

# Inclusive jet production at Tevatron in the Regge limit of QCD

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## Abstract

We consider inclusive hadroproduction of jets, prompt photons,  $b$ -jets, and  $D$ -mesons in the quasi-multi-Regge-kinematics approach based on the hypothesis of gluon and quark Reggeization in  $t$ -channel exchanges at high energies. The data taken by the CDF and D0 Collaborations at the Fermilab Tevatron are well described without adjusting parameters. We find the main contribution to inclusive jet production to be the scattering of two Reggeized gluons, described by the effective Reggeon-Reggeon-gluon vertex, and to  $b$ -jet and  $D$ -meson production — the scattering of a Reggeized gluon and a Reggeized quark to an ordinary quark, being expressed in terms of the Reggeon-Reggeon-quark vertex. The main contribution to prompt photon production arises from Reggeized quark-Reggeized antiquark annihilation, which is described by the effective Reggeon-Reggeon-photon vertex. Our analysis is based on the Kimber-Martin-Ryskin prescription for unintegrated gluon and quark distribution functions using as input the Martin-Roberts-Stirling-Thorne collinear parton distribution functions of the proton.

## 1 Introduction

The study of inclusive jet and prompt photon production with large transverse momenta coming from a hard interaction between two partons in hadron collisions at high-energy colliders is of great interest for the test of perturbative quantum chromodynamics (QCD). Also these processes provide an opportunity to extract information on the parton densities within protons.

The total center-of-mass (CM) energy at the Tevatron Collider,  $\sqrt{S} = 1.8$  TeV in Run I and  $\sqrt{S} = 1.96$  TeV in Run II, sufficiently exceeds the scale  $\mu$  of the relevant hard processes, so that  $\sqrt{S} \gg \mu \gg \Lambda_{\text{QCD}}$ , where  $\Lambda_{\text{QCD}}$  is the asymptotic scale parameter of QCD. In such a high-energy regime, the contributions from subprocesses involving  $t$ -channel exchanges of partons (gluons and quarks) to the production cross section may become dominant. Then, the transverse momenta of the incoming partons and their off-shell properties can no longer be neglected, and we deal with *Reggeized*  $t$ -channel partons. In this so-called quasi-multi-Regge kinematics (QMRK), the particles (multi-Regge) or groups of particles (quasi-multi-Regge) produced in the collision are strongly separated in rapidity space. In the case of inclusive jet and prompt photon production, this implies the following: a single jet or photon is produced in the central region of rapidity, while other particles are produced at large rapidities. The QMRK approach [1] is particularly appropriate for this kind of high-energy phenomenology. It is based on an effective quantum field theory implemented with the non-Abelian gauge-invariant action

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including fields of Reggeized gluons [2] and quarks [3]. Roughly speaking, the Reggeization of amplitudes is a trick that offers an opportunity to take into account efficiently large radiation corrections to the processes under Regge limit condition beyond the collinear approximation of the parton model. The particle Reggeization is known in high-energy quantum electrodynamics (QED) for electrons only [4] and for gluons and quarks in QCD [5, 6, 7].

In this paper, we consider inclusive hadroproduction of jets,  $b$ -jets,  $D$ -mesons and prompt photons in the QMRK approach applying the hypothesis of gluon and quark Reggeization [5, 6, 7].

## 2 Effective vertices and amplitudes of $2 \rightarrow 1$ subprocesses

Throughout our analysis we will use the following definitions:  $\mathcal{Q}$  — Reggeized quark,  $\mathcal{R}$  — Reggeized gluon,  $q$  — ordinary quark,  $g$  — Yang-Mills gluon and indicate the 4-momenta of the particles in parenthesis after their denotations.

We consider high transverse momenta particle production in proton-antiproton collisions at the Tevatron Collider. In the CM frame the 4-momenta of the incoming proton and antiproton are written down as  $P_1 = E_1(1, 0, 0, 1)$  and  $P_2 = E_2(1, 0, 0, -1)$ , correspondingly.

We also introduce the ancillary vectors  $n_\mu^\pm = (1, 0, 0, \mp 1)$  and for any 4-momentum  $k^\mu, q^\mu, p^\mu$  define:

$$k^\pm = k \cdot n^\pm, \quad \gamma_\mu^{(\pm)}(q, p) = \gamma_\mu + \not{n}_\mu^\pm \frac{p^\pm}{p^\pm}, \quad (1)$$

$$\gamma_\mu^{(+)}(q_1, q_2) = \gamma_\mu - \frac{\not{n}_\mu^-}{q_2^-} - \frac{\not{n}_\mu^+}{q_1^+}. \quad (2)$$

In the CM frame the 4-momenta of a Reggeized particle coming from initial hadron may be represented in the form:  $q_i = x_i P_i + q_{iT}$ ,  $i = 1, 2$ , where  $x_i$  — the longitudinal momentum fraction of initial hadron carried by the Reggeized particle,  $q_{iT} = (0, \mathbf{q}_{iT}, 0)$  — the transverse momentum of the latter. To make subsequent formulas shorter we define  $t_i = -q_{iT}^2 = \mathbf{q}_{iT}^2$ .

Below we present effective vertices and squared amplitudes of the  $2 \rightarrow 1$  subprocesses with two Reggeized partons in the initial state, they read:  $\mathcal{Q} + \mathcal{R} \rightarrow q$ ,  $\mathcal{R} + \mathcal{R} \rightarrow g$ ,  $\mathcal{Q} + \mathcal{Q} \rightarrow g$ ,  $\mathcal{Q} + \bar{\mathcal{Q}} \rightarrow \gamma$ . Thus, the transverse momenta of the particles involved in these subprocesses are related by the condition:  $\mathbf{k}_T^2 = t_1 + t_2 + 2\sqrt{t_1 t_2} \cos \phi_{12}$ , where  $k = (k_0, \mathbf{k}_T, k_z)$  is a 4-momentum of the final quark or gluon,  $\phi_{12}$  — the azimuthal angle between the vectors  $\mathbf{q}_{1T}$  and  $\mathbf{q}_{2T}$ .

At first, we consider hard subprocess of a single quark production via Reggeized gluon-Reggeized quark scattering:

$$\mathcal{Q}(q_1) + \mathcal{R}(q_2) \rightarrow q(k), \quad (3)$$

As the subject of our investigation is a high transverse momenta jet production, we are allowed to neglect quark masses, so that the effective vertex of the subprocess (3) may be displayed like following, according to [3]:

$$C_{\mathcal{Q}\mathcal{R}}^q(q_1, q_2) = i\sqrt{4\pi\alpha_s} T^a \bar{U}(k) \gamma^{(-)\mu}(q_1, q_2) \Pi_\mu^{(+)}(q_2), \quad (4)$$

where  $\alpha_s$  is the strong coupling constant,  $T^a$  — the color gauge group SU(3) generators,  $a = 1, \dots, N_c^2 - 1$ ,  $\Pi_\mu^{(+)}(q_2) = -\frac{q_2^- \not{n}_\mu^+}{2\sqrt{t_2}}$ . The corresponding squared amplitude has a simple form [8]:

$$|\overline{\mathcal{M}(\mathcal{Q} + \mathcal{R} \rightarrow q)}|^2 = \frac{2}{3} \pi \alpha_s \mathbf{k}_T^2. \quad (5)$$

The effective vertex of a transition of two Reggeized gluons to Yang-Mills gluon  $\mathcal{R} + \mathcal{R} \rightarrow g$  may be written as [1, 5, 9]:

$$C_{\mathcal{R}\mathcal{R}}^{g,\mu}(q_1, q_2) = -\sqrt{4\pi\alpha_s} f^{abc} \frac{q_1^+ q_2^-}{2\sqrt{t_1 t_2}} \left[ (q_1 - q_2)^\mu + \frac{(n^+)^\mu}{q_1^+} (q_2^2 + q_1^+ q_2^-) - \frac{(n^-)^\mu}{q_2^-} (q_1^2 + q_1^+ q_2^-) \right], \quad (6)$$

where  $a$  and  $b$  — color indices of Reggeized gluons with 4-momenta  $q_1$  and  $q_2$ , correspondingly. The squared amplitude of the subprocess  $\mathcal{R} + \mathcal{R} \rightarrow g$  can be obtained by simple calculations:

$$|\overline{\mathcal{M}(\mathcal{R} + \mathcal{R} \rightarrow g)}|^2 = \frac{3}{2} \pi \alpha_s \mathbf{k}_T^2. \quad (7)$$

The effective vertex of the transition of a Reggeized quark with 4-momenta  $q_1$  and a Reggeized antiquark with 4-momenta  $q_2$  to photon or Yang-Mills gluon looks like following [6, 7]:

$$C_{\mathcal{Q}\bar{\mathcal{Q}}}^{\gamma(g),\mu}(q_1, q_2) = \mathcal{C}_{\gamma(g)} \left[ \gamma^\mu - \not{q}_1 \frac{(n^-)^\mu}{q_1^- + q_2^-} - \not{q}_2 \frac{(n^+)^\mu}{q_1^+ + q_2^+} \right], \quad (8)$$

where  $\mathcal{C}_\gamma = -i\sqrt{4\pi\alpha} Z_q$  for photon,  $\alpha \simeq 1/137$  — the fine structure constant and  $Z_q$  — quark electric charge. In the case of gluon  $\mathcal{C}_g = -i\sqrt{4\pi\alpha_s} T^a$ . The squared amplitude of the concerned transition has the form:

$$|\overline{\mathcal{M}(\mathcal{Q} + \bar{\mathcal{Q}} \rightarrow \gamma(g))}|^2 = \mathcal{B}_{\gamma(g)}(t_1 + t_2), \quad (9)$$

where the factor  $\mathcal{B}_\gamma = \frac{4}{3} \pi \alpha Z_q^2$  for photon and  $\mathcal{B}_g = \frac{16}{3} \pi \alpha_s$  for gluon.

It is evident from cited above formulas, that squared amplitudes of  $2 \rightarrow 1$  subprocesses are equal to zero in the collinear approximation, in which case we need to start from  $2 \rightarrow 2$  subprocesses, being of the next order in  $\alpha_s$ .

Exploiting the hypothesis of high-energy factorization, we may write a hadronic cross sections  $d\sigma$  as convolutions of partonic cross sections  $d\hat{\sigma}$  with unintegrated parton distribution functions (PDFs)  $\Phi_a^h$  of Reggeized partons  $a$  in the hadrons  $h$ . The unintegrated PDFs  $\Phi_a^h(x, t, \mu^2)$  are related to their collinear counterparts  $F_a^h(x, \mu^2)$  by the normalization condition

$$xF_a^h(x, \mu^2) = \int^{\mu^2} dt \Phi_a^h(x, t, \mu^2), \quad (10)$$

which yields the correct transition from formulas in the QMRK approach to those in the collinear parton model, where the transverse momenta of the partons are neglected. In our numerical analysis, we adopt the prescription proposed by Kimber, Martin, and Ryskin [10] to obtain unintegrated gluon and quark distribution functions for the proton from the conventional integrated ones, as implemented in Watt's code [11]. As input for this procedure, we use the Martin-Roberts-Stirling-Thorne [12] proton PDFs.

In our analysis the renormalization and factorization scales are identified and chosen to be  $\mu = \xi k_T$ , where  $\xi$  is varied between 1/2 and 2 about its default value 1 to estimate the theoretical uncertainty. The resulting errors are indicated as shaded bands in the figures.

As we consider production of particles with high transverse momenta  $k_T \gg m_q$ , it is allowed to work in massless approximation.

### 3 Inclusive jet production

The first subject of our investigation is inclusive jet production. It was experimentally studied by CDF [13] and D0 [14] Collaborations in several rapidity intervals at jet transverse momenta up to 700 GeV. The main contribution to such processes in leading order (LO) of QMRK comes from  $2 \rightarrow 1$  subprocess of Reggeized gluon fusion producing Yang-Mills gluon, which is described by the effective vertex (6) and squared amplitude (7). Using the hypothesis of high-energy factorization, we have

$$d\sigma(p\bar{p} \rightarrow jetX) = \int \frac{dx_1}{x_1} \int \frac{d^2q_{1T}}{\pi} \int \frac{dx_2}{x_2} \int \frac{d^2q_{2T}}{\pi} \Phi_g^p(x_1, t_1, \mu^2) \Phi_g^{\bar{p}}(x_2, t_2, \mu^2) d\hat{\sigma}(\mathcal{R}\mathcal{R} \rightarrow g) \quad (11)$$

For the reader's convenience, we present here compact formula for the differential cross section (11):

$$\frac{d\sigma}{dk_T dy}(p\bar{p} \rightarrow jetX) = \frac{1}{k_T^3} \int d\phi_1 \int dt_1 \Phi_g^p(x_1, t_1, \mu^2) \Phi_g^{\bar{p}}(x_2, t_2, \mu^2) \overline{|\mathcal{M}(\mathcal{R}\mathcal{R} \rightarrow g)|^2}, \quad (12)$$

where  $y$  is the rapidity,  $\phi_1$  is the azimuthal angle enclosed between the vectors  $\mathbf{q}_{1T}$  and  $\mathbf{k}_T$ ,

$$x_{1,2} = \frac{k_T \exp(\pm y)}{\sqrt{S}}, \quad t_2 = t_1 + k_T^2 - 2k_T \sqrt{t_1} \cos \phi_1. \quad (13)$$

In the Figs. 1, top, and 1, bottom, our predictions obtained in the QMRK approach are compared with CDF [13] and D0 [14] data, correspondingly. The theoretical predictions nicely agree with experimental data up to 200 GeV. The discrepancy at  $k_T > 200$  GeV arises because the average values of the scaling variables  $x_1$  and  $x_2$  in the unintegrated PDFs exceed 0.2, so that, strictly speaking, the QMRK approach ceases to be valid.

### 4 Inclusive $b$ -jet production

Recently CDF Collaboration presented preliminary data on inclusive single  $b$ -jet production in  $p\bar{p}$ -collisions at Tevatron Run II [15]. The measurement was performed in the kinematic range  $38 < k_T < 400$  GeV and  $|y| < 0.7$ . As the modulus of the  $b$ -quark transverse momentum  $k_T \geq 32$  GeV [15], sufficiently exceeds its mass  $m_b$ , it is justified to assume beauty to be an active flavor in the proton. Such a way we are allowed to examine the process under consideration in the fixed-flavor-number scheme with  $n_f = 5$  active quark flavors. To LO in the QMRK approach, there is only one partonic subprocess, namely

$$\mathcal{Q}_b(q_1) + \mathcal{R}(q_2) \rightarrow b(k), \quad (14)$$

where  $\mathcal{Q}_b$  — the Reggeized  $b$  quark. Neglecting Reggeized-quark masses, the effective vertex and squared amplitude of the concerned transition are given by (4) and (5). The high-energy factorization formula for the cross section takes the form:

$$\begin{aligned} d\sigma(p\bar{p} \rightarrow bX) = & \int \frac{dx_1}{x_1} \int \frac{d^2q_{1T}}{\pi} \int \frac{dx_2}{x_2} \int \frac{d^2q_{2T}}{\pi} [\Phi_b^p(x_1, t_1, \mu^2) \Phi_b^{\bar{p}}(x_2, t_2, \mu^2) + \\ & + \Phi_g^p(x_1, t_1, \mu^2) \Phi_b^{\bar{p}}(x_2, t_2, \mu^2)] d\hat{\sigma}(\mathcal{Q}_b\mathcal{R} \rightarrow b), \end{aligned} \quad (15)$$

and it may be written compactly in the way similar to (12).

In the Fig. 2 we present the results of our calculation in comparison with CDF data [15]. One can obtain that they nicely agree with the CDF data throughout the entire  $k_T$  range.

The detailed investigation of single and associated  $b$ -jet production in QMRK including these results can be found in our recent work [16].

## 5 Inclusive open charm production

In this section we study  $D$ -meson production via charm-quark fragmentation under Tevatron experimental conditions for the first time in the framework of the QMRK approach complemented with the quark Reggeization hypothesis [17]. The high-energy factorization formulas for these processes are analogous to ones for inclusive jet production and contain an extra integral arising from  $c \rightarrow D$  fragmentation function  $D_{c \rightarrow D}$ . As for the latter, we adopt the non-perturbative  $D$ -meson sets determined in the zero-mass variable-flavor-number scheme with initial evolution scale  $\mu_0 = m_c$  [18] from fits to OPAL data from CERN LEP1.

The CDF Collaboration [19] measured the  $k_T$  distributions of  $D^0$ ,  $D^\pm$ ,  $D^*$ , and  $D_s$  mesons with rapidity  $|y| \leq 1$  inclusively produced in hadroproduction in Run II at the Tevatron, with  $\sqrt{S} = 1.96$  TeV. To LO in the QMRK approach there is only one partonic subprocess  $Q_c^{p(\bar{p})} \mathcal{R}^{\bar{p}(p)} \rightarrow c$ , where the top subscript indicates the mother particle. This subprocess is described via the Reggeized-quark–Reggeized gluon effective vertex (4).

In Fig. 3, our results for  $D^*$  and  $D_s$  mesons are compared with the CDF data [19]. We find that the theoretical predictions generally agree rather well with the experimental data, except perhaps for the slope. In fact, the predictions exhibit a slight tendency to undershoot the data at small values of  $k_T$  and to overshoot them at large values of  $k_T$  [17]. However, we have to bear in mind that these are just LO predictions, so that there is room for improvement by including higher orders.

## 6 Prompt photon production

In this part we consider an inclusive production of prompt photons at Fermilab Tevatron Run I [20] and Run II [21] measured by D0 Collaboration. In the QMRK approach the LO contribution comes from the Reggeized quark–Reggeized antiquark annihilation, and we can write the factorization formula as follows [8]:

$$d\sigma(p\bar{p} \rightarrow \gamma X) = \int \frac{dx_1}{x_1} \int \frac{d^2 q_{1T}}{\pi} \int \frac{dx_2}{x_2} \int \frac{d^2 q_{2T}}{\pi} \Phi_q^p(x_1, t_1, \mu^2) \Phi_{\bar{q}}^{\bar{p}}(x_2, t_2, \mu^2) d\hat{\sigma}(Q_q \bar{Q}_{\bar{q}} \rightarrow \gamma), \quad (16)$$

where  $q = u, d, s, c, b$ , and the charge conjugated subprocesses are taken into account.

In the Fig. 4 we present the results of our calculation in comparison with the D0 Collaboration data [20, 21]. We see that the agreement between the data and the theoretical calculation is well again except in the region of  $p_T > 100$  GeV, where theoretical results overestimate the experimental data. Note, that in this region of the photon transverse momentum the parton longitudinal momentum fractions become non-small and so long do not satisfy the conditions of particle Reggeization. At very large  $p_T$  one has  $x_{1,2} \geq 0.1$  and so far the collinear parton model should be applied, where the squared Reggeized amplitude  $|\overline{M(Q\bar{Q} \rightarrow \gamma)}|^2 \rightarrow 0$  and the  $2 \rightarrow 2$  parton subprocesses ( $qg \rightarrow q\gamma$ ,  $q\bar{q} \rightarrow g\gamma$ , etc.) are needed to take into account.

## 7 Conclusions

We studied the inclusive hadroproduction of jets,  $D$ -mesons,  $b$ -jets and prompt photons at LO in the QMRK approach, applying the quark Reggeization hypothesis, under Tevatron experimental conditions. Despite the great simplicity of our formulas, our theoretical predictions turned out to describe recent measurements of transverse momenta cross section distributions of different final particles at the Tevatron surprisingly well, without any ad-hoc adjustments of input parameters. By contrast, in the collinear parton model of QCD, such a degree of agreement can only be achieved by taking next-to-leading corrections into account and performing soft-gluon resummation. In conclusion, the QMRK approach is once again [8, 16, 17, 22] proven to be a powerful tool for the theoretical description of QCD processes in the high-energy limit.

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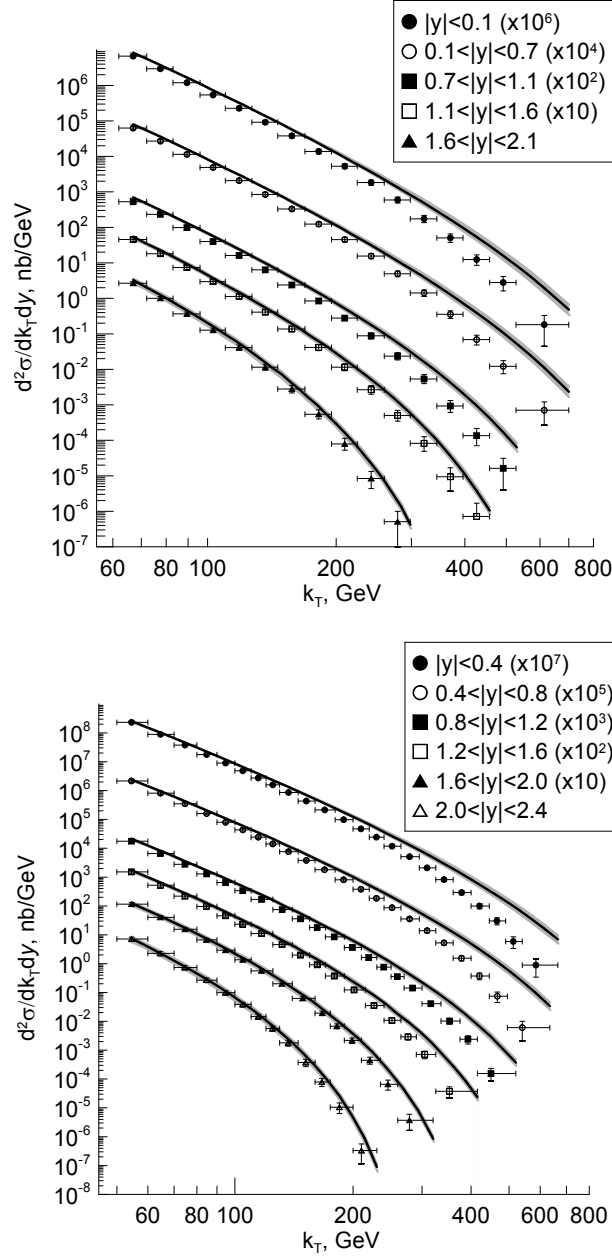


Figure 1: QMRK predictions on inclusive jet production in comparison with CDF [13] (top) and D0 [14] (bottom) data.

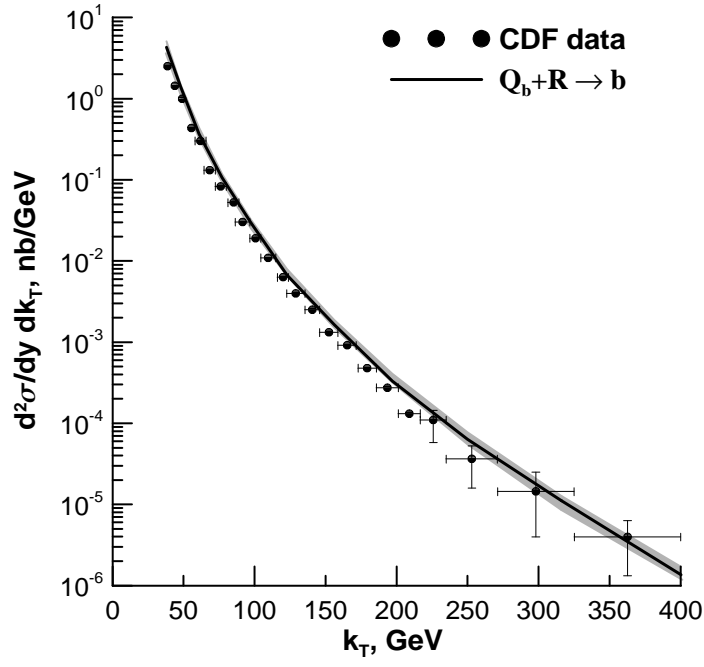


Figure 2: QMRK predictions on inclusive  $b$ -jet production in comparison with CDF data [15].

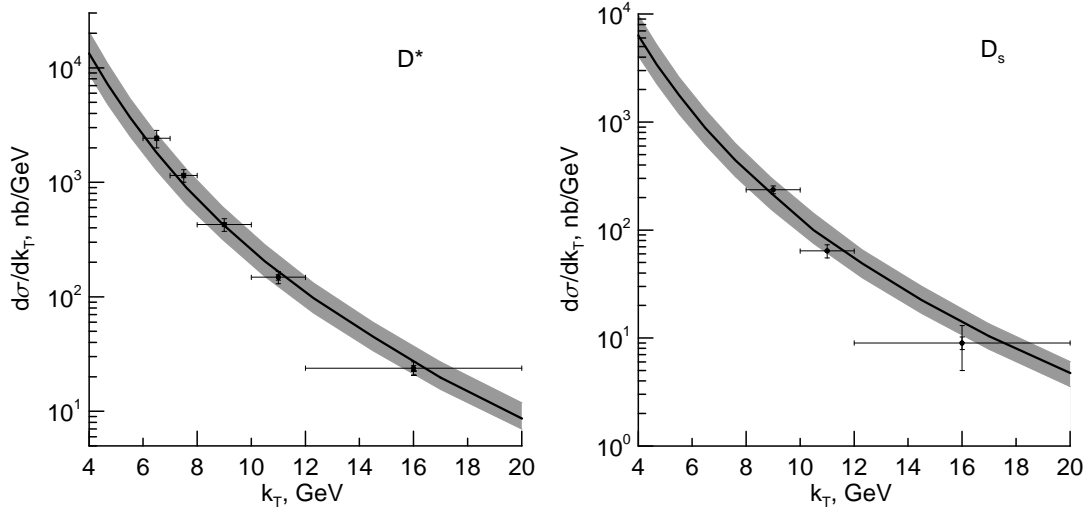


Figure 3: The  $k_T$  distributions of inclusive  $D^*$  (left) and  $D_s$  (right) hadroproduction for  $\sqrt{S} = 1.96$  TeV and  $|y| \leq 1$ . The CDF data from Ref. [19] are compared with LO predictions from the QMRK approach within the quark Reggeization hypothesis.



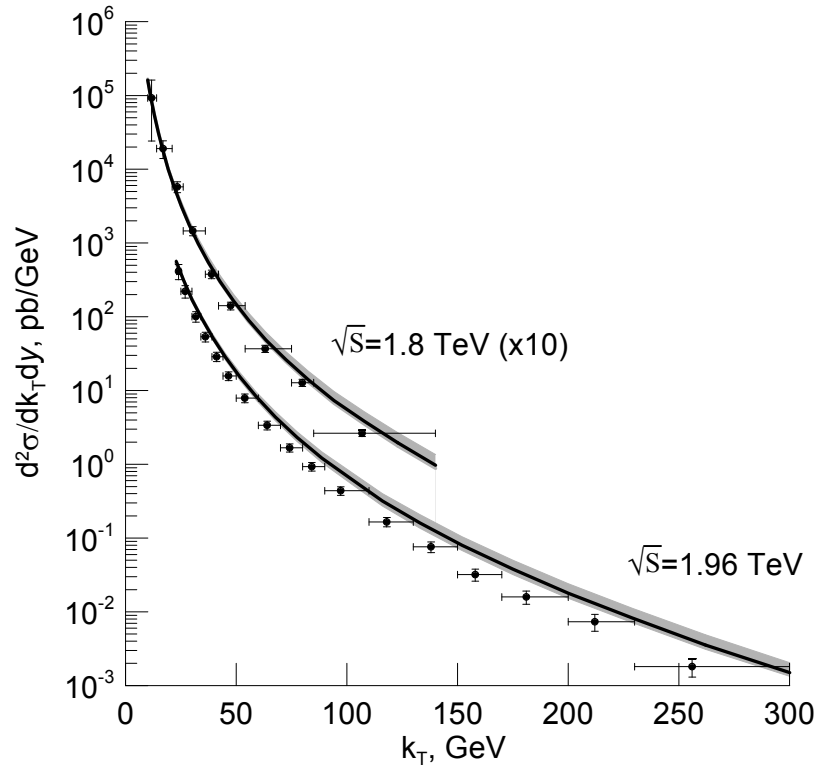


Figure 4: QMRK predictions on prompt photon production in comparison with D0 data [20, 21].