Analytical properties of the $\pi\pi$ scattering amplitude and the light scalar mesons

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Abstract

The $\pi\pi$ scattering amplitude with regular analytical properties in the s complex plane has been constructed. It describes simultaneously the data on the $\pi\pi$ scattering, $\phi \to \pi^0\pi^0\gamma$ decay and $\pi\pi \to K\bar{K}$ reaction. The chiral shielding of the $\sigma(600)$ meson and it's mixing with the $f_0(980)$ meson are taken into account also. The data agrees with the four-quark nature of the $\sigma(600)$ and $f_0(980)$ mesons.

The amplitude in the range $-5m_\pi^2 < s < 0.64$ GeV^2 also agrees with results, obtained on the base of the chiral expansion, dispersion relations and the Roy equations.

1 Introduction

Study of light scalar resonances is one of the central problems of non-perturbative QCD, it is important for understanding both the confinement physics and the chiral symmetry realization way in the low energy region. The commonly suggested nonet of light scalar mesons is $f_0(600)$ (or $\sigma(600)$), $K_0^*(800)$ (or $\kappa(800)$), $f_0(980)$ and $a_0(980)$ Ref. [1].

In the 1987 it was suggested in [2] to investigate light scalar mesons in radiative ϕ decays. Chiral one-loop mechanism of the transition $\phi \to K^+K^- \to \gamma f_0(a_0)$ (Kaon Loop model) was also proposed there.

Ten years later, in the 1998, the decays $\phi \to \eta\pi^0\gamma$ and $\phi \to \pi^0\pi^0\gamma$ were experimentally discovered in Budker INP [3]. Then in the 2002 KLOE group published the high-statistical data on these decays [4, 5] (800 $\phi \to \eta\pi^0\gamma$ events and 2400 $\phi \to \pi^0\pi^0\gamma$ events).

In [6] the KLOE data on the $\phi \to \pi^0\pi^0\gamma$ decay were described simultaneously with the data on the $\pi\pi$ scattering and the $\pi\pi \to K\bar{K}$ reaction. The description was carried out taking into account the chiral shielding [7, 8] of $\sigma(600)$ and the $\sigma(600) - f_0(980)$ mixing. The data didn't contradict the existence of the $\sigma(600)$ meson and yielded evidence in favor of the four-quark nature of the $\sigma(600)$ and $f_0(980)$ mesons. These experiments ratified Kaon Loop mechanism .

This description revealed new goals. The point is that at the same time it was calculated in [9] the $\pi\pi$ scattering amplitude in the s complex plane, basing on chiral expansion, dispersion relations and Roy equations, see Figs. 1,2. In particular, the pole in $s = M_\sigma^2$ was obtained in [9], where

$$M_\sigma = 441^{+16}_{-8} - i272^{+9}_{-12.5} \text{ MeV} \tag{1}$$

and was assigned to the σ resonance.

Aiming the comparison of the results of [6] and [9] it is necessary to build the $\pi\pi$ scattering amplitude with correct analytical properties in the complex s plane. The point is that in [6] S-matrix of the $\pi\pi$ scattering is the product of the "resonance" and "background" parts:

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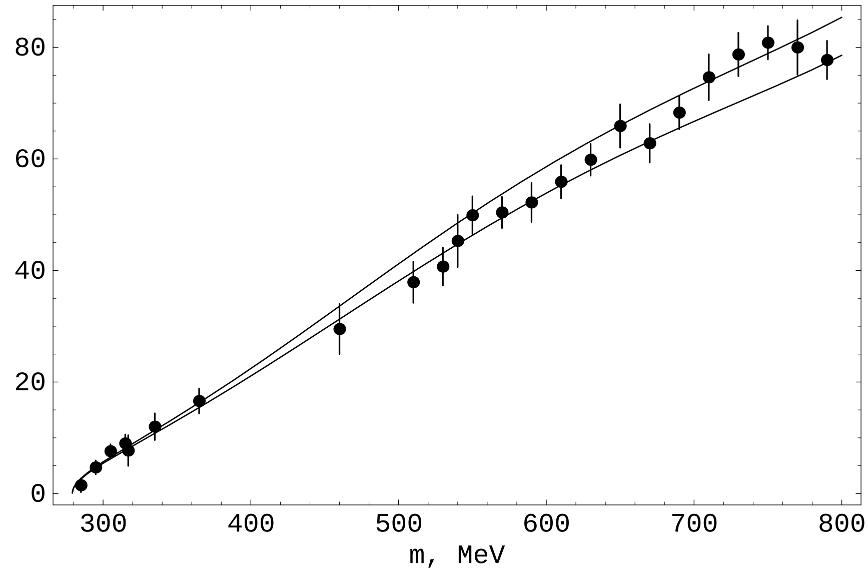

Figure 1: The corridor in the phase δ_0^0 of the $\pi\pi$ scattering [9] and the experimental data [10]

$$S_{\pi\pi} = S_{back}\, S_{res}\,,$$

and the S_{res} had desired analytical properties, while analytical properties of the S_{back} in the whole complex s plane were not essential for the aims of [6], where mainly physical region was investigated, and Adler zero existence together with poles absence on the real axis of the s complex plane were demanded.

In this paper we present the $\pi\pi$ scattering amplitude with desired analytical properties and the data description obtained with this amplitude. The comparison with [9] results is presented also.

2 Some theory

The S-wave amplitude T_0^0 of the $\pi\pi$ scattering with I=0 [11] is

$$T_0^0 = \frac{\eta_0^0 e^{2i\delta_0^0} - 1}{2i\rho_{\pi\pi}(m)} = \frac{e^{2i\delta_B^{\pi\pi}} - 1}{2i\rho_{\pi\pi}(m)} + e^{2i\delta_B^{\pi\pi}} \sum_{R,R'} \frac{g_{R\pi\pi} G^{-1}_{RR'} g_{R'\pi\pi}}{16\pi}\,. \tag{2}$$

Here $\eta_0^0 \equiv \eta_0^0(m)$ is the inelasticity, δ_0^0 is the scattering phase, $\delta_B^{\pi\pi}$ is the phase of the elastic background, and

$$G_{RR'}(m) = \begin{pmatrix} D_{f_0}(m) & -\Pi_{f_0\sigma}(m) \\ -\Pi_{f_0\sigma}(m) & D_\sigma(m) \end{pmatrix}$$

The desired analytical properties of the $\pi\pi$ scattering amplitude are: two cuts in the s-complex plane, Adler zero, absence of poles on the physical sheet of the Riemannian surface, resonance poles on the second sheet of the Riemannian surface. This applies curtain restrictions on the $\delta_B^{\pi\pi}$.

The inverse propagator of scalar R [12]

$$D_R(m^2) = m_R^2 - m^2 + Re\left(\Pi_R(m_R^2)\right) - \Pi_R(m^2) \tag{3}$$

where

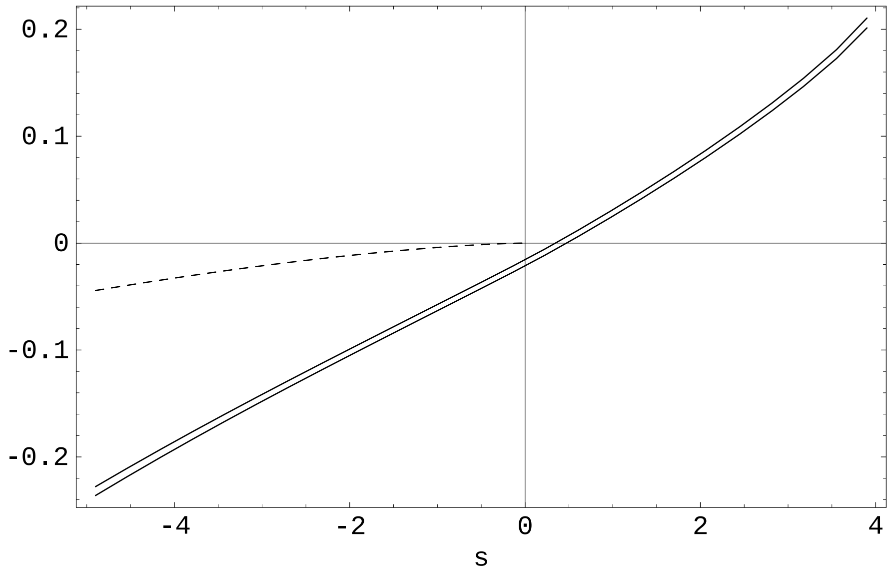

Figure 2: The real and the imaginary parts of the amplitude T_0^0 of the $\pi\pi$ scattering (s in units of m_π^2) [9]

$$\Pi_R^{ab}(m^2) = \frac{1}{\pi}[m^2 - (m_a + m_b)^2] \int_{(m_a+m_b)^2}^{\infty} \frac{\bar{m}\Gamma(R \to ab, \bar{m})\, d\bar{m}^2}{[\bar{m}^2 - (m_a + m_b)^2](\bar{m}^2 - m^2 - i\varepsilon)}$$

So, following [13], we have

$$Im\,(D_R(z)) = -y\left(1 + \sum_{ab} \frac{1}{\pi} \int_{(m_a+m_b)^2}^{\infty} \frac{\bar{m}\Gamma(R \to ab, \bar{m})}{|\bar{m}^2 - z|^2}\, d\bar{m}^2\right)$$

and, since expression in brackets is positive, Im$D_R(z)$ may be equal to zero only when $y \equiv$Im $z = 0$.

Let's use the same mechanism and take $e^{2i\delta_B^{\pi\pi}}$ in the form

$$e^{2i\delta_B^{\pi\pi}} = \frac{P^*_{\pi 1}(s) P^*_{\pi 2}(s)}{P_{\pi 1}(s) P_{\pi 2}(s)}\,,$$

where

$$P_{\pi 1}(s) = a_1 - a_2 \frac{s}{4m_\pi^2} - \Pi_{\pi\pi}(s) + a_3\, \Pi_{\pi\pi}(4m_\pi^2 - s) - a_4 Q_1(s)\,,$$

$$Q_1(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{s - 4m_\pi^2}{s' - 4m_\pi^2}\, \frac{\rho_{\pi\pi}(s')}{s' - s - i\varepsilon} K_1(s')$$

$$Im(Q_1(s)) = K_1(s)\rho_{\pi\pi}(s)$$

Since Im$(Q_1(s)) = K_1(s)\rho_{\pi\pi}(s)$ we require that $K_1(s)$ should be positive and have no singularities on the physical sheet. This is provided if we take

$$K_1(s) = \frac{L_1(s)}{D_1(4m_\pi^2 - s) D_2(4m_\pi^2 - s) D_3(4m_\pi^2 - s) D_4(4m_\pi^2 - s)}\,,$$

$$D_i(s) = m_i^2 - s - g_i \Pi_{\pi\pi}(s)\,,$$

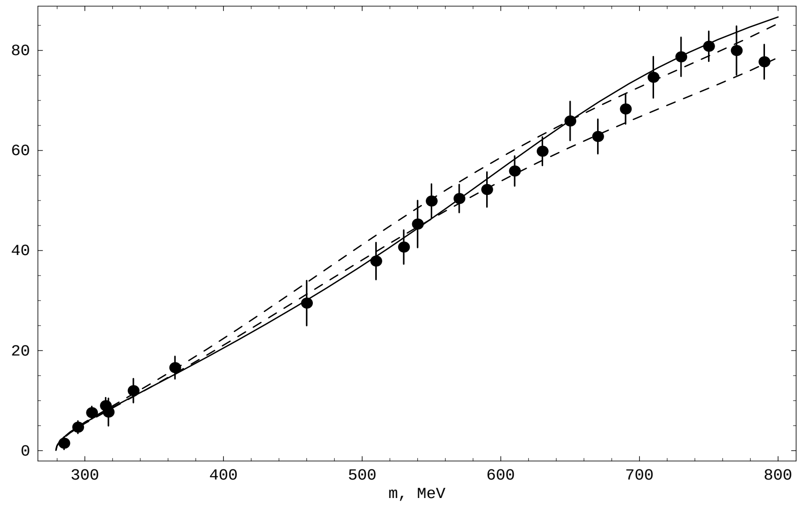

Figure 3: The phase δ_0^0 of the $\pi\pi$ scattering. Solid line is our description, dashed lines mark borders of the corridor [9], points are experimental data

$$L_1(s) = (s-4m_\pi^2)^4 + \alpha_1 4m_\pi^2(s-4m_\pi^2)^3 + \alpha_2(4m_\pi^2)^2(s-4m_\pi^2)^2 +$$

$$+\alpha_3(4m_\pi^2)^3(s-4m_\pi^2) + q_1^8 + \sqrt{s}\Bigg(c_1(2m_\pi)^7 + c_2(2m_\pi)^5(s-4m_\pi^2) +$$

$$+c_3(2m_\pi)^3(s-4m_\pi^2)^2 + c_4(2m_\pi)(s-4m_\pi^2)^3\Bigg)$$

$$P_{\pi 2}(s) = \frac{\Lambda^2 + s - 4m_\pi^2}{4m_\pi^2} + k_2 Q_2(s)\,,$$

$$Q_2(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{s-4m_\pi^2}{s'-4m_\pi^2}\,\frac{\rho_{\pi\pi}(s')}{s'-s-i\varepsilon} K_2(s')\,,$$

$$K_2(s) = \frac{L_2(s)}{D_{1A}(4m_\pi^2-s)D_{2A}(4m_\pi^2-s)D_{3A}(4m_\pi^2-s)}\,,$$

$$L_2(s) = 4m_\pi^2\left(s^2 + \beta(4m_\pi^2)s + \gamma_1(2m_\pi)^3 s^{1/2} + \gamma_2(2m_\pi)s^{3/2}\right)$$

With the help of the above formulas we obtain the results, shown in Figs 3,4,5. One can see that the experimental data and the corridor from [9] are described well.

3 Poles of the resonances

The σ pole is not at (1) but at

$$M_\sigma = 617 - i318 \text{ MeV}$$

This deviation is not a crime, because Roy equations are approximate, they are one-channel $(\pi\pi)$ [1].

[1]Note that for one-channel σ inverse propagator

$$D_\sigma(m) = m_\sigma^2 - m^2 + Re\big(\Pi_\sigma^{\pi\pi}(m_\sigma^2)\big) - \Pi_\sigma^{\pi\pi}(m^2)$$

the pole at (1) means violation of the Källen – Lehmann representation, see [13].

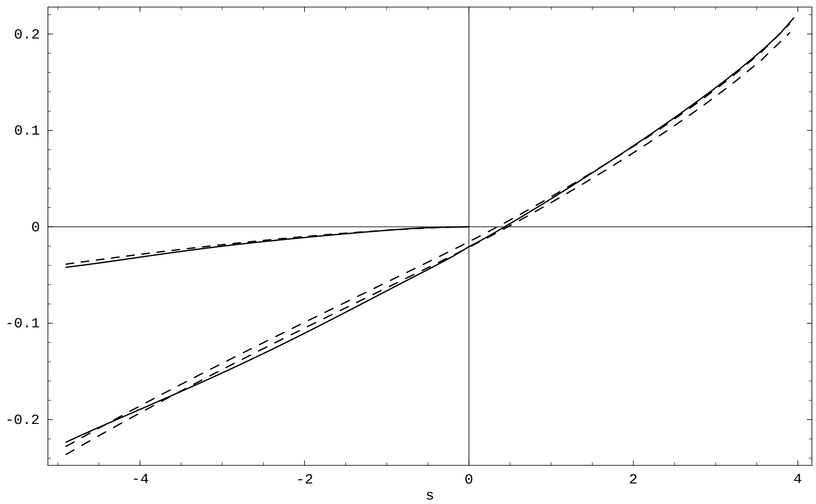

Figure 4: The real and the imaginary parts of the amplitude T_0^0 of the $\pi\pi$ scattering (s in units of m_π^2). Solid lines show our description, dashed lines mark borders of the real part corridor and the imaginary part for $s < 0$ [9]

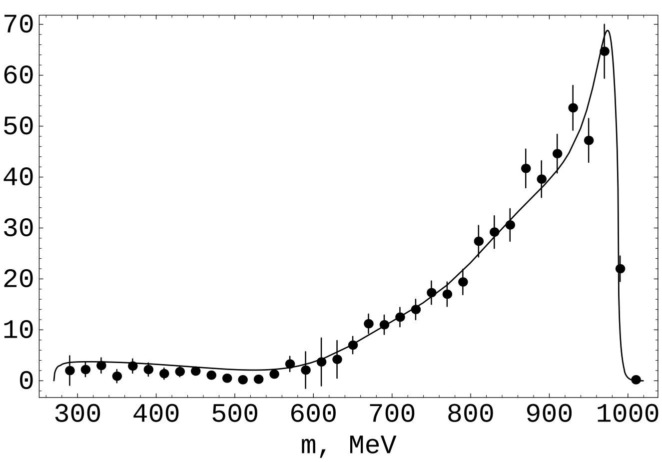

Figure 5: The $\pi^0\pi^0$ spectrum in $\phi \to \pi^0\pi^0\gamma$ decay, solid line is our description

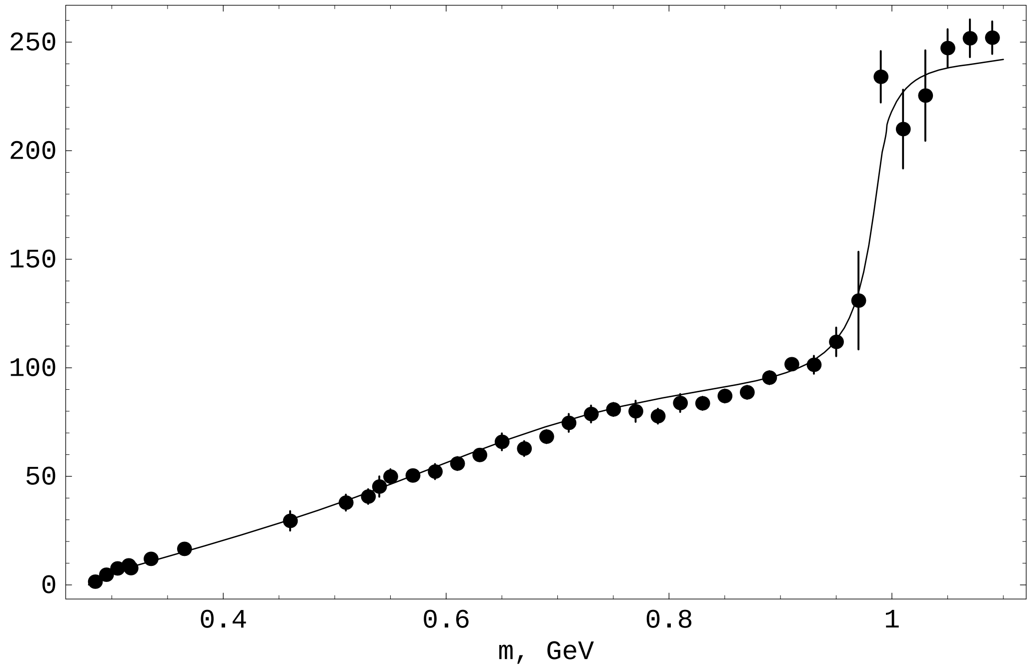

Figure 6: The phase δ_0^0 of the $\pi\pi$ scattering, solid line is our description

Also $M_{f_0} = 1188 - i780$ MeV, while $m_{f_0} = 986$ MeV, $\Gamma(f_0 \to \pi\pi, m_{f_0}) = 85$ MeV.

We find out that the role of high channels is very important for pole positions. For R=$\sigma(600)$, $f_0(980)$ we take

$$\Pi_R = \Pi_R^{\pi^+\pi^-} + \Pi_R^{\pi^0\pi^0} + \Pi_R^{K^+K^-} + \Pi_R^{K^0\bar{K}^0} + \Pi_R^{\eta\eta} + \Pi_R^{\eta'\eta} + \Pi_R^{\eta'\eta'}$$

But a fit without f_0 coupling to $\eta\eta, \eta\eta'$, and $\eta'\eta'$ and small σ coupling to these channels gives us
$M_\sigma = 639 - i313$ MeV (instead of $617 - i318$ MeV),
$M_{f_0} = 984 - i423$ MeV (instead of $1188 - i780$ MeV).

Note that if we additionally neglect σ coupling to the $K\bar{K}$ in the obtained results, the pole is transferred to $M_\sigma = 562 - i233$ MeV.

4 Perspectives

We plan to use results of this work for study of other reactions, including decays of heavy quarkoniums and pion polarization.

Special words should be said about refinement of the obtained results. New precise measurement of the inelasticity η_0^0 near $K\bar{K}$ threshold would be crucial, it can improve our knowledge about light scalar mesons a lot. In fact, commonly used data on η_0^0 were obtained forty years ago, and they are not enough. As far as we know, some collaborations even have raw data (for example, VES Collaboration), and all they have to do is to process these data.

New precise data on the $\phi \to \pi^0\pi^0\gamma$ decay would also clarify the situation. The KLOE Collaboration has measured this reaction with excellent precision, and hopefully they will publish final results.

5 Summary

1. The $\pi\pi$ scattering amplitude with correct analytical properties has been built.

2. This amplitude describes experimental data and the results based on Roy equations on the real s axis.
3. A contradiction in position of σ pole may be an indication of the important role of high channels in the $\pi\pi$ scattering amplitude and its analytical continuation even for $|s|$ much less than m_{f_0}.
4. The f_0 pole is situated far from the position, predicted by Breit-Wigner approximation.
5. New experiments are important for the study of light scalar mesons.

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