Light meson spectroscopy and Regge trajectories in the relativistic quark model

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Abstract

Masses of light tetraquarks are obtained in the diquark-antidiquark picture. Such exotic states can be identified with the known light scalar mesons forming an SU(3) nonet (σ meson $f_0(600)$, κ meson $K_0^*(800)$, $f_0(980)$ and $a_0(980)$). Mass spectra of light (u, d, s) quark-antiquark mesons are calculated within the QCD-motivated relativistic quark model. Their Regge trajectories exhibit linearity and equidistance.

The consistent theoretical understanding of the light meson sector remains an important problem already for many years. An extensive analysis of the data on highly excited light non-strange meson states up to a mass of 2400 MeV collected by Crystal Barrel experiment at LEAR (CERN) has been published. Classification of these new data requires better theoretical description of light meson mass spectra. This is especially important, since light exotic states (such as tetraquarks, glueballs, hybrids) within quantum chromodynamics (QCD) are expected to have masses in this range. Particular interest is focused on scalar mesons, their properties and abundance. A generally accepted consistent picture has not yet emerged. Experimental and theoretical evidence for the existence of $f_0(600)(\sigma)$, $K_0^*(800)(\kappa)$, $f_0(980)$ and $a_0(980)$ indicates that lightest scalars form a complete $SU(3)$ flavour nonet. A peculiar feature of their mass spectrum is the inversion of the mass ordering, which cannot be naturally understood in the $q\bar{q}$ picture. This fact stimulated various alternative interpretations of light scalars as four quark states (tetraquarks) in particular diquark-antidiquark bound states. The proximity of f_0/a_0 to the $K\bar{K}$ threshold led to the $K\bar{K}$ molecular picture.

In the quasipotential approach and diquark-antidiquark picture of tetraquarks the interaction of two quarks in a diquark and the diquark-antidiquark interaction in a tetraquark are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the tetraquark wave function (Ψ_T) of the bound diquark-antidiquark state, respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type [1]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_{d,T}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \tag{2}$$

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Table 1: Masses of light unflavored diquark-antidiquark ground state ($\langle\mathbf{L}^2\rangle$=0) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

State J^{PC}	Diquark content	Theory mass	Experiment $I=0$	mass	$I=1$	mass
$(qq)(\bar q\bar q)$						
0^{++}	$S\bar S$	596	$f_0(600)$ (σ)	400-1200		-
$1^{+\pm}$	$(S\bar A\pm\bar S A)/\sqrt{2}$	672				
0^{++}	$A\bar A$	1179	$f_0(1370)$	1200-1500		
1^{+-}	$A\bar A$	1773				
2^{++}	$A\bar A$	1915	$f_2(1910)$	1903(9)		
			$f_2(1950)$	1944(12)		
$(qs)(\bar q\bar s)$						
0^{++}	$S\bar S$	992	$f_0(980)$	980(10)	$a_0(980)$	984.7(12)
1^{++}	$(S\bar A+\bar S A)/\sqrt{2}$	1201	$f_1(1285)$	1281.8(6)	$a_1(1260)$	1230(40)
1^{+-}	$(S\bar A-\bar S A)/\sqrt{2}$	1201	$h_1(1170)$	1170(20)	$b_1(1235)$	1229.5(32)
0^{++}	$A\bar A$	1480	$f_0(1500)$	1505(6)	$a_0(1450)$	1474(19)
1^{+-}	$A\bar A$	1942	$h_1(1965)$	1965(45)	$b_1(1960)$	1960(35)
2^{++}	$A\bar A$	2097	$f_2(2010)$	2011(70)	$a_2(1990)$	2050(45)
			$f_2(2140)$	2141(12)	$a_2(2080)$	2100(20)
$(ss)(\bar s\bar s)$						
0^{++}	$A\bar A$	2203	$f_0(2200)$	2189(13)		-
1^{+-}	$A\bar A$	2267	$h_1(2215)$	2215(40)		-
2^{++}	$A\bar A$	2357	$f_2(2340)$	2339(60)		-

and E_1, E_2 are given by

$$E_1=\frac{M^2-m_2^2+m_1^2}{2M}, \quad E_2=\frac{M^2-m_1^2+m_2^2}{2M}. \tag{3}$$

Here, $M=E_1+E_2$ is the bound-state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks ($q=u,d$ and s) which form the diquark or of the diquark (d) and antidiquark ($\bar d'$) which form the light tetraquark (T), and $\mathbf{p}$ is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. For the quark-quark interaction in a diquark we use the relation $V_{qq}=V_{q\bar q}/2$ arising under the assumption of an octet structure of the interaction from the difference in the qq and $q\bar q$ colour states. An important role in this construction is played by the Lorentz structure of the confining interaction. In our analysis of mesons, while constructing the quasipotential of the quark-antiquark interaction, we assumed that the effective interaction is the sum of the usual one-gluon exchange term and a mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli term. We use the same conventions for the construction of the quark-quark and diquark-antidiquark interactions in the tetraquark.

At the first step, we take the masses and form factors of the light diquarks from the previous consideration of light diquarks in heavy baryons. At the second step, we calculate the masses of light tetraquarks considered as the bound states of a light diquark and antidiquark.

Table 2: Masses of strange diquark-antidiquark ground state ($\langle \mathbf{L}^2\rangle$=0) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

State J^P	Diquark content	Theory mass	Experiment $I=\frac{1}{2}$	Experiment mass
$(qq)(\bar s\bar q)$ or $(sq)(\bar q\bar q)$				
0^+	$S\bar S$	730	$K_0^*(800)$ (κ)	672(40)
1^+	$(S\bar A \pm \bar S A)/\sqrt{2}$	1057		
0^+	$A\bar A$	1332	$K_0^*(1430)$	1425(50)
1^+	$A\bar A$	1855		
2^+	$A\bar A$	2001	$K_2^*(1980)$	1973(26)

The obtained mass spectra of ground state light tetraquarks are presented in Tables 1 and 2. We see that the diquark-antidiquark picture can provide a natural explanation for the inversion of masses of light scalar 0^+ mesons. Indeed all lightest experimentally observed scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be interpreted in our model as light tetraquarks composed from a scalar diquark and antidiquark ($S\bar S$). Therefore, the $f_0(980)$ and $a_0(980)$ tetraquarks contain, in comparison to the $q\bar q$ picture, an additional pair of strange quarks which gives a natural explanation why their masses are heavier than the strange $K_0^*(800)$ (κ).

The other scalar tetraquark states can be composed from an axial vector diquark and antidiquark ($A\bar A$). Their masses are predicted to be approximately 600 MeV heavier than the $S\bar S$ tetraquarks (S is a scalar diquark). The diquark-antidiquark composition also naturally explains the experimentally observed proximity of masses of the unflavored $a_0(1450)$, $f_0(1500)$ and strange $K_0^*(1430)$ scalars. Note that quark-antiquark scalar states are predicted in our model to have masses around 1200 MeV ($q\bar q$) and 1400 MeV ($q\bar s$).

The calculated masses of light unflavoured and strange ($q\bar q$) mesons are given in Tables I and II of Ref. [2]. Some of the Regge trajectories both in (M^2,J) and (M^2, n_r) planes (M is the mass, J is the spin and n_r is the radial quantum number of the meson state) are shown in Figs. 1-6.

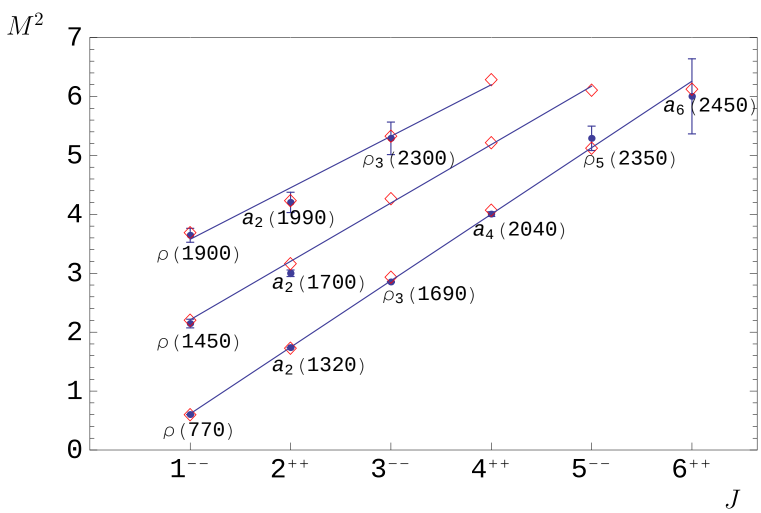

Figure 1: Parent and daughter (J, M^2) Regge trajectories for isovector light mesons with natural parity (ρ). Diamonds are predicted masses. Available experimental data are given by dots with error bars and particle names. M^2 is in GeV^2.

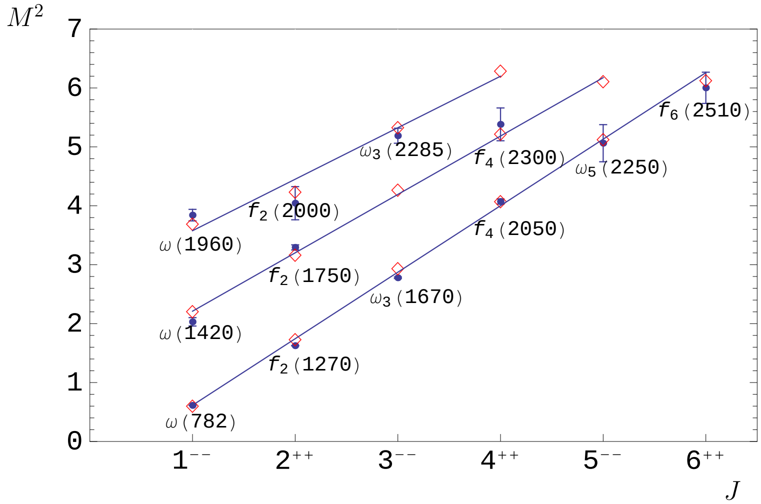

Figure 2: Same as in Fig. 1 for isoscalar light $q\bar{q}$ mesons with natural parity (ω).

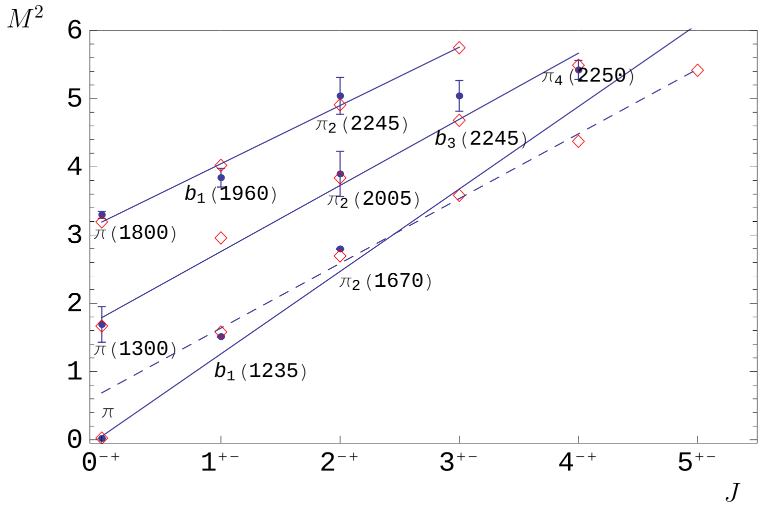

Figure 3: Same as in Fig. 1 for isovector light mesons with unnatural parity (π). Dashed line corresponds to the Regge trajectory, fitted without π.

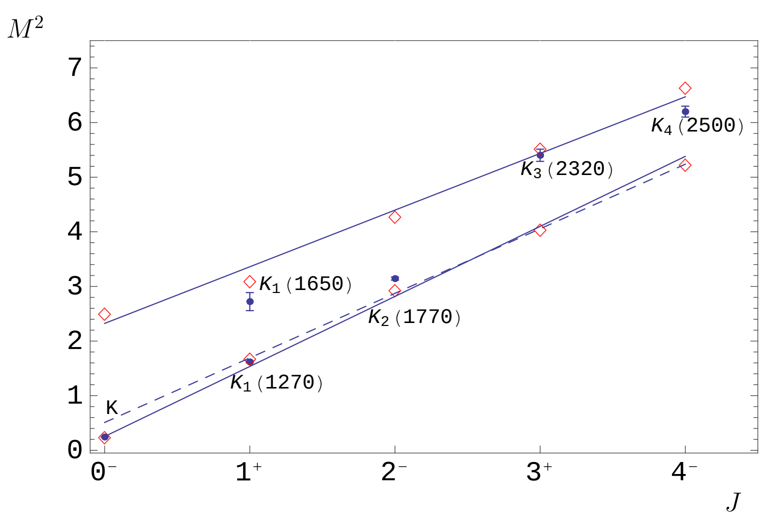

Figure 4: Same as in Fig. 1 for isodublet light mesons with unnatural parity (K). Dashed line corresponds to the Regge trajectory, fitted without K.

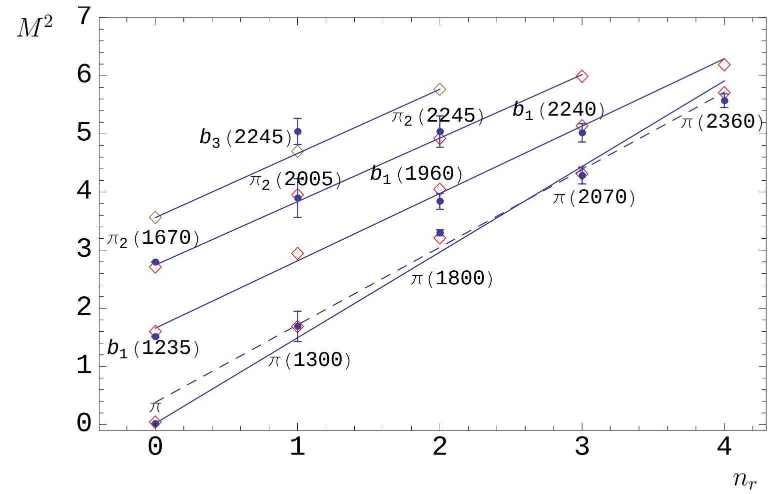

Figure 5: The (n_r, M^2) Regge trajectories for spin-singlet isovector mesons π, b_1, π_2 and b_3 (from bottom to top). Notations are the same as in Fig. 1. The dashed line corresponds to the Regge trajectory, fitted without π.

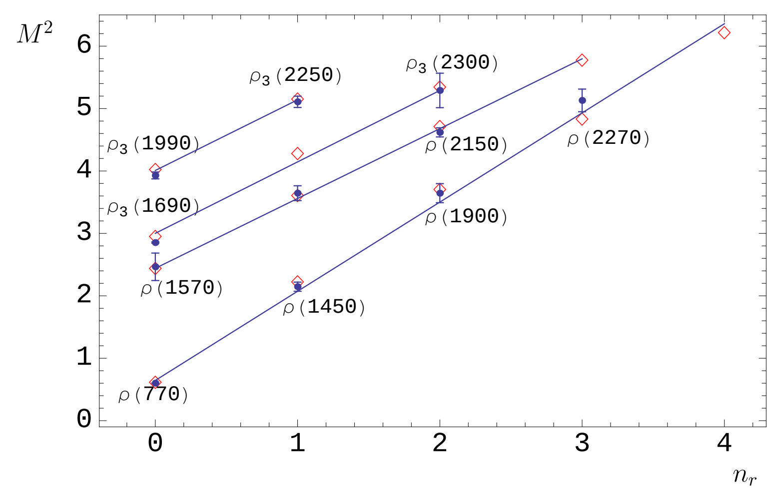

Figure 6: The (n_r, M^2) Regge trajectories for spin-triplet isovector mesons $\rho(^3S_1)$, $\rho(^3D_1)$, $\rho_3(^3D_3)$ and $\rho_3(^3G_3)$ (from bottom to top). Notations are the same as in Fig. 1.

We see that the calculated light meson masses fit nicely to the linear trajectories. These trajectories are almost parallel and equidistant. It is important to note that the quality of fitting the π meson Regge trajectories is significantly improved if the ground state π is excluded from the fit. In the kaon case omitting the ground state also improves the fit but not so dramatically as for the pion. The corresponding trajectories are shown in Figs. 3 and 4 by dashed lines. This indicates the special role of the pion originating from the chiral symmetry breaking.

In summary, we presented the masses of the ground state light tetraquarks in the diquark-antidiquark picture, the mass spectra and Regge trajectories of light $q\bar{q}$ mesons. In distinction with previous phenomenological treatments, we used the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equations. The diquark structure was taken into account by using diquark-gluon form factors in terms of diquark wave functions. It is important to emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their values fixed from our previous considerations of hadron properties. It was found that the lightest scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be naturally described in our model as diquark-antidiquark bound systems, while the lightest scalar (1^3P_0) $q\bar{q}$ states have masses above 1 GeV.

This work was supported in part by the Deutsche Forschungsgemeinschaft (DFG) under contract Eb 139/6-1 and the Russian Foundation for Basic Research (RFBR) grants No.08-02-00582 and No.10-02-91339.

References

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[2] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **79**, 114029 (2009).