as classical or as quantum. In the former case one makes use of the expression (1) as classical equation. The quantum nature of the problem here is hidden in a theory for μ_5 and corresponding correlators and fluctuations for this effective chiral chemical potential. In the later case one is to consider quantum averages like $\langle\Omega|j_\mu|\Omega\rangle$, $\langle\Omega|j_\mu j_\nu|\Omega\rangle$ etc. and to understand (1) as operator relation. However if one takes diagonal matrix element of (1) in the vacuum the answer is of course trivial: $\langle 0|\mathbf{j}|0\rangle = 0$ even for nonzero external magnetic field. The absence of net electric current is directly related to the fact that fundamental QCD Lagrangian contains no such quantities as μ_L or μ_R.

We discuss three basic complementary ways to address quantum nature of CME in this paper:

1. To make use of P-parity violation probed by local order parameters

2. To consider CME in quantum measurement theory framework

3. To study P-odd $\times$ P-odd contributions to P-even observables.

We discuss all these approaches in the present paper and start with the first one in the next Section which is phenomenologically the simplest.

2 P-parity violation probed by local order parameters

As is well known quantum field theoretical averages of local operators have typically the following leading contribution:

$$\langle\Omega|\mathcal{O}(x)|\Omega\rangle \propto c \cdot \Lambda^{d_{\mathcal{O}}} \tag{5}$$

where Λ is ultraviolet cutoff and numerical constant c is generally non-vanishing if $c = 0$ is not protected by some symmetry. Therefore the crucial step in the discussed problem is to model transition from local microscopic current j_μ to nonlocal macroscopic one J_μ. It is done by taking the matrix elements of the current j_μ over the medium degrees of freedom $|\Phi\rangle$ from full state vector $|\Omega\rangle = |\Phi\rangle \otimes |\phi\rangle$:

$$j_\mu(x) = \bar{\psi}\gamma_\mu\psi(x) \leftrightarrow J_\mu \propto \langle\Phi| \int dx \rho_V(x) j_\mu(x)|\Phi\rangle \tag{6}$$

Here the function $\rho_V(x)$ defines the measure of integration over "physically infinitesimal volume", as is usual in condensed matter physics.

The second important ingredient is the existence of the medium itself. For phenomenological purposes it is not important what particular kind of microscopic description for the medium is chosen. What does matter is Lorentz symmetry breaking following from the existence of a distinguished frame - the medium rest frame. In the simplest cases of uniform medium characterized by nonzero temperature/density it is usually parameterized by a unit vector u_μ - the medium four-velocity, so that for applied uniform electromagnetic field one has the standard text-book answer for induced current

$$\langle\phi|J_\mu|\phi\rangle \propto u^\nu F_{\mu\nu} \tag{7}$$

We say about local parity violation in a state $|\Omega\rangle$ when a local parity-odd operator $\mathcal{O}(x) = -P\mathcal{O}(x)P^\dagger$ has nonzero expectation value in this state

$$\langle\Omega|\mathcal{O}(x)|\Omega\rangle \neq 0 \tag{8}$$

for example $\langle\psi^\dagger\gamma_5\psi\rangle \neq 0$. The condition of locality here is important. Operationally it means that the operators and their products are defined at the scale $a \sim \Lambda^{-1}$ where Λ is ultraviolet cutoff. For nonlocal averages, on the other hand, it is not a problem to have nonzero P-odd matrix element, e.g. $\langle j_0(x)j_3(y)\rangle$. The medium, characterized by finite coherence length, brings physical meaning to this nonlocality. For example, in a medium with applied uniform electromagnetic field nothing forbids to have P-odd correlation between axial current divergence and the vector current:

$$\langle\phi|J_\mu\,\partial J^5|\phi\rangle \propto u^\nu\tilde{F}_{\mu\nu} \tag{9}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$.

To feel the physical meaning of (9) let us imagine radial distribution of velocities $\mathbf{v}$ of the matter in a uniform magnetic field $\mathbf{B}$. If the divergence ∂J^5 is also uniform in the ("fireball") volume, the charge density is to be of different sign above and below the reaction plane:

$$\langle\phi|J_0\,\partial J^5|\phi\rangle \propto \mathbf{v}\cdot\mathbf{B} \tag{10}$$

In medium rest frame characterized by $u_\mu = (1,0,0,0)$ for uniform magnetic background, the electric current $\mathbf{J}$ flows along the magnetic field $\mathbf{B}$.

It seems quite natural to interpret (9) in the following way: as soon as the concept of a medium can be applied to the discussed problem one can easily construct classical nonzero local P-odd parameters without specifying any particular "chiral microscopy". The medium (manifested by existence of the selected frame) is crucial in two aspects: first, it allows to consider meaningful local objects and not badly divergent quantities like (5) and second, by Lorentz invariance breaking it provides invariant meaning for the electric and magnetic fields, thus making possible correlations between local (in macroscopic sense!) operators of different parities. We also see here the importance of the uniformity condition: if ∂J^5 is short-correlated, there is no net effect. This brings us back to the question about dynamical scales hierarchy.

3 CME in quantum measurement theory framework

It is possible to understand (1) as a correlation between preferred direction of outgoing electric charge distribution asymmetry and the magnetic field in a particular event. The sign of this P-parity odd asymmetry is fixed by the sign of effective μ_5 in this event (and of course varies randomly from event to event due to topological neutrality of QCD vacuum). The quantitative theory would require information about distribution function of effective μ_5.

Since detailed picture of the discussed microscopic quantum/classical interplay is beyond us, our attitude here is purely phenomenological. We define the effective η-dependent current $J_\mu(x,\eta)$ as

$$J_\mu(x,\eta) = \langle\Omega_\eta|j_\mu(x)|\Omega_\eta\rangle \tag{11}$$

where electric current $j_\mu(x) = \bar{\psi}(x)Q\gamma_\mu\psi(x)$ with quarks charge matrix $Q = \mathrm{diag}(2/3,-1/3,-1/3)$. The state $|\Omega_\eta\rangle$ is characterized by

$$\langle\Omega_\eta|\int_V d^4y\,\partial j^5(y)|\Omega_\eta\rangle = \eta \tag{12}$$

It is physically obvious that $J_\mu(x,\eta)$ must be an odd function in η and

$$\int_{-\infty}^{\infty} d\eta J_\mu(x,\eta) = 0 \tag{13}$$

Since by assumption each event is characterized by some value of η, positive or negative with equal probability, this corresponds to "averaging to zero" over many events.

To proceed further it is convenient to use the formalism of partial partition functions:

$$Z = \int D\Phi \exp(-S[\Phi]) \prod_i \int d\eta_i \, \tilde{\delta}(\eta_i - O_i[\Phi]) \tag{14}$$

where $S[\Phi]$ is the standard Euclidean QCD action, Φ stays for dynamical quark and gluon fields $A, \bar{\psi}, \psi$ and $O_i[\Phi]$ is a gauge-invariant operator made of these fields. We approximate the real detector with finite resolution by the choice of the "detector function" $\tilde{\delta}(x)$ in Gaussian form:

$$\tilde{\delta}(\eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\lambda \exp(-\lambda^2 l^2/2 + i\lambda\eta) \tag{15}$$

so that $\int_{-\infty}^{\infty} d\eta \tilde{\delta}(\eta) = 1$.

We are interested in a value of the electric current (11). For exactly conserved axial current $\partial j^5 = 0$ one would have $\langle\Omega| j_\mu(x) \cdot \partial j^5(y) |\Omega\rangle = 0$. Due to (electromagnetic) anomaly however the result reads (for isovector components)

$$i \int dx \, e^{iq(x-y)} \, \langle\Omega| j_\mu(x) \cdot \partial j^{5,a}(y) |\Omega\rangle = \mathrm{Tr}\,[Q^2 t^a] \cdot \left(-\frac{N_c}{4\pi^2}\right) \cdot q_\nu \tilde{F}_{\mu\nu} \tag{16}$$

where t^a are generators of flavour $SU(2)$ or $SU(3)$.

For singlet current the anomaly gets gluon contribution

$$\partial j^5 = -\mathrm{Tr}[Q^2] \, \frac{N_c}{4\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{N_f}{16\pi^2} \mathrm{Tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu} \tag{17}$$

(notice that for uniform magnetic field $F_{\mu\nu}\tilde{F}^{\mu\nu} = 0$) and computing

$$J_\mu(\eta, x) = \frac{1}{Z} \int D\Phi \, j_\mu(x) \, \tilde{\delta}\,(\eta - n_V) \, \exp(-S[\Phi]) \tag{18}$$

where

$$n_V = \int_V d^4 y \partial j^5 = -\frac{N_f}{16\pi^2} \int_V d^4 y \mathrm{Tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu} \tag{19}$$

at the leading order of the cluster expansion

$$\langle A \exp B \rangle \approx \langle AB \rangle \exp(\langle B^2 \rangle/2) \tag{20}$$

valid for $\langle A \rangle = 0$ and $\langle B \rangle = 0$, one gets in this approximation

$$J_\mu(x, \eta) = -\mathrm{Tr}[Q^2] \, \frac{N_c}{4\pi^2} \, \frac{\eta e^{-\eta^2/2L^2}}{\sqrt{2\pi L^6}} \cdot \left[\int \frac{d^4 q}{(2\pi)^4} e^{iqx} f_V(q) i q^\nu \right] \cdot \tilde{F}_{\mu\nu} \tag{21}$$

Here $L^2 = l^2 + \langle n_V^2 \rangle$ and the formfactor is given by $f_V(q) = \int_V d^4 y \exp(-iqy)$. In the infinite volume limit $\chi = \lim_{V\to\infty} \langle n_V^2 \rangle / V N_f^2$ where χ is the standard topological susceptibility.

The expression (21) deserves a few comments. First, the right hand side of (21) is odd function of η as it should be, and at small η the current is linear in η. If the point x is far from $y \in V$ the current vanishes due to formfactor $f_V(q)$, i.e. the current flows only in the interaction volume V. On the other hand, if $x \in V$ and V is large enough to neglect surface terms, the current also vanishes as it should be for any finite-volume effect. The volume scaling $\langle n_V^2 \rangle \sim V$ for the phase with finite correlation length is another manifestation of the same fact.

It is worth mentioning that the maximal current is reached at $\eta \sim L$ and decrease as $J^{max} \propto B/\tau L^2$ (where $\tau \sim V^{1/4}$). This result seems counter-intuitive. Indeed, a naive picture would suggest that stronger fluctuations of topological charge $\langle n_V^2 \rangle$ are to correspond to stronger currents $J_\mu(x,\eta)$. This in fact is not the case. Rough physical explanation follows from (16): since the product of j_μ and ∂j^5 is fixed by electromagnetic anomaly (i.e. by the magnitude of external abelian field $F_{\mu\nu}$) large ∂j^5 corresponds to small j_μ and vice versa. Let us remind that according to the lattice data [42] the magnitude of topological charge fluctuations experience rather sharp drop above the deconfinement transition. According to the above it means the effective *enhancement* of maximal possible electric current fluctuations! Of course at too small $\langle n_V^2 \rangle$ Gaussian approximation (neglect of higher order correlators) we have used is to break down.

It is seen that the discussed effect is a result of subtle interplay between strong and electromagnetic anomalies (see related remarks in [8]). While the later one is responsible for correlation between vector and axial currents, the former anomaly provides non-conservation of axial charge due to topological nonperturbative gluon fluctuations. The question about μ_5 distribution addressed in the introduction is translated here into the question about η distribution for experimental events.

4 Charge fluctuations asymmetry and polarization operator

Perhaps the most logically consistent way is to study transition matrix elements of (1) between states of opposite P-parity. This corresponds to:

$$\langle \Omega | j_i \, j_k | \Omega \rangle \to \sum_A \langle \Omega | j_i | A \rangle \langle A | j_k | \Omega \rangle \tag{22}$$

where the states $|\Omega\rangle$ and $|A\rangle$ have opposite P-parities and $\langle A|\Omega\rangle = 0$. Of course the expression (22) is nothing but the electromagnetic polarization operator in the state $|\Omega\rangle$ saturated by particular states in spectral expansion.

This line of reasoning has been addressed in the literature before. Local averages like $\langle j_\mu^2(x) \rangle$ were computed in pioneering studies of CME on the lattice [43, 44] and many interesting patterns were found. Later nonlocal averages $\langle j_\mu(x) j_\nu(y) \rangle$ are computed [45, 46]. We find it worth reminding once again that since the typical correlators we are interested in are given by dimension six operators, their local matrix elements are strongly UV-singular

$$\langle j_\mu^2(x) \rangle_F \propto \Lambda^6 + F^2 \Lambda^2 + \text{UV-finite} \tag{23}$$

where Λ is UV-cutoff and F-external field strength. Even subtracted average $\langle j_\mu^2(x)\rangle_F - \langle j_\mu^2(x)\rangle_0$ is divergent. This problem is overcome in numerical lattice calculations, but present analytical challenge for any attempt to describe CME in terms of local matrix elements. To our view this is a clear signal about intrinsic nonlocal nature of the discussed phenomenon.

Polarization operator in the CME context is studied in [22]. There are two main differences between our approach and that of the cited paper. First the regular contribution (given by polarization operator in magnetic field) and CME-contribution (proportional to μ_5) are separated from the beginning in [22] (in some sense, quantum currents are superimposed on top of the classical current (1)). We follow another logic and consider polarization operator as the only source of asymmetric charge fluctuations, but extract a particular formfactor from it, which corresponds to negative parity intermediate states. Second, the expression for charge fluctuations observable as a functional depending on polarization operator is different in our paper from that of [22]. We will make more comments on that below.

In this section we discuss P-odd $\times$ P-odd contributions to P-even observable, the role of which is played by current correlator $\langle j_\mu j_\nu \rangle$. It seems physically clear that this object should contain some information about charge distribution (4). The exact form of this correspondence is however far from trivial. One could think of several ways to relate these quantities. Before presenting our approach to this problem let us mention other methods used in the literature. First, we notice that the current in ϕ-direction is given by

$$\mathbf{e}_z j_3 + \mathbf{e}_y j_2 = \sqrt{j_3^2 + j_2^2}(\mathbf{e}_z \sin\phi + \mathbf{e}_y \cos\phi) \qquad (24)$$

and the corresponding charge difference from (4) is

$$\left\langle \int \frac{d(N_+ - N_-)}{d\phi}\, d\phi \int \frac{d({N'}_+ - {N'}_-)}{d\phi'}\, d\phi' \right\rangle_e \qquad (25)$$

where by the brackets $\langle ... \rangle_e$ we denote the average over events. One has $\langle (a_+ - a_-)^2 \rangle_e \propto \langle j_3^2 + j_2^2 \rangle$ where the current product is assumed to be local. This is very close (but different) to the definition used in [43]. It is natural to expect that positive definite $\langle (a_+ - a_-)^2 \rangle_e$ should be nonzero even without any magnetic field.

Another relation is suggested in [22]. It is written in terms of event average of the cosine, where $\alpha, \beta = +, -$ and $N_\pm$ is the total number of outgoing particles of a given charge:

$$\langle \cos(\phi_\alpha + {\phi_\beta}') \rangle_e \propto \frac{\alpha\beta}{N_\alpha N_\beta}(j_2^2 - j_3^2) \qquad (26)$$

where, up to some background terms

$$\langle \cos(\phi_\alpha + {\phi_\beta}') \rangle_e = \langle v_{1,\alpha} v_{1,\beta} \rangle_e - \langle a_\alpha a_\beta \rangle_e \qquad (27)$$

Assuming charge independence of $v_{1,\alpha}$ and equal numbers of particle species $N_+ = N_- = N$ one gets $\langle (a_+ - a_-)^2 \rangle_e \propto \langle j_3^2 - j_2^2 \rangle$ if one neglects $v_{1,\alpha}$ term with respect to a_α term. In fact, the leading term, which is always contained in j_3 component, coincides for both expressions, while the procedure of taking into account fluctuations in the reaction plane is different.

In this paper we use alternative signature provided by charge density fluctuations and not spatial components of the currents. An attractive feature of this quantity is that it is well defined even in the static limit. To this end consider electric charge in some spatial volume V at temperature T:

$$eQ_V = e \int_V d\mathbf{x}\, j_0(x) \qquad (28)$$

Since we work in zero density approximation the quantum average of this object vanish:

$$\langle Q_V \rangle = 0 \qquad (29)$$

This is not the case for its square:

$$\langle Q_V^2 \rangle = -\hat{\kappa} \int_V d\mathbf{x} \int_V d\mathbf{x}'\, \Pi_{44}(x, x') \qquad (30)$$

where $\Pi_{44}(x, x')$ is Euclidean polarization operator in constant external field $F_{\mu\nu}$ and at temperature T Wick-rotated from the standard Minkowski expression $\Pi_{00}^{(M)}(x, x')$:

$$\Pi_{\mu\nu}^{(M)}(x, x') = i \langle T\{ j_\mu(x) j_\nu(x') \} \rangle_{F,T} \qquad (31)$$

with $j_\mu = \bar\psi Q \gamma_\mu \psi$; $\Pi_{\mu\nu}^{(M)} \leftrightarrow \Pi_{\mu\nu}^{(E)}$, notice the sign convention (30) corresponding to positive-definite $\langle Q_V^2 \rangle$ in the static limit. In the standard way we denote

$$\Pi_{\mu\nu}(q) = \int d^4x\, e^{-iq(x-x')}\, \Pi_{\mu\nu}(x, x') \qquad (32)$$

with $\mu, \nu = 1, 2, 3, 4$ and $\mathbf{q} = (q_1, q_2, q_3)$, $q_\perp = (q_1, q_2)$.

The operator $\hat{\kappa}$ in (30) accounts for temporal profile of the process. In terms of momentum space components, (30) takes the following form

$$\langle Q_V^2\rangle = -\int \frac{dq_4}{2\pi}\kappa(q_4)\int \frac{d\mathbf{q}}{(2\pi)^3}|F_V(\mathbf{q})|^2\Pi_{44}(\mathbf{q},q_4) \tag{33}$$

where the form-factor $F_V(\mathbf{q}) = \int_V d\mathbf{x}\exp(i\mathbf{q}\mathbf{x})$ keeps information about spatial profile of the volume V, while the temporal factor $\kappa(q_4) = \int d\tau g(\tau)\exp(iq_4\tau)$ encodes temporal (in Euclidean sense) profile. For finite temperature case we consider here the standard Matsubara replacements $q_4 \to \omega_n = 2\pi nT$ and $(2\pi)^{-1}\int dq_4 \to T\sum_n$ are to be performed. The choice $g(\tau) = T$ we will adopt in the rest of the paper physically corresponds to the static limit where only the lowest Matsubara frequency $n = 0$ contributes:

$$\langle Q_V^2\rangle_{st} = -T\int \frac{d\mathbf{q}}{(2\pi)^3}|F_V(\mathbf{q})|^2\Pi_{44}(\mathbf{q},0) \tag{34}$$

It can be checked that in thermodynamic limit $V\to\infty$ without external field one reproduces standard Stefan-Boltzmann answer for elementary fermions $\lim\limits_{V\to\infty}\langle e^2Q_V^2\rangle_{st}/V = e^2T^3/3$. In case of quarks one should of course understand eB as $q_f eB$ and introduce additional trace over flavors with the factor N_cQ^2: $\Pi_{44}^{eB,T} \to N_c\sum_f q_f^2\Pi_{44}^{q_feB,T}$. For the sake of brevity we will use the simple notation as for elementary fermions of unit electric charge having in mind the necessity to make the replacement discussed above in the final answers.

In the limiting case of no background $B = 0, T = 0$ one has $\Pi_{44}(\mathbf{q},q_4) = \mathbf{q}^2\Pi(q^2)$ and, at the leading order, for large 4-volumes V_4:

$$\langle Q_V^2\rangle_{B=0,T=0} \propto \Pi'(0)\cdot V_4^{-1/2} \tag{35}$$

where the condition of gauge invariance $\Pi(0) = 0$ has been taken into account and the volume $V_4 = R^3\times t$ is assumed to be uniform: $R\sim t$. Thus the expression (30) is UV-safe and vacuum charge fluctuations in a given space-time region is purely finite-volume effect.

We can now come back to the definition (30) and rewrite the coordinate integration in cylinder coordinates with the axis 1 as the polar axis and angle ϕ defined in the 23 plane. This is the same notation as in (4), notice that in the standard setup azimuthal angle is usually defined in the plane 12. This allows to represent the form-factor $F_V(\mathbf{q})$ as

$$F_V(\mathbf{q}) = \int dx_1\, e^{iq_1x_1}\int_0 \rho d\rho\int_0^{2\pi} d\phi\, e^{i\bar{q}\rho} \tag{36}$$

where $\bar{q}\rho = q_2x_2 + q_3x_3 = q_2\rho\cos\phi + q_3\rho\sin\phi$ and the structure of integration upper limit is determined by the chosen model for spatial distribution (sharp boundary, smoothed boundary, Gaussian shape, exponential shape etc). The $\sin\phi$ - mode in Fourier expansion of (36) is multiplied by the following coefficient

$$c_1 = (1/\pi)\int_0^{2\pi} d\phi\sin\phi e^{i\bar{q}\rho} = \frac{2iq_3}{\hat{q}}\, J_1(\hat{q}\rho) \tag{37}$$

where $\hat{q} = \sqrt{q_2^2+q_3^2}$. Thus we have for expansion of (34) in harmonics:

$$\langle Q_V^2\rangle = ... + \int_0^{2\pi} d\phi\sin\phi\int_0^{2\pi} d\phi'\sin\phi'\langle (q_V^a)^2\rangle + ... \tag{38}$$

where $\langle (q_V^a)^2\rangle$ is given by the same expression (34) with the change $F_V(\mathbf{q}) \to f_V(q_1,q_2,q_3)$ where

$$f_V(q_1,q_2,q_3) = \frac{2iq_3}{\hat{q}}\int dx_1\, e^{iq_1x_1}\int_0 \rho J_1(\hat{q}\rho)d\rho \tag{39}$$

In the same way $\langle (q_V^{v_1})^2 \rangle$ corresponds to the exchange $q_3 \leftrightarrow q_2$ and $\sin\phi \leftrightarrow \cos\phi$. Making use of (4), (25) and (38) we obtain the following relation for the asymmetry

$$\langle q_V^2 \rangle = \langle (q_V^a)^2 \rangle - \langle (q_V^{v_1})^2 \rangle = - \sum_{\alpha,\beta=\pm} \alpha\beta \cos(\phi_\alpha + \phi_\beta{}') \tag{40}$$

$$\langle q_V^2 \rangle = N^2 \cdot \left(\langle (a_+ - a_-)^2 \rangle_e - \langle (v_{1,+} - v_{1,-})^2 \rangle_e \right) =$$
$$= T \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{q_3^2 - q_2^2}{q_3^2 + q_2^2} \left| \int dx_1 \, e^{iq_1 x_1} \int_0 \rho J_1(\hat{q}\rho) d\rho \right|^2 \Pi_{44}(\mathbf{q}, 0) \tag{41}$$

It is obvious that the above expression has to be proportional to magnetic field since there is no other $O(3)$-violating factors in the problem. The effect we are looking for corresponds to *strong enhancement* of (41) in external magnetic field and hence, from experimental point of view, strong dependence of (41) on centrality. It is to be stressed that the multiplicity factor N^2 is by itself strongly centrality-dependent. This dependence is kinematical and has nothing to do with magnetic field dependence of $\langle q_V^2 \rangle$. Only the latter lies at the heart of CME.

5 General structure of polarization operator

In this section we analyze general structure of polarization operator in the background of nonzero temperature and magnetic field. As is clear from the above discussion, this is a necessary prerequisite before one can compute the charge fluctuation asymmetry (41).

First of all let us make a few general comments about space-time dependence of current-current correlator. In confinement phase (i.e. at sufficiently low temperatures) at large distances and for weak magnetic field one expects general structure of Euclidean polarization operator of the following form

$$\langle j(x) j(x') \rangle \propto e^{-m_\rho |x-x'|} + C(B) \cdot e^{-m_\pi |x-x'|} \tag{42}$$

with $C(B) \propto B^2$. This interesting effect of different parity states mixing in external field is similar to the one observed long time ago in [47] at finite temperature. The long-distance correlations are thus saturated by the lightest degrees of freedom (i.e. pions in the confinement phase). On the other hand, in deconfinement phase at strong fields, if Larmor radius is much smaller than Λ_{QCD}^{-1} no quarks can propagate in transverse direction at all:

$$\langle j(x) j(x') \rangle \propto e^{-eB(x-x')_\perp^2/2} \tag{43}$$

Large-N_c suppressed transverse correlations are possible only due to gluon degrees of freedom.

We confine our attention in what follows to a particular case of purely magnetic constant abelian background field $F_{\mu\nu}$ in the thermal bath rest frame at nonzero temperature T. We have chosen $F_{12} = -F_{21} = B$, i.e. magnetic field is directed along the third axis. The temperature effects break Lorentz-invariance and the physical answers depend on 4-vector u_μ which represents four-velocity of the thermal bath. It is normalized as $u_\mu u^\mu = 1$. In the present paper we take zero chemical potential $\mu = 0$. It is to be noticed that many general conclusions concerning the structure of polarization operator stay intact for $\mu \neq 0$ since the latter is associated with the same four-vector u_μ given by $u_\mu = (1, 0, 0, 0)$ in the medium rest frame.

The polarization operator (32) is a rank two tensor depending on two polar vectors q_μ and u_μ and antisymmetric tensor $F_{\mu\nu}$. The general decomposition of (32) in terms of independent tensors was extensively studied in the literature starting from [48, 49], see [50] for recent exposition and [51] for a useful collection of references. Generally, one is to deal with $4 \times 4 = 16$ independent tensor structures, built by multiplying the four independent base vectors $q_\mu, u_\mu, q^\alpha F_{\alpha\mu}, q^\alpha F_\alpha^\beta F_{\beta\mu}$. It can be shown however that general requirements of being transversal

$$q^\mu \Pi_{\mu\nu}(q) = q^\nu \Pi_{\mu\nu}(q) = 0 \tag{44}$$

and Bose symmetric $\Pi_{\mu\nu}(q) = \Pi_{\nu\mu}(-q)$ together with generalized Furry's theorem [48]

$$\Pi_{\mu\nu}(q,u,F) = \Pi_{\mu\nu}(q,-u,-F) \tag{45}$$

reduce the number of independent tensor structures to six. Two of them are field-independent, the other two depend on $F_{\mu\nu}$ linearly and the last two - quadratically (notice that our numeration of the tensors is different from the one adopted in [48]). Their explicit form reads

$$\begin{aligned}
\Psi^{(1)}_{\mu\nu} &= q^2\delta_{\mu\nu} - q_\mu q_\nu \\
\Psi^{(2)}_{\mu\nu} &= (q^2u_\mu - q_\mu(uq))(q^2u_\nu - q_\nu(uq)) \\
\Psi^{(3)}_{\mu\nu} &= (uq)(q_\mu F_{\nu\rho}q^\rho - q_\nu F_{\mu\rho}q^\rho + q^2F_{\mu\nu}) \\
\Psi^{(4)}_{\mu\nu} &= (u_\mu F_{\nu\rho}q^\rho - u_\nu F_{\mu\rho}q^\rho + (uq)F_{\mu\nu}) \\
\Psi^{(5)}_{\mu\nu} &= F_{\mu\rho}q^\rho F_{\nu\sigma}q^\sigma \\
\Psi^{(6)}_{\mu\nu} &= (q^2\delta_{\mu\rho} - q_\mu q_\rho)F^\rho_\alpha F^{\alpha\sigma}(q^2\delta_{\sigma\nu} - q_\sigma q_\nu)
\end{aligned} \tag{46}$$

The coefficient functions of the decomposition

$$\Pi_{\mu\nu}(q,u,F) = \sum_{i=1}^{6} \pi^{(i)} \cdot \Psi^{(i)}_{\mu\nu} \tag{47}$$

depend on q^2, mixed invariants $(uq)^2$, $(qF)^2$, $(uF)^2$, $(qFu)^2$, pure field invariants $F^2, F\tilde{F}$ and also the temperature T and particle data, encoded in matrices Q and M. The expression (47) allows to discuss current correlations asymmetries in invariant way in any theory where the expression for polarization operator can be obtained.

Having these general prerequisites let us come back to analysis of correlation patterns. For our choice $F_{12} = B$ the invariants $(uF)^2$, $(qFu)^2$ and $F\tilde{F}$ equal to zero. In what follows we will be especially interested in a particular type of contribution to $\Pi_{\mu\nu}(q)$ proportional to the tensor structure $\Psi^{(7)}_{\mu\nu}$ given by the product of two axial vectors

$$\Psi^{(7)}_{\mu\nu} = \tilde{F}_{\mu\rho}q^\rho \tilde{F}_{\nu\sigma}q^\sigma \tag{48}$$

It is not independent and one easily checks that $\Psi^{(7)}_{\mu\nu}$ can be expressed as a linear combination of (46):

$$q^2\Psi^{(7)}_{\mu\nu} = \left(q^2F^2/2 - (qF)^2\right)\Psi^{(1)}_{\mu\nu} + q^2\Psi^{(5)}_{\mu\nu} + \Psi^{(6)}_{\mu\nu} \tag{49}$$

Let us consider tensor structure of the polarization operator in more details. First of all, since we are interested only in diagonal 11, 22, 33, 44 and also 34 components in this paper, we have no contributions from $\pi^{(3)}$ and $\pi^{(4)}$ because the tensors $\Psi^{(3)}_{\mu\nu}$ and $\Psi^{(4)}_{\mu\nu}$ are antisymmetric and also vanish for $\mu = 3, \nu = 4$ in the chosen background field. Second, we notice that for μ, ν equal to 3 or 4, one has identically $\Psi^{(5)}_{\mu\nu} = 0$. Adopting conventional notation: $q_\perp = (q_1, q_2)$, $q_{||} = (q_3, q_4)$ we can rewrite (47) using (49) as

$$\Pi_{||}(q) = \pi^{(Q)} \cdot \Psi^{(1)}_{||} + \pi^{(T)} \cdot \Psi^{(2)}_{||} + \tilde{\pi}^{(F)} \cdot \Psi^{(7)}_{||} \tag{50}$$

where the new invariant functions are given by

$$\begin{aligned}
\pi^{(Q)} &= \pi^{(1)} - \left(q^2F^2/2 - (qF)^2\right)\pi^{(6)} \\
\pi^{(T)} &= \pi^{(2)} \ ; \ \tilde{\pi}^{(F)} = q^2\pi^{(6)}
\end{aligned} \tag{51}$$

As for the diagonal correlators in 12-plane, one has

$$\Pi_\perp(q) = \pi^{(Q)} \cdot \Psi_\perp^{(1)} + \pi^{(T)} \cdot \Psi_\perp^{(2)} + \pi^{(F)} \cdot \Psi_\perp^{(5)} \tag{52}$$

where $\pi^{(Q)}$ and $\pi^{(T)}$ are defined by the same expressions (51) while $\pi^{(F)}$ form-factor reads

$$\pi^{(F)} = \pi^{(5)} - q^2 \pi^{(6)} \tag{53}$$

It is seen that the correlators of our interest can be decomposed into just three independent structures. The first, $\pi^{(Q)}$ corresponds to purely quantum fluctuations. It has nonzero limit at both $B \to 0$ and $T \to 0$, which coincides in this case with the textbook expression for polarization operator. The second structure, $\pi^{(T)}$ is responsible for thermal fluctuations. It vanishes at $T \to 0$. It is worth mentioning that both functions $\pi^{(Q)}$ and $\pi^{(T)}$ depend on temperature and external field (since the pattern of both quantum and thermal fluctuations is sensitive to the external conditions) and our notation corresponds rather to the limiting form of these functions.

We notice that the terms proportional to $\pi^{(Q)}$ and $\pi^{(T)}$ are identical in (50) and (52) up to obvious change of notation $|| \leftrightarrows \perp$. This is to be expected since quantum and thermal fluctuation are $\mathbb{O}(3)$-isotropic. The only non-isotropic term (and the most interesting for us here) is the last terms: $\tilde{\pi}^{(F)}$ in (50) and $\pi^{(F)}$ in (52). The former one takes into account charge (and also the current component j_3) fluctuations induced by the external magnetic field. P-parity structure of this term is given by

$$\begin{array}{ccccccc} \delta_B \langle j_3 j_3 \rangle & = & \tilde{\pi}^{(F)} & \times & \tilde{F}_{3\rho} p^\rho & \times & \tilde{F}_{3\sigma} p^\sigma \\ \text{P-even} & = & \text{P-even} & \times & \text{axial} & \times & \text{axial} \end{array}$$

It is to be compared with the thermal contribution proportional to $\Psi_{||}^{(2)}$:

$$\begin{array}{ccccccc} \delta_T \langle j_3 j_3 \rangle & = & \pi^{(T)} & \times & p_3(up) & \times & p_3(up) \\ \text{P-even} & = & \text{P-even} & \times & \text{vector} & \times & \text{vector} \end{array}$$

This directly corresponds to our discussion in the introduction: in the latter case the thermal fluctuations are distributed isotropically in the thermal bath rest frame, while in the former one there are electric currents fluctuating along the magnetic field. The magnitude of these fluctuations is measured by the function $\tilde{\pi}^{(F)}$, and no physical principle forces it to vanish either below or above critical temperature. Physically $\tilde{\pi}^{(F)}$ corresponds to P-odd intermediate states in the polarization operator.

The function $\pi^{(F)}$ entering (52) is a sum of two terms according to (53). This also is to be expected. Charged particles flowing in the plane perpendicular to the magnetic field are deflected by the Lorentz force, and this *diamagnetic* effect is taken into account by the form-factor $\pi^{(5)}$. It is absent in $\Pi_{||}$. But particle's spin interacts with the field by means of $\sigma_{\alpha\beta}\mathbf{F}^{\alpha\beta}$ term in $\Pi_{||}$ as well as in $\Pi_\perp$ which results in the factor $q^2\pi^{(6)}$ in both expressions (50) and (52). It is worth noting that according to our general logic the electric charge asymmetry is computed for the full expression for Π_{44}, not just from some part of it, proportional to $\tilde{\pi}^{(F)}$. Thus it is legitimate to speak about CME-interpretation of the answer (41) only in the limiting case when $\tilde{\pi}^{(F)}$ provides dominant contribution. We discuss that in more details below.

6 Model examples

We analyze in this section two limiting cases where one can construct $\tilde{\pi}^{(F)}$ in explicit way. The first one corresponds to weak magnetic fields in the confinement phase. In this case the intermediate states are hadron resonances of negative P-parity (see closely related discussion in [52]). However to select explicitely physical states making dominant contribution is far from trivial and the answer strongly depends on kinematics. We confine ourselves in this paper to the simplest case keeping only three neutral 0^{-+} intermediate states: π^0, η, η'. Technically it is more convenient to consider from the very beginning matrix elements of vector currents between vacuum and these states in external field. Making use of the definition of off-shell vector-vector-axial form-factor $\mathcal{F}_\pi \equiv \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q^2, q_1^2, q_2^2)$ (see, e.g. [53]) with $q = q_1 + q_2$

$$\int dx \int dy e^{iq_1 x + iq_2 y} \langle 0| \mathrm{Tr}\ \{j_\mu(x) j_\nu(y)\} |\pi^0(q)\rangle = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_\pi(q^2, q_1^2, q_2^2) \qquad (54)$$

one gets at the leading order in constant external field:

$$\langle 0| j_\mu(-q) |\pi^0(q)\rangle_F = ieq^\rho \tilde{F}_{\rho\mu}\, \mathcal{F}_\pi(q^2, q^2, 0). \qquad (55)$$

The expressions for η and η' contributions are completely analogous with the replacement of $\mathcal{F}_\pi$ by $\mathcal{F}_\eta$ and $\mathcal{F}_{\eta'}$.

Thus the q^2-dependence of polarization operator in external field is determined in this approximation by the form-factors $\mathcal{F}_\phi(q^2, q^2, 0)$ with one on-shell leg (corresponding to external field vertex). These form-factors are essentially nonperturbative QCD objects. Let us remind that on-shell (i.e. at the point $\mathcal{F}_\phi(m_\phi^2, 0, 0)$) they are fixed by triangle anomaly, for example for pion:

$$\mathcal{F}_\pi(m_\pi^2, 0, 0) = -\frac{N_c}{12\pi^2 F_\pi} \qquad (56)$$

Another important case is large $q^2 \to \infty$ limit where one has (for chiral fermions) $\mathcal{F}_\phi(q^2, q^2, 0) \to \chi_F F_\pi/3$ where χ_F is QCD quark condensate magnetic susceptibility, defined by $\langle 0|\bar{q}\sigma_{\mu\nu} q|0\rangle_F = e_q \chi_F \langle \bar{q}q\rangle F_{\mu\nu}$. Different approximation schemes valid at intermediate momenta are discussed in the literature (see, e.g. [54]).

Having written the field-dependent matrix element (55) one is able to express the invariant function $\tilde{\pi}^{(F)}(q^2)$ as follows:

$$\tilde{\pi}^{(F)}(q^2) = \sum_{\phi = \pi, \eta, \eta'} \frac{|\mathcal{F}_\phi(q^2, q^2, 0)|^2}{q^2 - m_\phi^2} \qquad (57)$$

From the point of view of expression (41) the dominant contribution to asymmetry is this phase comes from the lightest degree of freedom, i.e. massless in the chiral limit pion (to be more precise, we assume the limit $m_\pi R \ll 1$). Choosing for concretness Gaussian boundary condition (i.e. introducing the factors $\exp(-q_i^2 R^2/2)$ into (41) one obtains

$$\langle q_V^2 \rangle = \gamma \left(\frac{eB}{F_\pi}\right)^2 T R^3 \qquad (58)$$

where the numerical factor $\gamma = 1.6 \cdot 10^{-4}$ is of course specific for this boundary choice. Certainly the result trivially follows from dimensional considerations. We see $\langle q_V^2 \rangle \ll 1$ for phenomenologically reasonable choice of parameters. Contributions of mass gapped states bring additional suppression (and, in particular, break $\sim R^3$ scaling).

As the second example we consider free fermions in strong field limit. This regime would correspond to deconfinement phase where proper dynamical degrees of freedom are quarks and gluons with perturbatively weak interaction between each other. To compute polarization operator under external conditions in perturbation theory one usually makes use of Schwinger proper-time technique and there is extensive literature on the subject [55, 56, 57, 58, 59] where different kinds of external backgrounds were studied. The polarization operator in constant magnetic field and at nonzero temperature was calculated in [60] in imaginary time formalism.

Our aim here is to put these results in a charge fluctuations asymmetry prospective. For the reader's convenience we reproduce the explicit one-loop expressions for polarization operator $\Pi_{||}$ given by [60].

It is convenient to present the Euclidean polarization operator in the following form

$$\Pi_{\mu\nu}(q_\perp, q_3, n) = \sum A_{\mu\nu}(q)\, e^{-\phi(q)} + Q_{\mu\nu}(q) \tag{59}$$

where the sum includes integration over proper-times and summation over Matsubara frequencies, the functions $A_{\mu\nu}[q]$ polynomially depend on momenta components q. The contact terms $Q_{\mu\nu}(q)$ have no sensitivity to infrared parameters (like temperature or external field) and provide correct limit of $\Pi_{\mu\nu}$ at vanishing background.

One can notice that $\tilde{\pi}^{(F)}$ can be simply related to the polarization operator components. Namely, solving the system of three linear equations (50) for the choices $(\mu\nu) = 44, 33$ and 34 one finds all three invariant form-factors, including $\tilde{\pi}^{(F)}$:

$$B^2 \tilde{\pi}^{(F)} = -\frac{q_3 q_4 \Pi_{44} + (q_\perp^2 + q_3^2)\Pi_{34}}{q_\perp^2 q_3 q_4} \tag{60}$$

where $q_\perp^2 = q_1^2 + q_2^2$ and $q_4 \equiv \omega_n = 2\pi T n$.

The explicit expression for $\tilde{\pi}^{(F)}$ looks especially simple in small T regime. It reads

$$\tilde{\pi}^{(F)} = -\frac{1}{(4\pi)^2}\frac{1}{eB}\int_\epsilon^\infty du \int_{-1}^{+1} dv\ \big((1-v^2)\coth\bar{u} + f_\perp(\bar{u}, v)\big)\exp(-\phi^{(0)}) \tag{61}$$

where $\bar{u} = ueB$ and the functions $\phi^{(0)}$ and $f_\perp(\bar{u}, v)$ are given in the [60]. Notice that such form-factor was discussed in a different context in [49].

In the weak field limit one has

$$\lim_{B\to 0} \tilde{\pi}^{(F)} = \frac{1}{6\pi^2}\int_{-1}^{1} dv \frac{(1-v^2)(3-v^2)}{(4m^2 + (1-v^2)q^2)^2} \tag{62}$$

In the strong field limit (still at small T) the situation becomes more interesting - form-factor $\tilde{\pi}^{(F)}$ provides dominant contribution to the polarization operator:

$$\begin{aligned} \Pi_{44} &\to q_3^2 (eB)^2 \tilde{\pi}^{(F)} \to \\ &\to -\frac{eB}{4\pi^2}\, e^{-\frac{q_\perp^2}{2|eB|}} \int_{-1}^{1} dv \frac{(1-v^2)q_3^2}{4m^2 + (1-v^2)q_3^2} \end{aligned} \tag{63}$$

up to the terms $\mathcal{O}\left(q_\perp^2/eB\right)$. One can say that all asymmetry of charge fluctuations is due to CME-like formfactor in this limit.

We see another interesting effect - in the chiral limit (63) does not depend on q_3 at all, while the dependence on $q_\perp$ is suppressed by the field B. On the other hand, the essence of the asymmetry of interest is just different dependence of the polarization operator on different components of momentum. Since the polarization operator itself linearly rise with B for strong field it is nor a priori clear which effect is to win. Detailed calculation shows that in fact they balance each other and the asymmetry (41) is not asymptotically rising with B - there is an effect of saturation. It is reasonable to separate different regimes depending on ratios between basic parameters such as B, m, T and R where the latter one stays for the typical 3-dimensional size of the volume V_3. For two light flavors one can safely neglect quark masses m. Three other parameters are in the ballpark of 100 MeV (for large fireball one can think of phenomenologically realistic $eBR^2 = 5 \div 10$). Without intention to cook up numerical factors but just to get feeling of the numbers, pluging (63) into (41) we get

$$\langle q_V^2 \rangle = \gamma' \cdot RT \tag{64}$$

where again the numerical factor $\gamma' = 4.1 \cdot 10^{-2}$ corresponds to Gaussian boundary shape. Thus for asymptotically large B one reaches "kinematical limit" for the asymmetry in our picture, despite numerically it is still very small.

7 Conclusions

We have discussed three possible ways to study quantum physics behind chiral magnetic effect and electric charge fluctuation asymmetry observed in heavy ion collisions. For all approaches the importance of scale separation is stressed - there should by hierarchy of dynamical scales characterizing the life of quark-gluon phase after the collision and intrinsic QCD scales (perhaps field/temperature shifted) characterizing the nonabelian topological charge fluctuation pattern. The physical essence of CME as we tried to present it here is that the quark-gluon medium plays the role of a measuring device with respect to the topological QCD vacuum with the final particles electric charge asymmetry as an outcome. This is most clearly illustrated by the expression (21).

The third approach we have considered, i.e. the analysis of P-odd $\times$ P-odd contributions to P-even observables, is somewhat different because it provides nonzero results even for free fermions in magnetic field, i.e. without any "topological origin". We believe that this can be considered as a particular case of CME as well. Just nonzero matrix element of the vector current between vacuum and J^{-+} states in external magnetic field leads to asymmetric charge/current pattern as if there is fluctuating vector current collinear to $\mathbf{B}$. Of course the detailed picture depends on the actual quantum dynamics of these J^{-+} degrees of freedom, and we have shown that indeed it is strongly suppressed in the confinement phase. Nevertheless we find it legitimate to interpret this dynamics using the same CME-like language since namely this anomaly-driven vector-axial correlation is at the heart of the effect, while the concrete way of life of the axial degrees of freedom (distribution function for μ_5 in the standard CME analysis) is of secondary importance.

References

[1] D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, “Possibility of spontaneous parity violation in hot QCD,” Phys. Rev. Lett. **81**, 512 (1998) [arXiv:hep-ph/9804221].

[2] D. Kharzeev and R. D. Pisarski, “Pionic measures of parity and CP violation in high energy nuclear collisions,” Phys. Rev. D **61**, 111901 (2000) [arXiv:hep-ph/9906401].

[3] D. E. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, “Aspects of parity, CP, and time reversal violation in hot QCD,” arXiv:hep-ph/0012012.

[4] D. Kharzeev, A. Krasnitz and R. Venugopalan, “Anomalous chirality fluctuations in the initial stage of heavy ion collisions and parity odd bubbles,” Phys. Lett. B **545**, 298 (2002) [arXiv:hep-ph/0109253].

[5] D. Kharzeev, “Parity violation in hot QCD: Why it can happen, and how to look for it,” Phys. Lett. B **633**, 260 (2006) [arXiv:hep-ph/0406125].

[6] D. Kharzeev and A. Zhitnitsky, “Charge separation induced by P-odd bubbles in QCD matter,” Nucl. Phys. A **797**, 67 (2007) [arXiv:0706.1026 [hep-ph]].

[7] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, “The effects of topological charge change in heavy ion collisions: 'Event by event P and CP violation',” Nucl. Phys. A **803**, 227 (2008) [arXiv:0711.0950 [hep-ph]].

[8] K. Fukushima, D. E. Kharzeev and H. J. Warringa, “The Chiral Magnetic Effect,” Phys. Rev. D **78**, 074033 (2008) [arXiv:0808.3382 [hep-ph]].

[9] H. J. Warringa, “Implications of CP-violating transitions in hot quark matter on heavy ion collisions,” J. Phys. G **35**, 104012 (2008) [arXiv:0805.1384 [hep-ph]].

[10] D. E. Kharzeev, "Hot and dense matter: from RHIC to LHC: Theoretical overview," Nucl. Phys. A **827**, 118C (2009) [arXiv:0902.2749 [hep-ph]].

[11] H. J. Warringa, "The Chiral Magnetic Effect: Measuring event-by-event P- and CP-violation with heavy ion-collisions," arXiv:0906.2803 [hep-ph].

[12] D. E. Kharzeev, "Topologically induced local P and CP violation in hot QCD," arXiv:0906.2808 [hep-ph].

[13] D. E. Kharzeev and H. J. Warringa, "Chiral Magnetic conductivity," Phys. Rev. D **80**, 034028 (2009) [arXiv:0907.5007 [hep-ph]].

[14] D. E. Kharzeev, "Chern-Simons current and local parity violation in hot QCD matter," Nucl. Phys. A **830**, 543C (2009) [arXiv:0908.0314 [hep-ph]].

[15] H. U. Yee, "Holographic Chiral Magnetic Conductivity," JHEP **0911**, 085 (2009) [arXiv:0908.4189 [hep-th]].

[16] E. S. Fraga and A. J. Mizher, "Chiral symmetry restoration and strong CP violation in a strong magnetic background," PoS C **POD2009**, 037 (2009) [arXiv:0910.4525 [hep-ph]].

[17] S. i. Nam, "Chiral magnetic effect at low temperature," Phys. Rev. D **80**, 114025 (2009) [arXiv:0911.0509 [hep-ph]].

[18] M. Abramczyk, T. Blum, G. Petropoulos and R. Zhou, "Chiral magnetic effect in 2+1 flavor QCD+QED," arXiv:0911.1348 [hep-lat].

[19] L. McLerran, "Theoretical Concepts for Ultra-Relativistic Heavy Ion Collisions," arXiv:0911.2987 [hep-ph].

[20] D. E. Kharzeev, "Topologically induced local P and CP violation in QCD x QED," Annals Phys. **325**, 205 (2010) [arXiv:0911.3715 [hep-ph]].

[21] S. i. Nam, "Chiral magnetic effect (CME) at low temperature from instanton vacuum," arXiv:0912.1933 [hep-ph].

[22] K. Fukushima, D. E. Kharzeev and H. J. Warringa, "Electric-current Susceptibility and the Chiral Magnetic Effect," arXiv:0912.2961 [hep-ph].

[23] A. Bzdak, V. Koch and J. Liao, "Remarks on possible local parity violation in heavy ion collisions," arXiv:0912.5050 [nucl-th].

[24] W. j. Fu, Y. x. Liu and Y. l. Wu, "Chiral Magnetic Effect and Chiral Phase Transition," arXiv:1002.0418 [hep-ph].

[25] K. Fukushima, D. E. Kharzeev and H. J. Warringa, "Real-time dynamics of the Chiral Magnetic Effect," arXiv:1002.2495 [hep-ph].

[26] M. Giovannini and M. E. Shaposhnikov, "Primordial magnetic fields, anomalous isocurvature fluctuations and big bang nucleosynthesis," Phys. Rev. Lett. **80**, 22 (1998) [arXiv:hep-ph/9708303].

[27] G. Veneziano, "U(1) Without Instantons," Nucl. Phys. B **159**, 213 (1979).

[28] E. Witten, "Current Algebra Theorems For The U(1) Goldstone Boson," Nucl. Phys. B **156**, 269 (1979).

[29] S. A. Voloshin, "Discussing the possibility of observation of parity violation in heavy ion collisions," Phys. Rev. C **62**, 044901 (2000) [arXiv:nucl-th/0004042].

[30] S. A. Voloshin, "Parity violation in hot QCD: How to detect it," Phys. Rev. C **70**, 057901 (2004) [arXiv:hep-ph/0406311].

[31] I. V. Selyuzhenkov [STAR Collaboration], "Global polarization and parity violation study in Au + Au collisions," Rom. Rep. Phys. **58**, 049 (2006) [arXiv:nucl-ex/0510069].

[32] I. Selyuzhenkov [STAR Collaboration], "Azimuthal charged particle correlations as a probe for local strong parity violation in heavy-ion collisions," arXiv:0910.0464 [nucl-ex].

[33] S. A. Voloshin, "Local strong parity violation and new possibilities in experimental study of non-perturbative QCD," arXiv:1003.1127 [nucl-ex].

[34] S. A. Voloshin, "Anisotropic flow: Achievements, Difficulties, Expectations," J. Phys. G **35**, 104014 (2008) [arXiv:0805.1351 [nucl-ex]].

[35] S. A. Voloshin [STAR Collaboration], "Probe for the strong parity violation effects at RHIC with three particle correlations," arXiv:0806.0029 [nucl-ex].

[36] S. A. Voloshin, "Anisotropic collective phenomena in ultra-relativistic nuclear collisions," Nucl. Phys. A **827**, 377C (2009) [arXiv:0902.0581 [nucl-ex]].

[37] S. A. Voloshin [STAR Collaboration], "Experimental study of local strong parity violation in relativistic nuclear collisions," Nucl. Phys. A **830**, 377C (2009) [arXiv:0907.2213 [nucl-ex]].

[38] B. I. Abelev *et al.* [STAR Collaboration], "Observation of charge-dependent azimuthal correlations and possible local strong parity violation in heavy ion collisions," arXiv:0909.1717 [nucl-ex].

[39] B. I. Abelev *et al.* [STAR Collaboration], "Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation," Phys. Rev. Lett. **103**, 251601 (2009) [arXiv:0909.1739 [nucl-ex]].

[40] G. Wang [STAR Collaboration], "Highlights from STAR: probing the early medium in heavy ion collisions," Nucl. Phys. A **830**, 19C (2009) [arXiv:0907.4504 [nucl-ex]].

[41] D. T. Son and P. Surowka, "Hydrodynamics with Triangle Anomalies," Phys. Rev. Lett. **103**, 191601 (2009) [arXiv:0906.5044 [hep-th]].

[42] L. Del Debbio, H. Panagopoulos and E. Vicari, "Topological susceptibility of SU(N) gauge theories at finite temperature," JHEP **0409**, 028 (2004) [arXiv:hep-th/0407068].

[43] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, "Chiral magnetization of non-Abelian vacuum: a lattice study," Nucl. Phys. B **826**, 313 (2010) [arXiv:0906.0488 [hep-lat]].

[44] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, "Numerical evidence of chiral magnetic effect in lattice gauge theory," Phys. Rev. D **80**, 054503 (2009) [arXiv:0907.0494 [hep-lat]].

[45] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, "Lattice QCD in strong magnetic fields," arXiv:0909.1808 [hep-ph].

[46] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, "Numerical study of chiral magnetic effect in quenched SU(2) lattice gauge theory," arXiv:0910.4682 [hep-lat].

[47] M. Dey, V. L. Eletsky and B. L. Ioffe, "Mixing of vector and axial mesons at finite temperature: an Indication towards chiral symmetry restoration," Phys. Lett. B **252**, 620 (1990).

[48] H. Perez Rojas and A. E. Shabad, "Polarization Of Relativistic Electron And Positron Gas In A Strong Magnetic Field. Propagation Of Electromagnetic Waves," Annals Phys. **121**, 432 (1979).

[49] V. N. Baier, V. M. Katkov and V. M. Strakhovenko, "An Operator Approach To Quantum Electrodynamics In External Field. 2. Electron Loops," Zh. Eksp. Teor. Fiz. **68**, 405 (1975).

[50] A. E. Shabad and V. V. Usov, "Real and virtual photons in an external constant electromagnetic field of most general form," Phys. Rev. D **81**, 125008 (2010) [arXiv:1002.1813 [hep-th]].

[51] K. Bhattacharya, "Elementary particle interactions in a background magnetic field," arXiv:hep-ph/0407099.

[52] M. Asakawa, A. Majumder and B. Muller, "Electric Charge Separation in Strong Transient Magnetic Fields," Phys. Rev. C **81**, 064912 (2010) [arXiv:1003.2436 [hep-ph]].

[53] F. Jegerlehner and A. Nyffeler, "The Muon g-2," Phys. Rept. **477**, 1 (2009) [arXiv:0902.3360 [hep-ph]].

[54] A. S. Gorsky, "Higher Twist Effects In QCD Description Of Light Meson Exclusive Form-Factors: Twist Four Wave Functions And The Application To Pi0 Gamma Gamma Amplitude," Preprint ITEP-87-86

[55] W. Y. Tsai, "Vacuum Polarization In Homogeneous Magnetic Fields," Phys. Rev. D **10**, 2699 (1974).

[56] L. F. Urrutia, "Vacuum Polarization In Parallel Homogeneous Electric And Magnetic Fields," Phys. Rev. D **17**, 1977 (1978).

[57] S. L. Adler, "Photon splitting and photon dispersion in a strong magnetic field," Annals Phys. **67**, 599 (1971).

[58] A. Chodos, K. Everding and D. A. Owen, "QED With A Chemical Potential: 1. The Case Of A Constant Magnetic Field," Phys. Rev. D **42**, 2881 (1990).

[59] P. Elmfors, D. Persson and B. S. Skagerstam, "QED effective action at finite temperature and density," Phys. Rev. Lett. **71**, 480 (1993) [arXiv:hep-th/9305004].

[60] J. Alexandre, "Vacuum polarization in thermal QED with an external magnetic field," Phys. Rev. D **63**, 073010 (2001) [arXiv:hep-th/0009204].