Light-Cone Wave Functions of Heavy Baryons

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Abstract

A classification of the three-quark light-cone distribution amplitudes (LCDAs) for the ground state heavy baryons with the spin-parities $J^P = 1/2^+$ and $J^P = 3/2^+$ in QCD in the heavy quark limit is presented. Several lowest moments of LCDAs are calculated based on the QCD sum rules. Simple models for the heavy-baryon distribution amplitudes are analyzed with account of their scale dependence.

1 Introduction

B-meson factories at SLAC and KEK, after approximately a decade of their operation, have made a great impact on a clarification of CP-violation origin in the quark sector of the Standard Model (SM). Study of heavy bottom baryons at LHC can serve as an additional test of the Kobayashi-Maskawa mechanism. Specific processes with bottom baryons, such as rare decays involving flavor-changing neutral currents (FCNC) transitions, are potential sources of new physics beyond the SM. In a difference to B-mesons, a non-zero spin of baryons allows also an experimental study of spin correlations. The spectrum of heavy bottom baryons have been enlarged substantially thanks to the effort done by the CDF and D0 collaborations at the Tevatron collider during last several years. Unlike these progress, study of FCNC motivated decays of bottom baryons remains to be statistically limited. A grater effort is expected at the LHC where heavy baryons will be copiously produced, and their weak decays may be measured precisely enough to provide important clues on physics beyond the Standard Model.

The theory of bottom baryon decays into light hadrons is more complicated compared to the B-meson decays and, hence, was receiving less attention. Calculations of heavy-baryon decays into light particles based on the heavy quark expansion, see e. g. [1], or using sum rules of the type proposed in [2–4] require the primary non-perturbative objects — the distribution amplitudes of heavy baryons. For a long period, the only existed models of heavy-baryon distribution amplitudes [5,6] have been motivated by quark models and not consistent with QCD constraints. In the paper [7], the complete classification of three-quark light-cone distribution amplitudes (LCDAs) of the Λ_b-baryon in QCD in the heavy quark limit were given and the scale-dependence of the leading-twist LCDA is discussed. In addition, simple models of the LCDAs were suggested and their parameters were fixed based on estimates of the first few moments by the QCD sum rules method. The analysis of [7] can be extended on all the ground state b-baryons with the spin-parity both $J^P = 1/2^+$ and $J^P = 3/2^+$. The basic steps and

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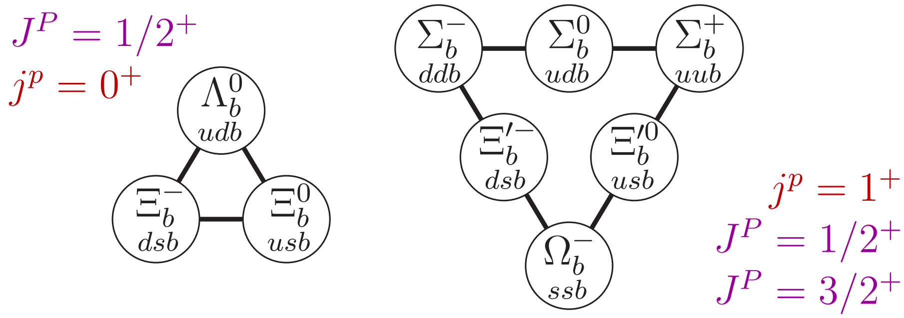

Figure 1: The $SU(3)_F$ flavor multiplets of bottom baryons. The $SU(3)_F$ triplet with the spin-parity $J^P = 1/2^+$ and scalar ($j^p = 0^+$) light-quark state is to the left. The $SU(3)_F$ sextet with the axial-vector ($j^p = 1^+$) light-quark state is to the right. The particle content on the diagram corresponds to the bottom baryons with the spin-parity $J^P = 1/2^+$. The ones with $J^P = 3/2^+$ have the $*$-modification of names.

main results of such an analysis are summarized in this talk and all the details are presented in the forthcoming paper [8].

2 Interpolating Currents of Heavy Baryons in HQET

Baryons with one heavy quark $Q = c, b$ in HQET (for a textbook about this topic see [9]) are classified according to the angular momentum ℓ and parity p of the light quark pair called diquark. The heavy quarks are non-relativistic particles which decouple from the diquark in the leading order of the $1/m_Q$ expansion.

The ground-state baryons ($\ell = 0$) with spin-parity J^P are characterized by the spin-parity j^p of the diquark. The spins of the light quarks produce two states with $j^p = 0^+$ and $j^p = 1^+$. In the state with $j^p = 0^+$ the spin wave-function is antisymmetric, while Fermi statistics of the baryon state and antisymmetry in color space require antisymmetric flavor wave-function. This results in a baryonic state with isospin $I = 0$ constructed from the light u- and d-quarks which is called the Λ_Q-baryon (the spin-parity is $J^P = 1/2^+$). When the spin-parity of the diquark is $j^p = 1^+$, the spin part of the baryon wave-function is symmetric which requires symmetry of the wave-function in the flavor space. In the case of light u- and d-quarks only, this gives two degenerate states with isospin $I = 1$, which are called Σ_Q- and Σ_Q^*-baryons having the spin-parities $J^P = 1/2^+$ and $J^P = 3/2^+$, respectively. Inclusion of the s-quark increases the number of heavy baryons in the multiplet, which is characterized by strangeness S. If $S = -1$, there are two baryonic states Ξ_Q and Ξ_Q' with $J^P = 1/2^+$ and Ξ_Q^*-baryon with $J^P = 3/2^+$. For $S = -2$, the baryons with $J^P = 1/2^+$ and $J^P = 3/2^+$ are called Ω_Q and Ω_Q^*. The $SU(3)_F$ multiplets of b-quark baryons are shown in Fig. 1.

The heavy baryon local currents have the following general structures [10, 11]:

$$J_1^{H_Q} = \varepsilon_{abc} \left[\psi^{aT} \mathcal{C} \Gamma \mathcal{T} \psi^b \right] \Gamma' Q_v^c, \qquad J_2^{H_Q} = \varepsilon_{abc} \left[\psi^{aT} \mathcal{C} \Gamma \not{v} \mathcal{T} \psi^b \right] \Gamma' Q_v^c, \tag{1}$$

where $a,\ b,\ c = 1, 2, 3$ are the color indices, ψ is the $SU(3)_{\mathrm{F}}$ triplet in the flavor space, v_μ is the four-velocity of the heavy quark, $\not{v} = (v\gamma)$, Q_v is the effective static field of the heavy quark satisfying $\not{v} Q_v = Q_v$, the index T means a transposition, $\mathcal{C}$ is the charge conjugation matrix with the properties $\mathcal{C} \gamma_\mu^T \mathcal{C}^{-1} = -\gamma_\mu$ and $\mathcal{C} \gamma_5 \mathcal{C}^{-1} = \gamma_5$, and $\mathcal{T}$ is a matrix in the flavor space.

For each of the ground-state baryons there are two independent interpolating local currents J_1 and J_2 with both having the appropriate quantum numbers. They can be constructed as suggested in Refs. [10–13]. For the states belonging to the $SU(3)_{\mathrm{F}}$ antitriplet $\bar{\mathbf{3}}$ (see Fig. 1a), the currents are:

$$J_1^{\Lambda_Q} = \varepsilon_{abc}\left[u^{aT}\mathcal{C}\gamma_5 d^b\right] Q_v^c, \qquad J_2^{\Lambda_Q} = \varepsilon_{abc}\left[u^{aT}\mathcal{C}\gamma_5 \not{v} d^b\right] Q_v^c, \tag{2}$$

$$J_1^{\Xi_Q} = \varepsilon_{abc}\left[q^{aT}\mathcal{C}\gamma_5 s^b\right] Q_v^c, \qquad J_2^{\Xi_Q} = \varepsilon_{abc}\left[q^{aT}\mathcal{C}\gamma_5 \not{v} s^b\right] Q_v^c, \tag{3}$$

where $q = u$ or d is one of the isodoublet quark fields. There are also two $SU(3)_\mathrm{F}$ sextet **6** states (see Fig. 1b) which differ by the total spin of the state. For the $1/2^+$ baryons, the local interpolating currents are:

$$J_1^{\Sigma_Q} = -\varepsilon_{abc}\left[q_1^{aT}\mathcal{C}\gamma_{t\mu} q_2^b\right]\gamma_t^\mu\gamma_5 Q_v^c, \qquad J_2^{\Sigma_Q} = -\varepsilon_{abc}\left[q_1^{aT}\mathcal{C}\gamma_{t\mu}\not{v} q_2^b\right]\gamma_t^\mu\gamma_5 Q_v^c, \tag{4}$$

$$J_1^{\Xi'_Q} = -\varepsilon_{abc}\left[q^{aT}\mathcal{C}\gamma_{t\mu} s^b\right]\gamma_t^\mu\gamma_5 Q_v^c, \qquad J_2^{\Xi'_Q} = -\varepsilon_{abc}\left[q^{aT}\mathcal{C}\gamma_{t\mu}\not{v} s^b\right]\gamma_t^\mu\gamma_5 Q_v^c, \tag{5}$$

$$J_1^{\Omega_Q} = -\varepsilon_{abc}\left[s^{aT}\mathcal{C}\gamma_{t\mu} s^b\right]\gamma_t^\mu\gamma_5 Q_v^c, \qquad J_2^{\Omega_Q} = -\varepsilon_{abc}\left[s^{aT}\mathcal{C}\gamma_{t\mu}\not{v} s^b\right]\gamma_t^\mu\gamma_5 Q_v^c, \tag{6}$$

where $\gamma_t^\mu = \gamma^\mu - \not{v}\, v^\mu$. For the $3/2^+$ baryons, the flavor structure is the same as for the $1/2^+$ baryons above, but the Dirac structure acting on the heavy-quark field is different. We exemplify such currents by the ones corresponding to the Σ_Q^*-baryon:

$$J_{1\mu}^{\Sigma_Q^*} = \varepsilon_{abc}\left[q_1^{aT}\mathcal{C}\gamma_t^\nu q_2'^b\right]\left(g_{\mu\nu} - \frac{1}{3}\gamma_{t\mu}\gamma_{t\nu}\right) Q_v^c, \tag{7}$$

$$J_{2\mu}^{\Sigma_Q^*} = \varepsilon_{abc}\left[q_1^{aT}\mathcal{C}\gamma_t^\nu \not{v} q_2'^b\right]\left(g_{\mu\nu} - \frac{1}{3}\gamma_{t\mu}\gamma_{t\nu}\right) Q_v^c.$$

These currents satisfy the condition $\gamma_t^\mu J_{i\mu}^{\Sigma_Q^*} = 0$ $(i = 1, 2)$.

Matrix elements of the local operators (2)-(7) define the baryonic couplings $f_{H_Q}^{(i)}$:

$$J^P = 1/2^+ : \qquad \langle 0|J_i^{H_Q}|H_Q(v)\rangle = f_{H_Q}^{(i)}\, u^{H_Q}(v), \tag{8}$$

$$J^P = 3/2^+ : \qquad \langle 0|J_{i\mu}^{H_Q^*}|H_Q^*(v)\rangle = \frac{1}{\sqrt{3}}\, f_{H_Q^*}^{(i)}\, u_\mu^{H_Q^*}(v), \tag{9}$$

where the coefficient in the matrix element $\langle 0|J_{i\mu}^{H_Q^*}|H_Q^*(v)\rangle$ is chosen such that $f_{H_Q}^{(i)} = f_{H_Q^*}^{(i)}$ in the heavy-quark symmetry limit. The Dirac spinors $u^{H_Q}(v)$ of the heavy baryon H_Q with the non-relativistic normalization $\bar{u}^{H_Q}(v)\, u^{H_Q}(v) = 1$ satisfy the condition $\not{v}\, u^{H_Q}(v) = u^{H_Q}(v)$. In the case of the H_Q^*-baryons, the wave-function is represented by the Rarita-Schwinger vector-spinor $u_\mu^{H_Q^*}(v)$, for which the following relations are valid: $\not{v}\, u_\mu^{H_Q^*}(v) = u_\mu^{H_Q^*}(v)$ and $v^\mu\, u_\mu^{H_Q^*}(v) = \gamma^\mu\, u_\mu^{H_Q^*}(v) = 0$.[1] The sums over their polarizations are as follows [14]:

$$\sum_{\lambda=1}^{2} u^{H_Q(\lambda)}(v)\,\bar{u}^{H_Q(\lambda)}(v) = \frac{1+\not{v}}{2} \equiv P_+, \tag{10}$$

$$\sum_{\lambda=1}^{4} u_\mu^{H_Q^*(\lambda)}(v)\,\bar{u}_\nu^{H_Q^*(\lambda)}(v) = P_+\left[-g_{\mu\nu} + v_\mu v_\nu + \frac{1}{3}\gamma_{t\mu}\gamma_{t\nu}\right]. \tag{11}$$

One can easily check the normalizations of these objects:

$$\mathrm{Sp}\sum_{\lambda=1}^{2} u^{H_Q(\lambda)}(v)\,\bar{u}^{H_Q(\lambda)}(v) = 2, \qquad -g^{\mu\nu}\,\mathrm{Sp}\sum_{\lambda=1}^{4} u_\mu^{H_Q^*(\lambda)}(v)\,\bar{u}_\nu^{H_Q^*(\lambda)}(v) = 4, \tag{12}$$

[1] A construction of the spin part of the excited baryons can be found in Ref. [14].

accounting for the numbers of the independent polarization states.

The LCDAs of a heavy baryon can be defined through the baryon-to-vacuum matrix elements of suitable non-local light-ray operators built from an effective heavy quark field $Q^c_v(0)$ and two light quark fields $q^a_i(t_i n)$ $(i = 1, 2)$. For the Λ_Q-baryon, the complete set of three-quark light-ray operators with respect to the diquark twist decomposition have been derived recently [7]. These currents can be easily adapted to the Ξ_Q-baryons by the replacement of one of the light u- or d-quarks by the s-quark and in general form can be written as follows:

$$\begin{aligned}
J_2(t_1,t_2) &= \varepsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_5 \not{n}\, q_2^b(t_2 n)\right] Q_v^c(0), \\
J_{3s}(t_1,t_2) &= \varepsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_5 q_2^b(t_2 n)\right] Q_v^c(0), \\
J_{3\sigma}(t_1,t_2) &= \frac{i}{2}\varepsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_5 (\bar{n}\sigma n) q_2^b(t_2 n)\right] Q_v^c(0), \\
J_4(t_1,t_2) &= \varepsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_5 \not{\bar{n}}\, q_2^b(t_2 n)\right] Q_v^c(0),
\end{aligned} \tag{13}$$

where $(\bar{n}\sigma n) = \bar{n}^\mu \sigma_{\mu\nu} n^\nu$ and the flavors of the light quarks $q = u,\ d,\ s$ are different $(q_1 \neq q_2)$ (for the quark content of real heavy baryons, see Fig. 1). The subscripts 2, 3, 4 refer to the twist of the diquark operator, n^μ and $\bar{n}^\mu$ are light-like vectors normalized as $(\bar{n}n) = 2$ which we choose such that $v^\mu = (n^\mu + \bar{n}^\mu)/2$ and $(nv) = (\bar{n}v) = 1$. The matrix elements of the operators (13) can be parametrized in accordance with [7]:

$$\begin{aligned}
\langle 0|J_2(t_1,t_2)|H_Q(v)\rangle &= f^{(2)}_{H_Q}\, \Psi_2(t_1,t_2)\, u^{H_Q}(v), \\
\langle 0|J_{3s}(t_1,t_2)|H_Q(v)\rangle &= f^{(1)}_{H_Q}\, \Psi_{3s}(t_1,t_2)\, u^{H_Q}(v), \\
\langle 0|J_{3\sigma}(t_1,t_2)|H_Q(v)\rangle &= f^{(1)}_{H_Q}\, \Psi_{3\sigma}(t_1,t_2)\, u^{H_Q}(v), \\
\langle 0|J_4(t_1,t_2)|H_Q(v)\rangle &= f^{(2)}_{H_Q}\, \Psi_4(t_1,t_2)\, u^{H_Q}(v).
\end{aligned} \tag{14}$$

The simplest way to construct the complete set of the three-particle LCDAs of baryons with the diquark spin-parity $j^p = 1^+$ is to switch off the spin of the heavy quark (we denote it as $\tilde{Q}_v$) and introduce the LCDAs for the "axial-vector baryon" state $\tilde{H}_Q(v,\eta)$, which in this case is characterized by the polarization vector η^μ satisfying the condition $(v\eta) = 0$. The LCDA definitions are borrowed from the light-cone analysis of the vector mesons [15]. Let us separate eight interpolating currents into two groups similar to the chiral-even and chiral-odd LCDAs of a vector meson [15]. The first set is:

$$J_{2V}(t_1,t_2) = \epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\not{n}\, q_2^b(t_2 n)\right] \tilde{Q}_v^c(0), \tag{15}$$

$$J_{4V}(t_1,t_2) = \epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\not{\bar{n}}\, q_2^b(t_2 n)\right] \tilde{Q}_v^c(0), \tag{16}$$

$$J^\mu_{3V}(t_1,t_2) = \epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_\perp^\mu q_2^b(t_2 n)\right] \tilde{Q}_v^c(0), \tag{17}$$

$$J^\mu_{3A}(t_1,t_2) = \epsilon_{abc}\varepsilon_\perp^{\mu\nu}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_{\perp\nu}\gamma_5 q_2^b(t_2 n)\right] \tilde{Q}_v^c(0), \tag{18}$$

where $\gamma_\perp^\mu = \gamma^\mu - (\not{n}\,\bar{n}^\mu + \not{\bar{n}}\, n^\mu)/2$, $\varepsilon_\perp^{\mu\nu} = (i/2)\,\varepsilon^{\mu\mu\rho\sigma}\, n_\rho \bar{n}_\sigma$ is the antisymmetric tensor in the plane perpendicular to the light cone (satisfying the condition $\varepsilon_{\perp\mu\nu}\varepsilon_\perp^{\nu\mu} = 2$). The second set is:

$$J^\mu_{2T}(t_1,t_2) = \epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_\perp^\mu \not{n}\, q_2^b(t_2 n)\right] \tilde{Q}_v^c(0), \tag{19}$$

$$J^\mu_{4T}(t_1,t_2) = \epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_\perp^\mu \not{\bar{n}}\, q_2^b(t_2 n)\right] \tilde{Q}_v^c(0), \tag{20}$$

$$J_{3T}(t_1,t_2) = \frac{i}{2}\epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\,(\bar{n}\sigma n)\, q_2^b(t_2 n)\right] \tilde{Q}_v^c(0), \tag{21}$$

$$J_{3S}(t_1,t_2) = \epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C} q_2^b(t_2 n)\right] \tilde{Q}_v^c(0). \tag{22}$$

It is easy to see that the linear combinations of the currents (15)-(17) and (19)-(21):

$$2J^\mu_{3V}(t_1,t_2) - \bar v^\mu\left[J_{2V}(t_1,t_2) - J_{4V}(t_1,t_2)\right] = 2\epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_t^\mu q_2^b(t_2 n)\right]\tilde Q_v^c(0), \tag{23}$$

$$J^\mu_{2T}(t_1,t_2) + J^\mu_{4T}(t_1,t_2) - 2\bar v^\mu\, J_{3T}(t_1,t_2) = 2\epsilon_{abc}\left[q_1^{aT}(t_1 n)\mathcal{C}\gamma_t^\mu \not v q_2^b(t_2 n)\right]\tilde Q_v^c(0), \tag{24}$$

reduce to the non-vanishing local currents with the matrix elements:

$$\left\langle 0\left|\epsilon_{abc}\left[q_1^a(0)\mathcal{C}\gamma_t^\mu q_2^b(0)\right]\tilde Q_v^c(0)\right|\tilde H_Q(v,\eta)\right\rangle = \tilde\lambda_1\,\eta^\mu, \tag{25}$$

$$\left\langle 0\left|\epsilon_{abc}\left[q_1^a(0)\mathcal{C}\gamma_t^\mu \not v q_2^b(0)\right]\tilde Q_v^c(0)\right|\tilde H_Q(v,\eta)\right\rangle = \tilde\lambda_2\,\eta^\mu. \tag{26}$$

Here, $\bar v^\mu = (n^\mu - \bar n^\mu)/2$ (with $\bar v^2 = -1$ and $(v\bar v) = 0$) and $\gamma_t^\mu = \gamma_\perp^\mu - \not\bar v\,\bar v^\mu$. With the definitions above, the matrix elements of the non-local operators can be determined as follows:

$$\begin{aligned}
\left\langle 0\left|J_{2V}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= \tilde\lambda_1\,(n\eta)\,\Psi_{2V}(t_1,t_2), &(27)\\
\left\langle 0\left|J^\mu_{3V}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= \tilde\lambda_1\,\eta_\perp^\mu\,\Psi_{3V}(t_1,t_2), &(28)\\
\left\langle 0\left|J_{4V}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= -\tilde\lambda_1\,(\bar n\eta)\,\Psi_{4V}(t_1,t_2), &(29)\\
\left\langle 0\left|J^\mu_{3A}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= \tilde\lambda_1\,\eta_\perp^\mu\,\Psi_{3A}(t_1,t_2), &(30)\\
\left\langle 0\left|J^\mu_{2T}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= \tilde\lambda_2\,\eta_\perp^\mu\,\Psi_{2T}(t_1,t_2), &(31)\\
\left\langle 0\left|J_{3T}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= \tilde\lambda_2\,(\bar v\eta)\,\Psi_{3T}(t_1,t_2), &(32)\\
\left\langle 0\left|J^\mu_{4T}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= \tilde\lambda_2\,\eta_\perp^\mu\,\Psi_{4T}(t_1,t_2), &(33)\\
\left\langle 0\left|J_{3S}(t_1,t_2)\right|\tilde H_Q(v,\eta)\right\rangle &= \tilde\lambda_2\,(\bar v\eta)\,\Psi_{3S}(t_1,t_2), &(34)
\end{aligned}$$

where $\eta_\perp^\mu = \eta^\mu + \bar v^\mu\,(\bar v\eta)$. In the $SU(3)_F$-symmetry limit, the LCDAs $\Psi_{iV}(t_1,t_2)$ and $\Psi_{iT}(t_1,t_2)$ are symmetric under the exchange $t_1 \leftrightarrow t_2$ and normalized as $\Psi_{iV}(0,0) = \Psi_{iT}(0,0) = 1$ while $\Psi_{3S}(t_1,t_2)$ and $\Psi_{3A}(t_1,t_2)$ are antisymmetric and, hence, satisfy the condition $\Psi_{3A}(0,0) = \Psi_{3S}(0,0) = 0$. The breaking of the $SU(3)_{\rm F}$-symmetry results in the violation of the LCDAs symmetry properties.

The transition to the real heavy-quark field $Q_v^c(0)$ in the non-local currents (15)-(22) is equivalent to the replacement $\tilde Q_v^c(0) \to \Gamma' Q_v^c(0)$, where the matrix Γ' gives the right quantum numbers of the specific baryon. This also improves the r.h.s. of the definitions (27)-(34) but the set of the LCDAs remains the same (for details see [8]).

3 Scale-Dependence of Matrix Elements

A non-relativistic constituent quark picture of the Λ_b suggests that $f^{(2)}_{H_Q} \simeq f^{(1)}_{H_Q}$ at low scales of order 1 GeV, and this expectation is supported by numerous QCD sum rule calculations [10,11, 16,17]. In fact, the difference between the two couplings is only obtained at the level of NLO perturbative corrections to the sum rules [11,17] and it is numerically small.

The scale dependence of the couplings is given by

$$f^{(i)}_{H_Q}(\mu) = f^{(i)}_{H_Q}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_1^{(i)}/\beta_0}\left[1 - \frac{\alpha_s(\mu_0)-\alpha_s(\mu)}{4\pi}\,\frac{\gamma_1^{(i)}}{\beta_0}\left(\frac{\gamma_2^{(i)}}{\gamma_1^{(i)}} - \frac{\beta_1}{\beta_0}\right)\right], \tag{35}$$

where the anomalous dimensions of the local interpolating operators:

$$\frac{d\ln f^{(i)}_{H_Q}(\mu)}{d\ln\mu} \equiv -\gamma^{(i)} = -\sum_k \gamma_k^{(i)}\,a^k(\mu), \qquad a(\mu) \equiv \frac{\alpha_s^{\overline{\rm MS}}(\mu)}{4\pi}, \tag{36}$$

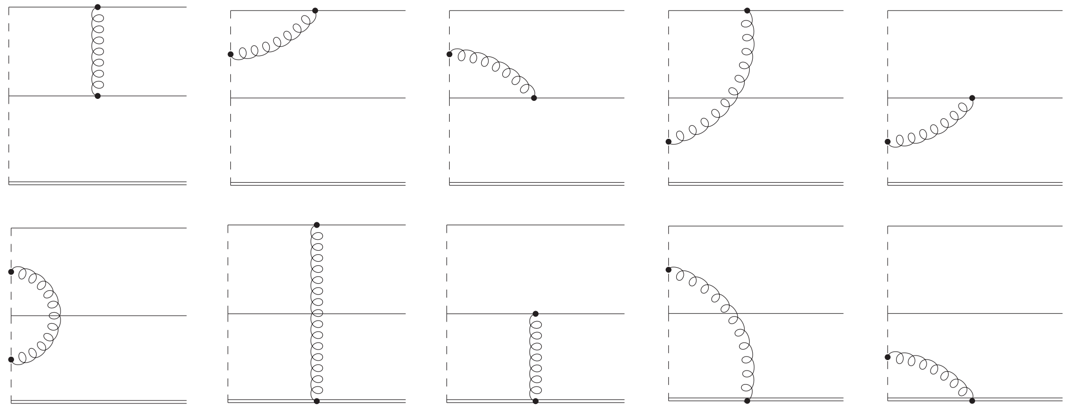

Figure 2: The total set of one-gluon-exchange diagrams for calculating the scale-dependence of LCDAs of the heavy baryon. Normal and bold solid lines correspond to light and heavy quarks, respectively. Dashed and curly lines describe the Wilson links and exchanged gluons.

are known to NLO [17], and the β-function expansion:

$$\frac{da(\mu)}{d\ln\mu} = -\beta_0\, a^2(\mu) - \beta_1\, a^3(\mu) + \cdots\,, \tag{37}$$

was used to NLO with the coefficients $\beta_0 = 2\,(11 - 2n_f/3)$ and $\beta_1 = 4\,(51 - 19n_f/3)$.

For the numerical value of the couplings, let us quote the result of the NLO QCD sum rule analysis in Ref. [7,11]:

$$f^{(1)}_{\Lambda_b}(1\ \mathrm{GeV}) \simeq f^{(2)}_{\Lambda_b}(1\ \mathrm{GeV}) \simeq (0.030 \pm 0.005)\ \ \mathrm{GeV}^3, \tag{38}$$

at the renormalization scale $\mu = 1$ GeV. Note that these couplings cannot coincide at all scales since the corresponding operators have different anomalous dimensions.

The LCDAs introduced in Sec. 2 are scale dependent. To work it out, it is convenient to make a Fourier transform of LCDAs into the momentum space:

$$\Psi_k(t_1,t_2) = \int\limits_0^\infty d\omega_1 \int\limits_0^\infty d\omega_2\, e^{-i(t_1\omega_1+t_2\omega_2)}\psi_k(\omega_1,\omega_2) = \int\limits_0^\infty \omega\, d\omega \int\limits_0^1 du\, e^{-i\omega(t_1u+t_2\bar{u})}\widetilde{\psi}_k(\omega,u), \tag{39}$$

so that $\widetilde{\psi}_k(\omega,u) = \psi_k(u\omega,\bar{u}\omega)$ with $\bar{u} = 1-u$. In the first representation ω_1 and ω_2 are the energies of the light quarks and in the second one $\omega = \omega_1+\omega_2$ is the total energy carried by light quarks (in the heavy-quark rest frame) whereas the dimensionless variable u corresponds to the energy fraction carried by the quark called q_1.

The leading-order (LO) evolution equation for the leading-twist LCDAs $\psi_2(\omega_1,\omega_2;\mu)$ can be derived following the usual procedure [7] by identifying the ultraviolet singularities of one-gluon-exchange diagrams presented in Fig 2. The result can be expressed in terms of the two-particle kernels familiar from the evolution equations of the B-meson and π-meson LCDAs [7]:

$$\begin{aligned} \mu\frac{d}{d\mu}\psi_2(\omega_1,\omega_2;\mu) = -\frac{\alpha_s(\mu)}{2\pi}\left(1+\frac{1}{N_c}\right)\Bigg\{ \int\limits_0^\infty d\omega_1'\,\gamma^{\mathrm{LN}}(\omega_1',\omega_1;\mu)\,\psi_2(\omega_1',\omega_2;\mu) \\ + \int\limits_0^\infty d\omega_2'\,\gamma^{\mathrm{LN}}(\omega_2',\omega_2;\mu)\,\psi_2(\omega_1,\omega_2';\mu) - \int\limits_0^1 dv\, V(u,v)\,\psi_2(v\omega,\bar{v}\omega;\mu) + \frac{3}{2}\,\psi_2(\omega_1,\omega_2;\mu)\Bigg\}, \end{aligned} \tag{40}$$

where the last term in the curly brackets, $3\psi_2/2$, is a result of the subtraction of the one-loop renormalization of the coupling $f^{(2)}_{H_Q}$.

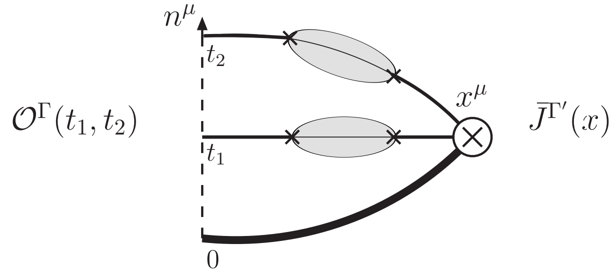

Figure 3: The correlation functions involving the non-local light-ray operator $\mathcal{O}^\Gamma(t_1, t_2)$ and a suitable local current $\bar{J}^{\Gamma'}(x)$. The bold solid line corresponds to the heavy-quark propagator and the solid lines with the blob insertion are the propagators of the light quarks which are modified by contributions of the non-local quark condensates.

The first two convolution integrals in Eq. (40) are associated with the heavy-light dynamics: each of them involves just one of the light quarks. Indeed, the kernel $\gamma^{\rm LN}(\omega', \omega; \mu)$ coincides with the one controlling the evolution of the B-meson distribution amplitude, the Lange-Neubert anomalous dimension [18]. In turn, the last convolution integral in Eq. (40) describes the interaction between the light quarks. $V(u, v)$ is the celebrated ER-BL kernel [19].

For small evolution ranges, $\ln(\mu/\mu_0) \lesssim 1$, it is sufficient to interpret the derivative on the l.h.s. of (40) as a finite difference $[\psi_2(\omega_1, \omega_2; \mu) - \psi_2(\omega_1, \omega_2; \mu_0)]/\ln(\mu/\mu_0)$ and substitute the initial condition $\psi_2(\omega_1, \omega_2; \mu_0)$ for $\psi_2(\omega_1, \omega_2; \mu)$ on the r.h.s. Obviously, this corresponds to taking into account one-loop renormalization only, neglecting the resummation of potentially large logarithms. As it was demonstrated in [7], this single-evolution-step (one-loop) approximation is quite good in practice, e. g. for $\mu_0 = 1$ GeV and $\mu \simeq m_b/2$. In order to go beyond the one-loop approximation, one possibility is to integrate the evolution equation (40) numerically. In Ref. [7], the other, semi-analytic, approach have been suggested which has an advantage that it allows one to understand the structure of the solution.

4 QCD Sum Rules

1. Models for the heavy-baryon LCDAs can be obtained by applying the QCD sum rules to the correlation functions involving the non-local light-ray operator $\mathcal{O}^\Gamma(t_1, t_2)$ and a suitable local current $\bar{J}^{\Gamma'}(x)$ which is shown in Fig. 3. For a suitable local current of the heavy baryons one can take

$$\bar{J}^{\Gamma'}(x) = A\, \bar{J}_1^{H_Q} + B\, \bar{J}_2^{H_Q} = \varepsilon_{abc} \left[\bar{q}_2^a(x)\, (A + B\not{v})\, \Gamma' \mathcal{C}^T \bar{q}_1^{bT}(x) \right] \bar{Q}_v^c(x), \tag{41}$$

where $\mathcal{C}^T$ is the matrix transpose to $\mathcal{C}$. Specific values of the coefficients A and B allow to account for the variation caused by the uncertainty in the choice of the local current. The currents of the type $\bar{J}_1^{H_Q}(x)$ and $\bar{J}_2^{H_Q}(x)$ have been used in Ref. [11] to write the diagonal, non-diagonal and constituent-type sum rules. For the baryons with the diquark spin-parity $j^p = 0^+$, one should take $\Gamma' = \gamma_5$ and for the $j^p = 1^+$ diquark in the baryon, there are two possibilities $\Gamma' = \gamma_\parallel$ or $\Gamma' = \gamma_\perp$. The current $\bar{J}_\Lambda(x)$ adopted in Ref. [7] for the Λ_b-baryon, having $j^p = 0^+$ of the diquark, is the current (41) with $A = B = 1/2$, which picks up the contributions of both even and odd dimensions, i.e. $\bar{J}_2^{\Lambda_b}(x)$ produces the perturbative theory and quartic condensate contributions, while $\bar{J}_1^{\Lambda_b}(x)$ results into the non-local quark condensates.

The procedure of constructing the QCD sum rules is well known and results the following general form:

$$f_k \left(A f_{H_Q}^{(1)} + B f_{H_Q}^{(2)} \right) \tilde{\psi}_k^{SR}(t_1, t_2)\, \mathrm{e}^{-\bar{\Lambda}/\tau} = \mathcal{B}[\Pi_k](t_1, t_2; \tau, s_0), \tag{42}$$

where $f_k = f_{H_Q}^{(2)}$ for the even twists and $f_k = f_{H_Q}^{(1)}$ for the twist-3 distribution amplitudes. The effective baryon mass is introduced as the difference $\bar{\Lambda} = m_{H_Q} - m_Q$ with m_Q being the mass of the heavy quark, τ is the Borel parameter, and s_0 is the continuum threshold. The r.h.s. in Eq. (42) is the Borel-transformed continuum-subtracted invariant function determined through the correlation function $\Pi_k(t_1, t_2; \tau)$.

For practical applications, one needs to know the LCDAs in the momentum space. The Fourier transform of the LCDAs has been defined in Eq. (39) and the one of the scalar correlation function is as follows:

$$\Pi_k(\omega, u; \tau) = \int\limits_{-\infty}^{\infty} \frac{dt_1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{dt_2}{2\pi} \, \mathrm{e}^{i\omega(ut_1 + \bar{u}t_2)} \, \Pi_k(t_1, t_2; \tau). \tag{43}$$

In the momentum space, the sum rules (42) can be rewritten in the following form:

$$f_k \left(A f_{H_Q}^{(1)} + B f_{H_Q}^{(2)} \right) \tilde{\psi}_k^{SR}(\omega, u) \, \mathrm{e}^{-\bar{\Lambda}/\tau} = \mathcal{B}[\Pi_k](\omega, u; \tau, s_0). \tag{44}$$

Taking into account the leading-order perturbative contribution to the sum rules only, one obtains:

$$\begin{aligned}
\tilde{\psi}_2(\omega, u) &= \frac{30\tau^4}{\mathcal{N}} \left[\hat{\omega}^2 \, u\bar{u} + \frac{A}{B} \hat{\omega} \left(\hat{m}_2 u + \hat{m}_1 \bar{u} \right) \right] E_1 \left(2\hat{s}_\omega \right) \mathrm{e}^{-\hat{\omega}}, && (45) \\
\tilde{\psi}_4(\omega, u) &= \frac{30\tau^4}{\mathcal{N}} \left[E_3 \left(2\hat{s}_\omega \right) + \frac{A}{B} \left(\hat{m}_1 + \hat{m}_2 \right) E_2 \left(2\hat{s}_\omega \right) \right] \mathrm{e}^{-\hat{\omega}}, && (46) \\
\tilde{\psi}_{3s}(\omega, u) &= \frac{15\tau^4}{\mathcal{N}} \left\{ \left[\hat{\omega} + \frac{B}{A} \left(\hat{m}_1 + \hat{m}_2 \right) \right] E_2 \left(2\hat{s}_\omega \right) \right. && (47) \\
&\qquad \left. + \frac{B}{A} \hat{\omega} \left(\hat{m}_2 u + \hat{m}_1 \bar{u} \right) E_1 \left(2\hat{s}_\omega \right) \right\} \mathrm{e}^{-\hat{\omega}}, \\
\tilde{\psi}_{3\sigma}(\omega, u) &= \frac{15\tau^4}{\mathcal{N}} \left\{ \left[\hat{\omega} \left(u - \bar{u} \right) + \frac{B}{A} \left(\hat{m}_1 - \hat{m}_2 \right) \right] E_2 \left(2\hat{s}_\omega \right) \right. && (48) \\
&\qquad \left. + \frac{B}{A} \hat{\omega} \left(\hat{m}_2 u - \hat{m}_1 \bar{u} \right) E_1 \left(2\hat{s}_\omega \right) \right\} \mathrm{e}^{-\hat{\omega}},
\end{aligned}$$

where $s_\omega = s_0 - \omega/2$, $\hat{\omega} = \omega/(2\tau)$, $\hat{s}_\omega = s_\omega/(2\tau)$, and $\hat{m}_{1,2} = m_{1,2}/(2\tau)$. The normalization integral $\mathcal{N}$ is introduced via the decay constant:

$$\left| f_{H_Q}^{(i)} \right|^2 \mathrm{e}^{-\bar{\Lambda}/\tau} = \frac{1}{20\pi^4} \int_0^{s_0} ds \, s^5 \, \mathrm{e}^{-s/\tau} \equiv \frac{\mathcal{N}}{20\pi^4}. \tag{49}$$

In Eqs. (45)-(48) it appears convenient to introduce the function:

$$E_a(x) = \frac{1}{\Gamma(a+1)} \int_0^x dt \, t^a \mathrm{e}^{-t} = 1 - \frac{\Gamma(a+1, x)}{\Gamma(a+1)}, \tag{50}$$

where $\Gamma(a+1, x) = \int_x^\infty dt \, t^a \mathrm{e}^{-t}$ is the incomplete Γ-function. For integer values of the parameter ($a = N$), this function is reduced to the well-known representation:

$$E_N(x) = 1 - \mathrm{e}^{-x} \sum_{n=0}^{N} \frac{x^n}{n!}, \tag{51}$$

In practical numerical estimations, it is more convenient to use the integration by parts

$$E_a(x) = E_{a+1}(x) + \frac{x^{a+1} \, \mathrm{e}^{-x}}{\Gamma(a+2)} \tag{52}$$

to re-express the negative value of the parameter a in $E_a(x)$ through a positive one, as done in [7].

In order to evaluate the non-perturbative contributions to the sum rules, one is forced to use the non-local quark condensates as explained in Refs. [20, 21] for the B-meson case. In the present analysis, we use the general parametrization [22, 23]

$$\langle \bar{q}(x) q(0) \rangle = \langle \bar{q} q \rangle \int\limits_0^\infty d\nu \, \mathrm{e}^{\nu x^2/4} \, f(\nu), \tag{53}$$

where $\langle \bar{q} q \rangle$ is the local quark condensate and $f(\nu)$ is the model function [21, 24]:

$$f(\nu) = \frac{\lambda^{a-2}}{\Gamma(a-2)} \, \nu^{1-a} \, \mathrm{e}^{-\lambda/\nu}, \qquad a - 3 = \frac{4\lambda}{m_0^2}. \tag{54}$$

The parameters λ and m_0^2 entering the model function $f(\nu)$ have the meanings of the correlation length and the ratio of the mixed quark-gluon and quark local condensates. A more comprehensive analysis of the non-local quark and gluon condensates has been undertaken in Ref. [25].

Taking into account the non-local quark condensates, the sum rules are as follows:

$$\begin{aligned} & f_{HQ}^{(2)} \left(A f_{HQ}^{(1)} + B f_{HQ}^{(2)} \right) \tilde{\psi}_2^{SR}(\omega, u) \, \mathrm{e}^{-\bar{\Lambda}/\tau} = \\ & \quad \frac{3\tau^4}{2\pi^4} \left[B\hat{\omega}^2 \, u\bar{u} + A \, \hat{\omega} \, (\hat{m}_2 u + \hat{m}_1 \bar{u}) \right] E_1(2\hat{s}_\omega) \mathrm{e}^{-\hat{\omega}} \\ & \quad - \frac{\langle \bar{q}_1 q_1 \rangle \tau^3}{\pi^2} \left[A\hat{\omega}\bar{u} + B\hat{m}_2 \right] f(2\tau\omega u) \, E_{2-a}(2\hat{s}_\kappa) \, \mathrm{e}^{-\hat{\omega}} \\ & \quad - \frac{\langle \bar{q}_2 q_2 \rangle \tau^3}{\pi^2} \left[A\hat{\omega} u + B\hat{m}_1 \right] f(2\tau\omega\bar{u}) \, E_{2-a}(2\hat{s}_{\bar{\kappa}}) \, \mathrm{e}^{-\hat{\omega}} \\ & \quad + \frac{2B}{3} \, \langle \bar{q}_1 q_1 \rangle \, \langle \bar{q}_2 q_2 \rangle \, \tau^2 \, f(2\tau\omega u) \, f(2\tau\omega\bar{u}) \, E_{3-2a}(2\hat{s}_{\kappa\bar{\kappa}}) \, \mathrm{e}^{-\hat{\omega}}, \end{aligned} \tag{55}$$

$$\begin{aligned} & f_{HQ}^{(2)} \left(A f_{HQ}^{(1)} + B f_{HQ}^{(2)} \right) \tilde{\psi}_4^{SR}(\omega, u) \, \mathrm{e}^{-\bar{\Lambda}/\tau} = \\ & \quad \frac{3\tau^4}{2\pi^4} \left[B \, E_3(2\hat{s}_\omega) + A \, (\hat{m}_1 + \hat{m}_2) \, E_2(2\hat{s}_\omega) \right] \mathrm{e}^{-\hat{\omega}} \\ & \quad - \frac{\langle \bar{q}_1 q_1 \rangle \tau^3}{\pi^2} \left[A \, E_{3-a}(2\hat{s}_\kappa) + B\hat{m}_2 \, E_{2-a}(2\hat{s}_\kappa) \right] f(2\tau\omega u) \, \mathrm{e}^{-\hat{\omega}} \\ & \quad - \frac{\langle \bar{q}_2 q_2 \rangle \tau^3}{\pi^2} \left[A \, E_{3-a}(2\hat{s}_{\bar{\kappa}}) + B\hat{m}_1 \, E_{2-a}(2\hat{s}_{\bar{\kappa}}) \right] f(2\tau\omega\bar{u}) \, \mathrm{e}^{-\hat{\omega}} \\ & \quad + \frac{2B}{3} \, \langle \bar{q}_1 q_1 \rangle \, \langle \bar{q}_2 q_2 \rangle \, \tau^2 \, f(2\tau\omega u) \, f(2\tau\omega\bar{u}) \, E_{3-2a}(2\hat{s}_{\kappa\bar{\kappa}}) \, \mathrm{e}^{-\hat{\omega}}, \end{aligned} \tag{56}$$

$$\begin{aligned} & f_{HQ}^{(1)} \left(A f_{HQ}^{(1)} + B f_{HQ}^{(2)} \right) \tilde{\psi}_{3s}^{SR}(\omega, u) \, \mathrm{e}^{-\bar{\Lambda}/\tau} = \\ & \quad \frac{3\tau^4}{4\pi^4} \left\{ \left[A\hat{\omega} + B \, (\hat{m}_1 + \hat{m}_2) \right] E_2(2\hat{s}_\omega) + B\hat{\omega} \, (\hat{m}_2 u + \hat{m}_1 \bar{u}) \, E_1(2\hat{s}_\omega) \right\} \mathrm{e}^{-\hat{\omega}} \\ & \quad - \frac{\langle \bar{q}_1 q_1 \rangle \tau^3}{2\pi^2} \left[B \, E_{3-a}(2\hat{s}_\kappa) + (B\hat{\omega}\bar{u} + 2A\hat{m}_2) \, E_{2-a}(2\hat{s}_\kappa) \right] f(2\tau\omega u) \, \mathrm{e}^{-\hat{\omega}} \\ & \quad - \frac{\langle \bar{q}_2 q_2 \rangle \tau^3}{2\pi^2} \left[B \, E_{3-a}(2\hat{s}_{\bar{\kappa}}) + (B\hat{\omega} u + 2A\hat{m}_1) \, E_{2-a}(2\hat{s}_{\bar{\kappa}}) \right] f(2\tau\omega\bar{u}) \, \mathrm{e}^{-\hat{\omega}} \\ & \quad + \frac{2A}{3} \, \langle \bar{q}_1 q_1 \rangle \, \langle \bar{q}_2 q_2 \rangle \, \tau^2 \, f(2\tau\omega u) \, f(2\tau\omega\bar{u}) \, E_{3-2a}(2\hat{s}_{\kappa\bar{\kappa}}) \, \mathrm{e}^{-\hat{\omega}}, \end{aligned} \tag{57}$$

Table 1: Experimental measurements and theoretical predictions (based on HQET [27] and Lattice QCD [28]) for the masses of the ground-state bottom baryons (in units of MeV). Here, $\bar{\Lambda} = m_{H_b} - m_b$ and the continuum thresholds s_0 in HQET for $m_b = 4.8$ GeV are also given (in units of GeV).

Baryon	J^P	Experiment [26]	HQET [27]	Lattice QCD [28]	$\bar{\Lambda}$	s_0
Λ_b	$1/2^+$	5620.0 ± 1.6	5637^{+68}_{-56}	$5641 \pm 21^{+15}_{-33}$	0.8	1.2
Σ_b^+	$1/2^+$	5807.8 ± 2.7	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$	1.0	1.3
Σ_b^-	$1/2^+$	5815.2 ± 2.0	5809^{+82}_{-76}	$5795 \pm 16^{+17}_{-26}$	1.0	1.3
Σ_b^{*+}	$3/2^+$	5829.0 ± 3.4	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$	1.0	1.3
Σ_b^{*-}	$3/2^+$	5836.4 ± 2.8	5835^{+82}_{-77}	$5842 \pm 26^{+20}_{-18}$	1.0	1.3
Ξ_b^-	$1/2^+$	5790.5 ± 2.7	5780^{+73}_{-68}	$5781 \pm 17^{+17}_{-16}$	1.0	1.3
Ξ_b'	$1/2^+$		5903^{+81}_{-79}	$5903 \pm 12^{+18}_{-19}$	1.1	1.4
$\Xi_b'^*$	$3/2^+$		5903^{+81}_{-79}	$5950 \pm 21^{+19}_{-21}$	1.1	1.4
Ω_b^-	$1/2^+$	6071 ± 40	6036 ± 81	$6006 \pm 10^{+20}_{-19}$	1.3	1.5
Ω_b^*	$3/2^+$		6063^{+83}_{-82}	$6044 \pm 18^{+20}_{-21}$	1.3	1.5

$$
\begin{aligned}
& f^{(1)}_{HQ}\left(A f^{(1)}_{HQ} + B f^{(2)}_{HQ}\right)\tilde{\psi}^{SR}_{3\sigma}(\omega,u)\,\mathrm{e}^{-\bar{\Lambda}/\tau} = \\
& \quad \frac{3\tau^4}{4\pi^4}\left\{\left[A\hat{\omega}\,(u-\bar{u}) + B\,(\hat{m}_1-\hat{m}_2)\right]E_2(2\hat{s}_\omega) + B\hat{\omega}\,(\hat{m}_2 u - \hat{m}_1\bar{u})E_1(2\hat{s}_\omega)\right\}\mathrm{e}^{-\hat{\omega}} \\
& \quad -\frac{B\langle\bar{q}_1 q_1\rangle\tau^3}{2\pi^2}\left[E_{3-a}(2\hat{s}_\kappa) - \hat{\omega}\bar{u}\,E_{2-a}(2\hat{s}_\kappa)\right]f(2\tau\omega u)\,\mathrm{e}^{-\hat{\omega}} \\
& \quad +\frac{B\langle\bar{q}_2 q_2\rangle\tau^3}{2\pi^2}\left[E_{3-a}(2\hat{s}_{\bar{\kappa}}) - \hat{\omega}u\,E_{2-a}(2\hat{s}_{\bar{\kappa}})\right]f(2\tau\omega\bar{u})\,\mathrm{e}^{-\hat{\omega}},
\end{aligned}
\tag{58}
$$

where $\hat{s}_\kappa = \hat{s}_\omega - \kappa/2$, $\hat{s}_{\bar{\kappa}} = \hat{s}_\omega - \bar{\kappa}/2$, $\hat{s}_{\kappa\bar{\kappa}} = \hat{s}_\omega - \kappa/2 - \bar{\kappa}/2$, and the short-hand notations are used:

$$
\kappa = \frac{\lambda}{2u\omega\tau}, \qquad \bar{\kappa} = \frac{\lambda}{2\bar{u}\omega\tau}. \tag{59}
$$

The QCD sum rules for the twist-2 LCDA (55) coincide with the ones in Ref. [7] in the limit of massless light quarks $\hat{m}_1 = \hat{m}_2 = 0$ when $A = B = 1/2$. The impact of the quark-hadron duality on the double condensate terms in Eqs. (55)-(57) is the appearance of the function $E_{3-2a}(2\hat{s}_{\kappa\bar{\kappa}})$ which in the local limit terns out to be unit, as originally obtained in Ref. [7].

In a similar way, the QCD sum rules can be obtained for heavy baryons containing the diquark with the spin-parity $j^p = 1^+$, and corresponding equations are presented in Ref. [8].

5 Numerical analysis

To perform the numerical analysis, it is necessary to specify required input parameters. The values of effective baryon masses $\bar{\Lambda} = m_{H_b} - m_b$ in HQET for $m_b = 4.8$ GeV are presented in Table 1 where experimental measurements [26] and theoretical predictions (based on HQET [27] and Lattice QCD [28]) for the masses (in units of MeV) of the ground-state bottom baryons are also shown. The comparative analysis of the predictions for the heavy baryon masses can be found in Refs. [27,29]. The continuum threshold values s_0 (the last column in Table 1) used by us are in agreement with ones from [27], used for the baryon mass evaluation to order $1/m_b$ within HQET. The other input parameters are presented in Table 2. For the discussion of these

Table 2: Numerical values of the parameters entering the QCD sum rules for the LCDAs of the bottom baryons.

τ	(0.6 ± 0.2) GeV	m_s (1 GeV)	(128 ± 21) MeV
$\langle \bar{q}q \rangle$ (1 GeV)	$-(242^{+28}_{-19})$ MeV^3	$\langle \bar{s}s \rangle / \langle \bar{q}q \rangle$	0.8 ± 0.2
m_0^2	(0.8 ± 0.2) GeV^2	λ	0.16 GeV^2

parameters see [30] and references therein. Note that the shape function $f(\nu)$ in the non-local quark condensate is assumed to be flavor independent for all light quarks.

These QCD sum rules constrain certain momentum fraction integrals (the moments). Let us define such moments as follows:

$$\langle f(\omega, u) \rangle_k^{H_Q} \equiv \int_0^{2s_0} \omega d\omega \int_0^1 du\, f(\omega, u)\, \tilde{\psi}_t^{\rm SR}(\omega, u), \tag{60}$$

where $t = 2, 3s, 3\sigma, 4$.

For the leading twist LCDAs ($t = 2$), one can fix the LCDAs normalization by

$$\int_0^{2s_0} \omega d\omega \int_0^1 du\, \tilde{\psi}_2^{\rm SR}(\omega, u) \equiv 1. \tag{61}$$

Estimates of the moments for the Λ_b- and Ξ_b-baryons are presented in Table 3. In this case, it is convenient to make an expansion in terms of the Gegenbauer polynomials $C_n^{3/2}(2u-1)$ which are orthogonal with respect to the asymptotic behavior $\sim u(1-u)$ of the leading twist LCDA (55) (in the massless limit). The errors correspond to the variation of the parameters A and B in the range: $0 \le A,\ B \le 1$, keeping the condition $A + B = 1$, and the central values of the other input parameters are given in Table 2. As a check, we reproduced the numerical values for the moments [7] corresponding to the leading twist LCDA of the Λ_b-baryon. All the moments of the Λ_b-baryon calculated with respect to the Gegenbauer polynomials of the odd order are equal to zero as a consequence of the symmetry of this function under the interchange $u \leftrightarrow 1-u$. This is not true anymore for the Ξ_b-baryon if the $SU(3)_{\rm F}$-breaking corrections are taken into account due to the non-vanishing s-quark mass and the difference between the strange $\langle \bar{s}s \rangle$ and non-strange $\langle \bar{q}q \rangle$ local condensates. From Table 3, one can easily see that these corrections yield typically $\sim 10\%$ effects.

For the twist-3 LCDAs $\tilde{\psi}_{3s}^{\rm SR}(\omega, u)$ and $\tilde{\psi}_4^{\rm SR}(\omega, u)$, the normalization condition (61) is used. As for the integral (61) with the LCDA $\tilde{\psi}_{3\sigma}^{\rm SR}(\omega, u)$, it turns out to be zero in the $SU(3)_{\rm F}$ symmetry limit, as this function is antisymmetric under the interchange $u \leftrightarrow 1-u$. To avoid this problem, we use for the $\tilde{\psi}_{3\sigma}^{\rm SR}(\omega, u)$ LCDA the following normalization condition:

$$\int_0^{2s_0} \omega d\omega \int_0^1 du\, C_1^{1/2}(2u-1)\, \tilde{\psi}_k^{\rm SR}(\omega, u) \equiv 1. \tag{62}$$

Note that for these LCDAs the expansion with respect to the $C_n^{1/2}(2u-1)$ Gegenbauer polynomials is more suitable. This is motivated by the analysis of the energetic π-meson for which the twist-3 $\phi_p(x)$ and twist-4 $\phi_3(x)$ LCDAs are described by the $C_n^{1/2}(2u-1)$ Gegenbauer polynomials [31–33].

Numerical estimates for the Λ_b- and Ξ_b-baryon moments based on the QCDSRs are presented in Table 3. As explained above, the errors correspond to the variation of the parameters A and B while the central values of the other input parameters are taken from Table 2 and kept fixed

Table 3: Numerical values of the first several moments of the heavy baryon LCDAs estimated by the QCDSRs. The moments $\langle\omega^{-1}\rangle$, $\langle\omega^{-1}C_n^{3/2}\rangle$ and $\langle\omega^{-1}C_n^{1/2}\rangle$ are dimensionful, with the entries below given in units of GeV^{-1}, while $\langle C_n^{3/2}\rangle$ and $\langle C_n^{1/2}\rangle$ are dimensionless.

H_Q	t	$\langle\omega^{-1}\rangle$	$\langle C_1^{3/2}\rangle$	$\langle\omega^{-1}C_1^{3/2}\rangle$	$\langle C_2^{3/2}\rangle$	$\langle\omega^{-1}C_2^{3/2}\rangle$	$\langle C_3^{3/2}\rangle$	$\langle\omega^{-1}C_3^{3/2}\rangle$
Λ_b	2	$1.65^{+0.91}_{-0.47}$	0	0	$1.00^{+0.54}_{-1.03}$	$0.61^{+0.76}_{-1.45}$	0	0
Ξ_b	2	$1.46^{+0.54}_{-0.34}$	$0.10^{+0.10}_{-0.06}$	$0.08^{+0.07}_{-0.05}$	$1.15^{+0.61}_{-0.98}$	$0.86^{+0.68}_{-1.10}$	$-0.02^{+0.32}_{-0.52}$	$-0.10^{+0.24}_{-0.38}$
H_Q	t	$\langle\omega^{-1}\rangle$	$\langle C_1^{1/2}\rangle$	$\langle\omega^{-1}C_1^{1/2}\rangle$	$\langle C_2^{1/2}\rangle$	$\langle\omega^{-1}C_2^{1/2}\rangle$	$\langle C_3^{1/2}\rangle$	$\langle\omega^{-1}C_3^{1/2}\rangle$
Λ_b	3s	$2.16^{+0.70}_{-0.36}$	0	0	$-0.032^{+0.022}_{-0.041}$	$-0.29^{+0.14}_{-0.27}$	0	0
	3σ	0	1	$1.54^{+0.14}_{-0.22}$	0	0	$-0.034^{+0.034}_{-0.021}$	$-0.027^{+0.027}_{-0.017}$
	4	$2.84^{+0.88}_{-0.46}$	0	0	$-0.108^{+0.035}_{-0.018}$	$-0.41^{+0.08}_{-0.15}$	0	0
Ξ_b	3s	$1.94^{+0.33}_{-0.21}$	$0.11^{+0.09}_{-0.05}$	$0.075^{+0.077}_{-0.047}$	$1.05^{+0.14}_{-0.23}$	$1.01^{+0.28}_{-0.46}$	$-0.014^{+0.051}_{-0.032}$	$-0.117^{+0.002}_{-0.005}$
	3σ	$0.0019^{+0.0014}_{-0.0019}$	1	$1.37^{+0.11}_{-0.14}$	$0.057^{+0.043}_{-0.057}$	$0.098^{+0.075}_{-0.098}$	$1.11^{+0.46}_{-0.35}$	$1.55^{+0.24}_{-0.32}$
	4	$2.73^{+0.61}_{-0.35}$	$0.12^{+0.09}_{-0.05}$	$0.05^{+0.09}_{-0.05}$	$0.55^{+0.18}_{-0.11}$	$0.99^{+0.16}_{-0.09}$	$-0.043^{+0.025}_{-0.015}$	$-0.18^{+0.02}_{-0.03}$

at their central values. It is worth noting, that for the Ξ_b-baryon, the integral (61) becomes non-zero because of the $SU(3)_F$-symmetry breaking, but numerically it is small:

$$\int_0^{2s_0} \omega d\omega \int_0^1 du\, \tilde{\psi}_k^{\rm SR}(\omega,u) = -0.0049^{+0.0049}_{-0.0037}. \tag{63}$$

The same is true also for the $\langle\omega^{-1}\rangle$ moment as seen from Table 3.

We propose the following simple models for the baryon LCDAs at the low scale $\mu = 1$ GeV:

$$\tilde{\psi}_2(\omega,u) = \omega^2 u(1-u) \sum_{n=0}^{2} \frac{a_n^{(2)}}{{\epsilon_n^{(2)}}^4} C_n^{3/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(2)}}, \tag{64}$$

$$\tilde{\psi}_{3s}(\omega,u) = \frac{\omega}{2} \sum_{n=0}^{2} \frac{a_n^{(3)}}{{\epsilon_n^{(3)}}^3} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(3)}}, \tag{65}$$

$$\tilde{\psi}_{3\sigma}(\omega,u) = \frac{\omega}{2} \sum_{n=0}^{3} \frac{b_n^{(3)}}{{\eta_n^{(3)}}^3} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\eta_n^{(3)}}, \tag{66}$$

$$\tilde{\psi}_4(\omega,u) = \sum_{n=0}^{2} \frac{a_n^{(4)}}{{\epsilon_n^{(4)}}^2} C_n^{1/2}(2u-1)\, \mathrm{e}^{-\omega/\epsilon_n^{(4)}}, \tag{67}$$

where $C_0^{1/2}(2u-1) = C_0^{3/2}(2u-1) = 1$. The Gegenbauer moments of the zeroth order are defined as $a_0^{(k)} \equiv 1$ which follows from the normalization:

$$\int_0^{\infty} \omega d\omega \int_0^1 du\, \tilde{\psi}_k(\omega,u) \equiv 1, \tag{68}$$

where $k = 2,\ 3s,\ 4$. In the construction of the models for the LCDAs, we have truncated the Gegenbauer expansion at the second non-asymptotic term.

Table 4: Estimates of the parameters entering the theoretical models for the heavy baryon LCDAs based on the QCDSRs values of the moments.

H_Q	t	$\varepsilon_0^{(t)}$	$\varepsilon_1^{(t)}$	$\varepsilon_2^{(t)}$	$a_1^{(t)}$	$a_2^{(t)}$
Λ_b	2	$0.201^{+0.143}_{-0.059}$	0	$0.551^{+\infty}_{-0.356}$	0	$0.391^{+0.279}_{-0.279}$
	3	$0.232^{+0.047}_{-0.056}$	0	$0.055^{+0.010}_{-0.020}$	0	$-0.161^{+0.108}_{-0.207}$
	4	$0.352^{+0.067}_{-0.083}$	0	$0.262^{+0.116}_{-0.132}$	0	$-0.541^{+0.173}_{-0.090}$
Ξ_b	2	$0.228^{+0.068}_{-0.061}$	$0.429^{+0.654}_{-0.281}$	$0.449^{+\infty}_{-0.473}$	$0.057^{+0.055}_{-0.034}$	$0.449^{+0.236}_{-0.380}$
	3	$0.258^{+0.031}_{-0.038}$	$0.750^{+0.308}_{-0.093}$	$0.520^{+0.229}_{-0.060}$	$0.339^{+0.261}_{-0.160}$	$5.244^{+0.696}_{-1.132}$
	4	$0.378^{+0.065}_{-0.080}$	$2.291^{+\infty}_{-0.842}$	$0.286^{+0.130}_{-0.150}$	$0.039^{+0.030}_{-0.018}$	$-0.090^{+0.037}_{-0.021}$
H_Q	t	$\eta_1^{(t)}$	$\eta_2^{(t)}$	$\eta_3^{(t)}$	$b_2^{(t)}$	$b_3^{(t)}$
Λ_b	3	$0.324^{+0.054}_{-0.026}$	0	$0.633^{+0.0??}_{-0.0??}$	0	$-0.240^{+0.240}_{-0.147}$
Ξ_b	3	$0.218^{+0.043}_{-0.047}$	$0.877^{+0.820}_{-0.152}$	$0.049^{+0.005}_{-0.012}$	$0.037^{+0.032}_{-0.019}$	$-0.027^{+0.016}_{-0.027}$

In terms of the LCDAs parameters, the moments of interest are:

$$\left\langle \omega^{-1} \right\rangle_k^{H_Q} = \left\{ \frac{1}{3\epsilon_0^{(2)}}, \frac{1}{2\epsilon_0^{(3)}}, \frac{b_0^{(3)}}{2\eta_0^{(3)}}, \frac{1}{\epsilon_0^{(4)}} \right\},$$

$$\left\langle C_1^{3/2} \right\rangle_k^{H_Q} = \left\{ \frac{9a_1^{(2)}}{5}, \frac{a_1^{(3)}}{3}, \frac{b_1^{(3)}}{3}, \frac{a_1^{(4)}}{3} \right\},$$

$$\left\langle \omega^{-1} C_1^{3/2} \right\rangle_k^{H_Q} = \left\{ \frac{3a_1^{(2)}}{5\epsilon_1^{(2)}}, \frac{a_1^{(3)}}{6\epsilon_1^{(3)}}, \frac{b_1^{(3)}}{6\eta_1^{(3)}}, \frac{a_1^{(4)}}{3\epsilon_1^{(4)}} \right\},$$

$$\left\langle C_2^{3/2} \right\rangle_k^{H_Q} = \left\{ \frac{18a_2^{(2)}}{7}, \frac{a_2^{(3)}}{5}, \frac{b_2^{(3)}}{5}, \frac{a_2^{(4)}}{5} \right\},$$

$$\left\langle \omega^{-1} C_2^{3/2} \right\rangle_k^{H_Q} = \left\{ \frac{6a_2^{(2)}}{7\epsilon_2^{(2)}}, \frac{a_2^{(3)}}{10\epsilon_2^{(3)}}, \frac{b_2^{(3)}}{10\eta_2^{(3)}}, \frac{a_2^{(4)}}{5\epsilon_0^{(4)}} \right\}.$$

In the limit of the exact $SU(3)_{\mathrm{F}}$ symmetry, all the above functions have definite symmetry properties: $\tilde{\psi}_2(\omega, u)$, $\tilde{\psi}_{3s}(\omega, u)$, and $\tilde{\psi}_4(\omega, u)$ are symmetric under the exchange $u \leftrightarrow 1-u$, while $\tilde{\psi}_{3\sigma}(\omega, u)$ is antisymmetric. As a result, only even Gegenbauer polynomials are entering the model functions for the LCDAs, i.e., $a_1^{(k)} = 0$ $(k = 1, 2, 3)$. If we restrict ourselves to the isospin symmetry only, then these symmetry conditions remain true for the Λ_Q-baryon but not for the Ξ_Q-baryon, as the s-quark contributes differently than the u- and d-quarks.

The detail numerical analysis of the baryons with the $j^p = 1^+$ diquark are presented in Ref. [8]. Here, we exemplify it by presenting the shape functions of the $J^P = 1/2^+$ baryons in Fig. 4. The uncertainty in the LCDAs is mainly dominated by an arbitrariness $(0 \leq A \leq 1)$ in the choice of the local interpolating current.

6 Conclusions

The total set of the non-local light-ray operators for the ground-state heavy baryons with $J^P = 1/2^+$ and $J^P = 3/2^+$ is constructed in QCD in the heavy quark limit. Matrix elements of these operators sandwiched between the heavy-baryon state and vacuum determine the LCDAs of different twist through the diquark current. The first several moments of LCDAs are calculated

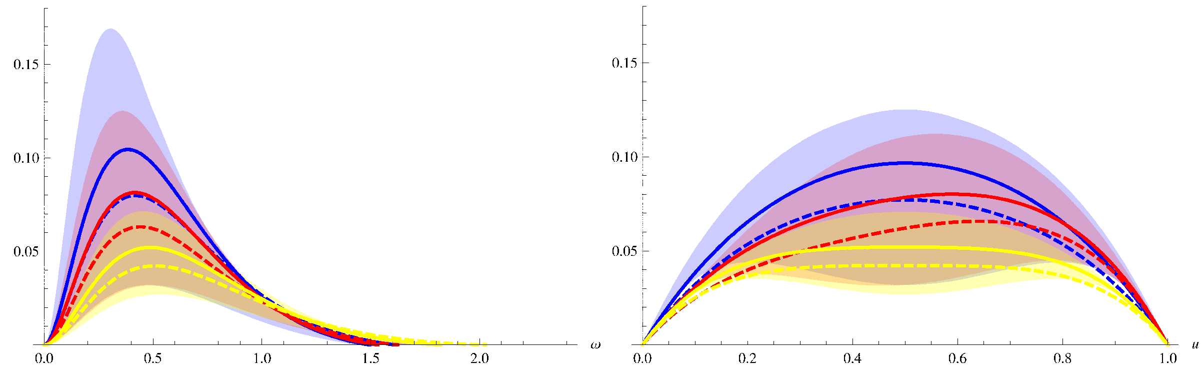

Figure 4: Twist 2 LCDAs of Σ_b (blue), Ξ_b (red) and Ω_b (yellow) at the energy scale $\mu = 1$ GeV (solid lines) and energy scale $\mu = 2.5$ GeV (dashed lines) including the most conservative error $A \in [0, 1]$ (light shade).

within the method of QCD sum rules using the non-local light-quark condensates. Simple theoretical models for the LCDAs have been proposed and their parameters are fitted based on the QCD sum rules estimations. $SU(3)_F$ breaking effects result the correction of order 10%.

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References

[1] X. G. He, T. Li, X. Q. Li and Y. M. Wang, Phys. Rev. D **74** (2006) 034026.

[2] A. Khodjamirian, T. Mannel and N. Offen, Phys. Rev. D **75** (2007) 054013.

[3] F. De Fazio, T. Feldmann and T. Hurth, Nucl. Phys. B **733** (2006) 1.

[4] F. De Fazio, T. Feldmann and T. Hurth, JHEP **0802** (2008) 031.

[5] W. Loinaz and R. Akhoury, Phys. Rev. D **53** (1996) 1416.

[6] F. Hussain, J. G. Korner, M. Kramer and G. Thompson, Z. Phys. C **51** (1991) 321.

[7] P. Ball, V. M. Braun and E. Gardi, Phys. Lett. B **665**, 197 (2008) [arXiv:0804.2424 [hep-ph]].

[8] A. Ali, C. Hambrock and A. Parkhomenko, (in preparation).

[9] A. G. Grozin, Springer Tracts Mod. Phys. **201**, 1 (2004).

[10] A. G. Grozin and O. I. Yakovlev, Phys. Lett. B **285**, 254 (1992) [arXiv:hep-ph/9908364].

[11] S. Groote, J. G. Korner and O. I. Yakovlev, Phys. Rev. D **56**, 3943 (1997) [arXiv:hep-ph/9705447].

[12] E. V. Shuryak, Nucl. Phys. B **198**, 83 (1982).

[13] Y. B. Dai, C. S. Huang, C. Liu and C. D. Lu, Phys. Lett. B **371**, 99 (1996) [arXiv:hep-ph/9602242].

[14] A. F. Falk, Nucl. Phys. B **378**, 79 (1992).

[15] P. Ball, V. M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. B **529**, 323 (1998) [arXiv:hep-ph/9802299].

[16] E. Bagan, M. Chabab, H. G. Dosch and S. Narison, Phys. Lett. B **301** (1993) 243.

[17] S. Groote, J. G. Korner and O. I. Yakovlev, Phys. Rev. D **55** (1997) 3016.

[18] B. O. Lange and M. Neubert, Phys. Rev. Lett. **91** (2003) 102001.

[19] G. P. Lepage and S. J. Brodsky, Phys. Lett. B **87** (1979) 359; Phys. Rev. D **22** (1980) 2157.

[20] A. G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997) [arXiv:hep-ph/9607366].

[21] V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, Phys. Rev. D **69**, 034014 (2004) [arXiv:hep-ph/0309330].

[22] S. V. Mikhailov and A. V. Radyushkin, JETP Lett. **43**, 712 (1986) [Pisma Zh. Eksp. Teor. Fiz. **43**, 551 (1986)].

[23] S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D **45**, 1754 (1992).

[24] V. Braun, P. Gornicki and L. Mankiewicz, Phys. Rev. D **51**, 6036 (1995) [arXiv:hep-ph/9410318].

[25] A. G. Grozin, Int. J. Mod. Phys. A **10**, 3497 (1995) [arXiv:hep-ph/9412238].

[26] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).

[27] X. Liu, H. X. Chen, Y. R. Liu, A. Hosaka and S. L. Zhu, Phys. Rev. D **77**, 014031 (2008) [arXiv:0710.0123 [hep-ph]].

[28] R. Lewis and R. M. Woloshyn, Phys. Rev. D **79**, 014502 (2009) [arXiv:0806.4783 [hep-lat]].

[29] J. R. Zhang and M. Q. Huang, Phys. Rev. D **78**, 094015 (2008) [arXiv:0811.3266 [hep-ph]].

[30] K. G. Chetyrkin, A. Khodjamirian and A. A. Pivovarov, Phys. Lett. B **661**, 250 (2008) [arXiv:0712.2999 [hep-ph]].

[31] V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989) [Sov. J. Nucl. Phys. **50**, 511 (1989), YAFIA **50**, 818 (1989)].

[32] V. M. Braun and I. E. Filyanov, Z. Phys. C **48**, 239 (1990) [Sov. J. Nucl. Phys. **52**, 126 (1990) YAFIA **52**, 199 (1990)].

[33] P. Ball, JHEP **9901**, 010 (1999) [arXiv:hep-ph/9812375].