Baryon resonances in the relativistic mean field approach

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Abstract

We suggest a new approach according to which baryon resonances can be viewed as collective excitations about 'intrinsic' one-quark excitations in a mean field of definite symmetry. This point of view is justified in the limit of large number of colours N_c. Although in the real world N_c is only three, we obtain a good agreement with the observed resonance spectrum up to 2 GeV. A possible implication of the scheme is the existence of new exotic charmed (and bottom) baryons that may be stable against strong decays.

1 Introduction

If the number of colours N_c is large the N_c quarks constituting a baryon can be considered in a mean (non-fluctuating) field which does not change as $N_c \to \infty$ [1]. While in the real world N_c is only three, we do not expect qualitative difference in the baryon spectrum with its large-N_c limit. The hope is that if one develops a clear picture at large N_c, its imprint will be visible at $N_c = 3$.

The advantage of the large-N_c approach is that at large N_c baryon physics simplifies considerably, which enables one to take into full account the important relativistic and field-theoretic effects that are often ignored. Baryons are not just three (or N_c) quarks but contain additional quark-antiquark pairs, as it is well known experimentally. The number of antiquarks in baryons is, theoretically, also proportional to N_c [2], which means that antiquarks cannot be obtained from adding one meson to a baryon: one needs $\mathcal{O}(N_c)$ mesons to explain $\mathcal{O}(N_c)$ antiquarks, implying in fact a classical mesonic field.

Baryon resonances may be formed not only from quark excitations as in the non-relativistic quark models, but also from particle-hole excitations and 'Gamov–Teller' transitions [4, 5]. At large N_c these effects are transparent and tractable. At $N_c = 3$ it is a mess called 'strong interactions'.

At the microscopic level quarks experience only colour interactions, however gluon field fluctuations are not suppressed if N_c is large; the mean field can be only 'colourless'. An example how originally colour interactions are Fierz-transformed into interactions of quarks with mesonic fields is provided by the instanton liquid model [3]. A non-fluctuating confining bag is another example of a 'colourless' mean field. A more modern example of a mean field is given by the 5- or 6-dimensional 'gravitational' background field in the ADS/QCD models.

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We shall assume that quarks in the large-N_c baryon obey the Dirac equation in a background mesonic field since there are no reasons to expect quarks to be non-relativistic, especially in excited baryons. All intrinsic quark Dirac levels in the mean field are stable in N_c. All negative-energy levels should be filled in by N_c quarks in the antisymmetric state in colour, corresponding to the zero baryon number state. Filling in the lowest positive-energy level by N_c 'valence' quarks makes a baryon. Exciting higher quark levels or making particle-hole excitations produces baryon resonances. The baryon mass is $\mathcal{O}(N_c)$, and the excitation energy is $\mathcal{O}(1)$. When one excites one quark the change of the mean field is $\mathcal{O}(1/N_c)$ that can be neglected to the first approximation.

Moreover, if one replaces one light (u, d or s) quark in light baryons by a heavy (c, b) one, as in charmed or bottom baryons, the change in the mean field is also $\mathcal{O}(1/N_c)$. Therefore, the spectrum of heavy baryons is directly related to that of light baryons. We shall see that these relations are in fair agreement with reality.

Our approach can be illustrated by the chiral quark soliton model [6] or by the chiral bag model [8] but actually the arguments of this paper are much more general. We argue that the mean field in baryons has a definite symmetry, namely it breaks spontaneously the symmetry under separate $SU(3)_{\rm flavour}$ and $SO(3)_{\rm space}$ rotations but does not change under simultaneous $SU(2)_{\rm iso+space}$ rotations in ordinary space and a compensating rotation in isospace [4, 5].

It implies that each intrinsic quark state, be it the ground state or a one-quark excitation in the Dirac spectrum, generates a band of resonances appearing as collective rotational excitations of a given intrinsic state. The quantum numbers of those resonances, their total number and their splittings are unequivocally dictated by the symmetry of the mean field. Assuming the $SU(2)_{\rm iso+space}$ symmetry of the mean field, we obtain exactly what is observed in Nature. Moreover, certain relations between resonance splittings that are satisfied with high accuracy, are dynamics-independent but follow solely from the particular symmetry of the mean field.

In this paper, we do not consider any specific dynamical model but concentrate mainly on symmetry. A concrete dynamical model would say what is the intrinsic relativistic quark spectrum in baryons. It may get it approximately correct, or altogether wrong. Instead of calculating the intrinsic Dirac spectrum of quarks from a model, we extract it from the experimentally known baryon spectrum by interpreting baryon resonances as collective excitations about the ground state and about the one-quark transitions. However, we show that the needed intrinsic quark spectrum can be obtained from a natural choice of the mean field satisfying the $SU(2)_{\rm iso+space}$ symmetry.

In summary, we show that it is possible to obtain a realistic spectrum of baryon resonances up to 2 GeV, starting from the large-N_c limit.

2 Symmetry of the mean field

In the mean field approximation, justified at large N_c, one looks for the solutions of the Dirac equation for single quark states in the background mean field. In a most general case the background field couples to quarks through all five Fermi variants. If the mean field is stationary in time, it leads to the Dirac eigenvalue equation for the u, d, s quarks in the background field, $H\psi = E\psi$, the Dirac Hamiltonian being schematically

$$H = \gamma^0\left(-i\partial_i\gamma^i + S(\mathbf{x}) + P(\mathbf{x})i\gamma^5 + V_\mu(\mathbf{x})\gamma^\mu + A_\mu(\mathbf{x})\gamma^\mu\gamma^5 + T_{\mu\nu}(\mathbf{x})\frac{i}{2}[\gamma^\mu\gamma^\nu]\right), \qquad (1)$$

where S, P, V, A, T are the scalar, pseudoscalar, vector, axial, tensor mean fields, respectively; all are matrices in flavour. In fact, the one-particle Dirac Hamiltonian (1) is generally nonlocal, however that does not destroy symmetries in which we are primarily interested. We include the current and the dynamically-generated quarks masses into the scalar term S.

The key issue is the symmetry of the mean field. We assume the chiral limit for u, d quarks, $m_u = m_d = 0$, which is an excellent approximation. As to the strange quark, from the large-N_c point of view the current strange quark mass is very small, $m_s = \mathcal{O}(1/N_c^2)$ [4], therefore a good starting point is exact $SU(3)$ flavour symmetry. It implies that baryons appear in degenerate $SU(3)$ multiplets **8**, **10**, ...; the splittings inside $SU(3)$ multiplets can be determined later on as a perturbation in m_s, see *e.g.* Ref. [9].

A natural assumption, then, would be that the mean field is flavour-symmetric, and spherically symmetric. However we know that baryons are strongly coupled to pseudoscalar mesons ($g_{\pi NN} \approx 13$). It means that there is a large pseudoscalar field inside baryons; at large N_c it is a classical mean field. There is no way of writing down the pseudoscalar field (it must change sign under inversion of coordinates) that would be compatible with the $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$ symmetry. The minimal extension of spherical symmetry is to write the "hedgehog" *Ansatz* "marrying" the isotopic and space axes [1]:

$$\pi^a(\mathbf{x}) = \begin{cases} n^a\, F(r), \quad n^a = \frac{x^a}{r}, & a = 1,2,3, \\ 0, & a = 4,5,6,7,8. \end{cases} \tag{2}$$

This *Ansatz* breaks the $SU(3)_{\text{flav}}$ symmetry. Moreover, it breaks the symmetry under independent space $SO(3)_{\text{space}}$ and isospin $SU(2)_{\text{iso}}$ rotations, and only a simultaneous rotation in both spaces remains a symmetry, since a rotation in the isospin space labeled by a, can be compensated by the rotation of the space axes. The *Ansatz* (2) implies a spontaneous (as contrasted to explicit) breaking of the original $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$ symmetry down to the $SU(2)_{\text{iso+space}}$ symmetry. It is analogous to the spontaneous breaking of spherical symmetry by the ellipsoid form of many nuclei; there are many other examples in physics where the original symmetry is spontaneously broken in the ground state.

We list here all possible structures in the S, P, V, A, T fields, compatible with the $SU(2)_{\text{iso+space}}$ symmetry and with the C, P, T quantum numbers of the fields [4, 5]. The fields below are generalizations of the 'hedgehog' Ansatz (2) to mesonic fields with other quantum numbers.

Since $SU(3)$ symmetry is broken, all fields can be divided into three categories:

I. Isovector fields acting on u, d quarks

$$\begin{aligned} \text{pseudoscalar}: P^a(\mathbf{x}) &= n^a\, P_0(r), \\ \text{vector, spacecomponents}: V_i^a(\mathbf{x}) &= \epsilon_{aik}\, n_k\, P_1(r), \\ \text{axial, spacecomponents}: A_i^a(\mathbf{x}) &= \delta_{ai}\, P_2(r) + n_a n_i\, P_3(r), \\ \text{tensor, spacecomponents}: T_{ij}^a(\mathbf{x}) &= \epsilon_{aij}\, P_4(r) + \epsilon_{bij}\, n_a n_b\, P_5(r). \end{aligned} \tag{3}$$

II. Isoscalar fields acting on u, d quarks

$$\begin{aligned} \text{scalar}: \quad S(\mathbf{x}) &= Q_0(r), \\ \text{vector, timecomponent}: \quad V_0(\mathbf{x}) &= Q_1(r), \\ \text{tensor, mixedcomponents}: \quad T_{0i}(\mathbf{x}) &= n_i\, Q_2(r). \end{aligned} \tag{4}$$

III. Isoscalar fields acting on s quarks

$$\begin{aligned} \text{scalar}: \quad S(\mathbf{x}) &= R_0(r), \\ \text{vector, timecomponents}: \quad V_0(\mathbf{x}) &= R_1(r), \\ \text{tensor, mixedcomponents}: \quad T_{0i}(\mathbf{x}) &= n_i\, R_2(r). \end{aligned} \tag{5}$$

[1] A. Hosaka informed us that historically, this *Ansatz* for the pion field in a nucleon appears for the first time in a 1942 paper by Pauli and Dancoff [10]; it reappears in 1961 in the seminal papers by Skyrme [11].

All the rest fields and components are zero as they do not satisfy the $SU(2)_{\rm iso+space}$ symmetry and/or the needed discrete C, P, T symmetries. The 12 'profile' functions $P_{0,1,2,3,4,5}$, $Q_{0,1,2}$ and $R_{0,1,2}$ should be eventually found self-consistently from the minimization of the mass of the ground-state baryon. We shall call Eqs. (3-5) the hedgehog *Ansatz*. However, even if we do not know those profiles, there are important consequences of this *Ansatz* for the baryon spectrum.

3 u, d, s quarks in the 'hedgehog' field

Given the $SU(2)_{\rm iso+space}$ symmetry of the mean field, the Dirac Hamiltonian for quarks actually splits into two: one for s quarks and the other for u, d quarks [4]. It should be stressed that the energy levels for u, d quarks on the one hand and for s quarks on the other are completely different, even in the chiral limit $m_s \to 0$.

The energy levels for s quarks are classified by half-integer J^P where P is parity under space inversion, and $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is quark angular momentum; all levels are $(2J+1)$-fold degenerate. The energy levels for u, d quarks are classified by integer K^P where $\mathbf{K} = \mathbf{T} + \mathbf{J}$ is the 'grand spin' (T is isospin), and are $(2K+1)$-fold degenerate.

All energy levels, both positive and negative, are probably discrete owing to confinement. Indeed, a continuous spectrum would correspond to a situation when quarks are free at large distances from the center, which contradicts confinement. One can model confinement *e.g.* by forcing the effective quark masses to grow linearly at infinity, $S(\mathbf{x}) \to \sigma r$.

The Dirac equation (1) for s quarks in the background field (5) takes the form of a system of two ordinary differential equations for two functions $f(r)$, $g(r)$ depending only on the distance from the center. The system of equations depends on the (half-integer) angular momentum of level under considerations, and on its parity. For s-quark levels with parity $P = (-1)^{J-\frac{1}{2}}$, *e.g.* for the levels $J^P = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, ..$, the system takes the form

$$\left\{ \begin{array}{lll} E\, f &=& -g' - \frac{J+\frac{3}{2}}{r}\, g + R_0\, f + R_1\, f + R_2\, g \\ E\, g &=& f' + \frac{-J+\frac{1}{2}}{r}\, f - R_0\, g + R_1\, g + R_2\, f. \end{array} \right. \tag{6}$$

To find an s-quark energy level E with these quantum numbers, one has to solve Eq. (6) with the initial condition $f(r) \sim r^{J-\frac{1}{2}}$, $g(r) \sim r^{J+\frac{1}{2}}$, and both functions decreasing at infinity.

For levels with opposite parity $P = (-1)^{J+\frac{1}{2}}$, *e.g.* $J^P = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, ..$, one has to solve another system:

$$\left\{ \begin{array}{lll} E\, f &=& -g' - \frac{-J+\frac{1}{2}}{r}\, g + R_0\, f + R_1\, f + R_2\, g \\ E\, g &=& f' + \frac{J+\frac{3}{2}}{r}\, f - R_0\, g + R_1\, g + R_2\, f. \end{array} \right. \tag{7}$$

We note that in the absence of the $R_{1,2}$ fields the energy spectrum is symmetric under simultaneous change of parity and energy signs.

Dirac equation for u, d quarks in the background fields (3,4) is more complicated: one has here a system of four ordinary differential equations. These equations are direct generalizations of the Dirac equations in the 'hedgehog' field [7], and can be derived similarly to how it is done in that reference.

The system of Dirac equations for the radial functions of the states with parity $(-1)^{K+1}$, namely $K^P = 1^+, 2^-, ...$ has the form

$$E\, f = -g' - \frac{1+K}{r}\, g + (Q_0+Q_1+P_2+P_4)f + (Q_2-P_1)g - \frac{P_0-P_1}{2K+1}(g+b_K\, h) + \frac{P_3+P_5}{2K+1}(f+b_K\, j), \tag{8}$$

$$E\, g = f' - \frac{K-1}{r}\, f + (Q_1-Q_0-P_2+P_4)g + (Q_2-P_1)f - \frac{P_0-P_1}{2K+1}(f+b_K\, j) + \frac{P_3-P_5+2P_2-2P_4}{2K+1}(g+b_K\, h), \tag{9}$$

$$E\,h = j'+\frac{2+K}{r}\,j+(Q_1-Q_0-P_2+P_4)h+(Q_2-P_1)j+\frac{P_0-P_1}{2K+1}(j-b_K\,f)-\frac{P_3-P_5+2P_2-2P_4}{2K+1}(h-b_K\,g), \tag{10}$$

$$E\,j = -h'+\frac{K}{r}\,h+(Q_0+Q_1+P_2+P_4)j+(Q_2-P_1)h+\frac{P_0-P_1}{2K+1}(h-b_K\,g)-\frac{P_3+P_5}{2K+1}(j-b_K\,f), \tag{11}$$

where $b_K = 2\sqrt{K(K+1)}$. The radial functions f, g, h, j refer to partial waves with $L = K-1, K, K, K+1$, respectively, and they behave at the origin as r^L. To find the energy levels for a given K^P, one has to solve these equations twice: once with the initial condition $f(r_{\rm min}) \sim r_{\rm min}^{K-1}$, all the rest functions being put to zero at the origin, and another time with the initial condition $h(r_{\rm min}) \sim r_{\rm min}^{K}$, with all the rest functions zeroes, $r_{\rm min} \to 0$. Evolving the functions according to the equations numerically up to some asymptotically large $r_{\rm max}$ one finds two sets of functions (f_1, g_1, h_1, j_1) and (f_2, g_2, h_2, j_2). The energy levels are found from the zeroes of two (equal) determinants $f_1h_2 - f_2h_1 = g_1j_2 - g_2j_1$.

For states with parity $(-1)^K$, namely $K^P = 1^-, 2^+, ..$ the system of Dirac equations is:

$$E\,f = -g'-\frac{1+K}{r}\,g+(Q_1-Q_0+P_2-P_4)f-(Q_2+P_1)g+\frac{P_0+P_1}{2K+1}(g+b_K\,h)+\frac{P_3-P_5}{2K+1}(f+b_K\,j), \tag{12}$$

$$E\,g = f'-\frac{K-1}{r}\,f+(Q_0+Q_1-P_2-P_4)g-(Q_2+P_1)f+\frac{P_0+P_1}{2K+1}(f+b_K\,j)+\frac{P_3+P_5+2P_2+2P_4}{2K+1}(g+b_K\,h), \tag{13}$$

$$E\,h = j'+\frac{2+K}{r}\,j+(Q_0+Q_1-P_2-P_4)h-(Q_2+P_1)j-\frac{P_0+P_1}{2K+1}(j-b_K\,f)-\frac{P_3+P_5+2P_2+2P_4}{2K+1}(h-b_K\,g), \tag{14}$$

$$E\,j = -h'+\frac{K}{r}\,h+(Q_1-Q_0+P_2-P_4)j-(Q_2+P_1)h-\frac{P_0+P_1}{2K+1}(h-b_K\,g)-\frac{P_3-P_5}{2K+1}(j-b_K\,f), \tag{15}$$

where again $f \sim r^{K-1}, g \sim r^K, h \sim r^K, j \sim r^{K+1}$, and the levels are found by the same trick. The fields $Q_{1,2}$ and $P_{0,2,3}$ break symmetry with respect to simultaneous change of parity and energy signs.

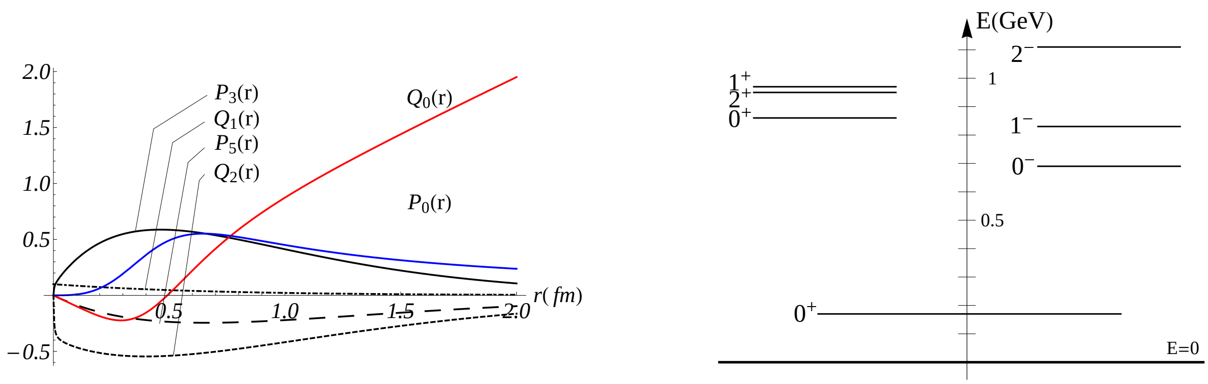

Figure 1: An illustrative example of intrinsic quark levels with quantum numbers K^P (right) generated by the mean fields shown in the left panel.

The case $K = 0$ is special, since the angular momentum is restricted to only one value $J = K + \frac{1}{2} = \frac{1}{2}$. It means that $g = h = 0$, and the system of eqs.(8)-(11) for the $K^P = 0^-$ level reduces to two equations:

$$\begin{aligned} E\,j &= -h' + (Q_0 + Q_1 + P_2 - P_3 + P_4 - P_5)j + (P_0 - 2P_1 + Q_2)h, \\ E\,h &= j' + \frac{2}{r}j + (-Q_0 + Q_1 - 3P_2 - P_3 + 3P_4 + P_5)h + (P_0 - 2P_1 + Q_2)j \end{aligned} \tag{16}$$

with $h \sim r^0, j \sim r^1$. Similarly, to find the $K^P = 0^+$ levels one has to solve only two equations:

$$\begin{aligned} E\,j &= -h' + (-Q_0 + Q_1 + P_2 - P_3 - P_4 + P_5)j - (P_0 + 2P_1 + Q_2)h, \\ E\,h &= j' + \frac{2}{r}j + (Q_0 + Q_1 - 3P_2 - P_3 - 3P_4 - P_5)h - (P_0 + 2P_1 + Q_2)j. \end{aligned} \tag{17}$$

In Fig. 1 we show an example of quark levels obtained from a 'natural' choice of external fields $Q_{0\text{–}2}, P_{0\text{–}5}$. We take a confining scalar field $S(r) = \sigma r$ with a standard string tension $\sigma = (0.44\,\mathrm{GeV})^2$, and a topological chiral angle field $P(r) = 2\arctan(r_0^2/r^2)$ such that the profile functions introduced in Eqs.(3,4) are $Q_0(r) = S(r)\cos P(r),\ P_0(r) = S(r)\sin P(r)$; the other profile functions are exponentially decaying al large distances. The external fields are shown in Fig. 1, left, and the resulting quark levels with various K^P are shown in Fig. 1, right. As we shall see, these or similar levels dictate the masses of baryon resonances.

4 Ground state baryons

According to the Dirac theory, all *negative*-energy levels, both for s and u, d quarks, have to be fully occupied, corresponding to the vacuum. It means that there must be exactly N_c quarks antisymmetric in colour occupying all degenerate levels with J_3 from $-J$ to J, or K_3 from $-K$ to K; they form closed shells. Filling in the lowest level with $E > 0$ by N_c quarks makes a baryon [6, 4], see Fig. 2. A similar picture arises in the chiral bag model [8].

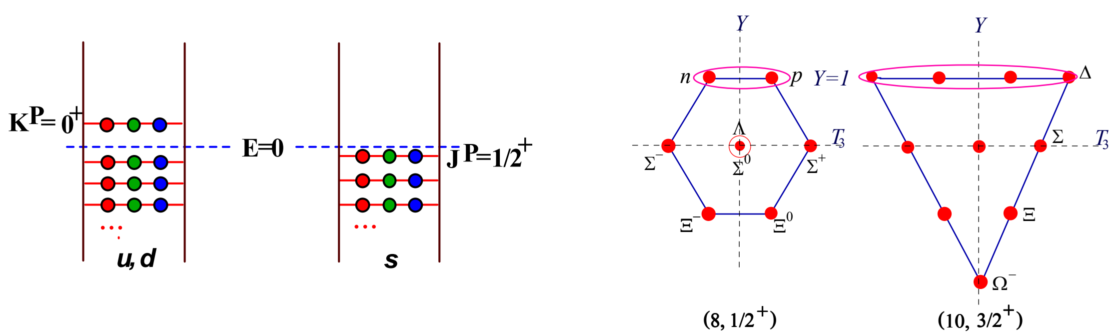

Figure 2: Filling u, d and s shells for the ground-state baryon (left), and the two lowest baryon multiplets that follow from quantizing the rotations of this filling scheme (right).

The mass of a baryon is the aggregate energy of all filled states, and being a functional of the mesonic field, it is proportional to N_c since all quark levels are degenerate in colour. Therefore quantum fluctuations of mesonic field in baryons are suppressed as $1/N_c$ so that the mean field is indeed justified.

Quantum numbers of the lightest baryons are determined from the quantization of the rotations of the mean field, leading to specific $SU(3)$ multiplets that reduce at $N_c = 3$ to the octet with spin $\frac{1}{2}$ and the decuplet with spin $\frac{3}{2}$, see *e.g.* [2]. Witten's quantization condition for the hypercharge of the upper line of the multiplet, $\tilde{Y} = \frac{N_c}{3}$ [12], follows trivially from the fact that there are N_c u, d valence quarks each with the hypercharge $\frac{1}{3}$ [9]. Therefore, the ground state shown in Fig. 2 entails in fact 56 rotational states. The splitting between the centers of the multiplets $(\mathbf{8}, \frac{1}{2}^+)$ and $(\mathbf{10}, \frac{3}{2}^+)$ is $\mathcal{O}(1/N_c)$, and the splittings inside multiplets can be determined as a perturbation in m_s [9].

The lowest baryon resonance beyond the rotational excitations of the ground state is the singlet $\Lambda(1405, \frac{1}{2}^-)$. Apparently, it can be obtained only as an excitation of the s quark, and its quantum numbers must be $J^P = \frac{1}{2}^-$ [4], see transition *1* in Fig. 3. Similarly, there must be a slightly higher s quark level with quantum numbers $J^P = \frac{3}{2}^-$ corresponding to $\Lambda(1520, 3/2^-)$, see the transition *2* in Fig. 3. These two baryons are the only known $SU(3)$ singlets, and there is no way of obtaining them other than from s quark excitations.

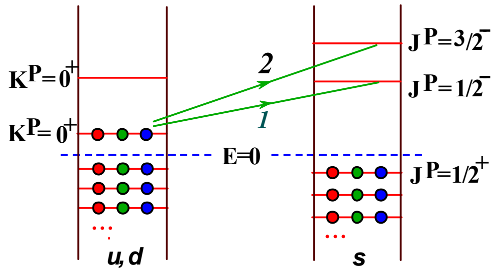

Figure 3: One-quark excitations of s quark levels, corresponding to the $SU(3)$ singlet resonances, $\Lambda(1405, 1/2^-)$ and $\Lambda(1520, 3/2^-)$.

5 Rotational bands about intrinsic quark levels

Assuming the chiral limit, $m_u = m_d = m_s = 0$, the original symmetry of the problem is $SU(3)_{\rm flav} \times SO(3)_{\rm space}$. This symmetry, however, is broken spontaneously by the 'hedgehog' *Ansatz* of the mean field. Correspondingly, a filling scheme of one-particle quark levels, be it for the ground state (Fig. 2) or for any one-quark excitations (see *e.g.* Fig. 3), treats u, d quarks and s quarks differently and therefore violates the $SU(3)_{\rm flav}$ and also the $SO(3)_{\rm space}$ symmetries. Only the $SU(2)_{\rm iso+space}$ symmetry of simultaneous isospin and compensating space rotations is preserved. The original $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ symmetry is restored when flavour and space rotations are accounted for.

In the chiral limit (which we assume) an arbitrary $SU(3)_{\rm flav}$ rotation of the mean field and hence of what we call u, d, s quarks does not change the energy of the state. The same is true for the $SO(3)_{\rm space}$ rotation. However, if $SU(3)_{\rm flav}$ and $SO(3)_{\rm space}$ rotations are slowly dependent on time, they generate a shift in the energy of the system; it is called the rotational energy.

According to Quantum Mechanics, these rotations have to be quantized. It leads to the appearance of a 'rotational band' atop every one-quark excitation, which, in this case, is a tower of $SU(3)_{\rm flav}$ multiplets with definite spin; these multiplets are specific for each one-quark excitation. Multiplets inside a band stemming from a given one-quark excitation, are split as $\mathcal{O}(1/N_c)$. In principle, the rotational states go up to infinity, however, in practice we terminate the rotational band when the rotational energy becomes $\mathcal{O}(1)$ since such high rotational states may mix with other one-quark excitations, which requires a more careful analysis.

We sketch below the quantization of $SU(3)_{\rm flav}$ and $SO(3)_{\rm space}$ rotations about a one-quark excitation from the ground state with $K^P = 0^+$ to a level with certain grand spin K. To the best of our knowledge, such quantization has not been considered before, and it is of some interest.

Let $R(t)$ be an $SU(3)$ matrix for slow rotations in flavour space, and $S(t)$ be an $SU(2)$ matrix for slow space (and spin) rotations. We introduce angular velocities $\tilde{\Omega}^a = -i{\rm Tr}(R^\dagger \dot{R} \lambda^a)$ and $\tilde{\omega}^i = -i{\rm Tr}(S^\dagger \dot{S} \sigma^i)$ where λ^a and σ^i are 8 Gell-Mann and 3 Pauli matrices, respectively. The rotational Lagrangian is (see *e.g.* [2])

$$\mathcal{L}^{(0)}_{\rm rot} = \frac{I_1}{2}\sum_{a=1}^{3}(\tilde{\Omega}^a - \tilde{\omega}^a)^2 + \frac{I_2}{2}\sum_{a=4}^{7}(\tilde{\Omega}^a)^2 - \frac{N_c}{2\sqrt{3}}\tilde{\Omega}^8, \tag{18}$$

where $I_{1,2} = \mathcal{O}(N_c)$ are the two moments of inertia. This is a well-known rotational Lagrangian for the ground state, but also for rotations about one-quark excitations of u, d quarks to the $K^P = 0^\pm$ levels.

For rotations about one-quark excitations to $K \neq 0$ levels of any parity, one has to add terms to the rotational Lagrangian, that are *linear* in angular velocities. The point is, a u, d quark level with $K \neq 0$ is a degenerate mixture of $2K + 1$ states with different projections K_3, $-K \le K_3 \le K$, characterized by the wave functions χ_{K_3}. When the background field is

R- or S rotated, the degeneracy of states with different K_3 is lifted, and one finds an additional one-quark contribution to the rotational Lagrangian:

$$\begin{aligned}
\mathcal{L}^{(1)}_{\rm rot} &= \chi^\dagger_{K_3} i\frac{\partial}{\partial t}\chi_{K_3} + \tilde{\mathbf{\Omega}}\cdot\mathbf{T}^{(1)} + \tilde{\boldsymbol{\omega}}\cdot\mathbf{J}^{(1)}, \\
\mathbf{T}^{(1)} &= \sum_{K_3,K_3'} \chi^\dagger_{K_3'}\chi_{K_3} < KK_3JL|\mathbf{T}|KK_3'JL >, \\
\mathbf{J}^{(1)} &= \sum_{K_3,K_3'} \chi^\dagger_{K_3'}\chi_{K_3} < KK_3JL|\mathbf{J}|KK_3'JL > .
\end{aligned} \tag{19}$$

$\mathbf{J}^{(1)}, \mathbf{T}^{(1)}$ are the angular momentum and isospin, respectively, of a given quark state $|KK_3JL>$. Apparently, they satisfy the relation $\mathbf{J}^{(1)}+\mathbf{T}^{(1)} = \mathbf{K}$ where $\mathbf{K}^2 = K(K+1)$ and K is the grand spin of the excited level. The full rotational Lagrangian is the sum,

$$\mathcal{L}_{\rm rot} = \mathcal{L}^{(0)}_{\rm rot} + \mathcal{L}^{(1)}_{\rm rot}. \tag{20}$$

To quantize the rotations about one-quark excitations, one introduces the canonically conjugated momenta

$$\begin{aligned}
&\tilde{\mathbf{T}} = \frac{\partial\mathcal{L}}{\partial\tilde{\mathbf{\Omega}}} = I_1(\tilde{\mathbf{\Omega}}-\tilde{\boldsymbol{\omega}}) + \mathbf{T}^{(1)}, \quad \tilde{\mathbf{J}} = \frac{\partial\mathcal{L}}{\partial\tilde{\boldsymbol{\omega}}} = -I_1(\tilde{\mathbf{\Omega}}-\tilde{\boldsymbol{\omega}}) + \mathbf{J}^{(1)}, \quad (\tilde{\mathbf{T}}+\tilde{\mathbf{J}} = \mathbf{K}), \\
&\tilde{T}^a = I_2\tilde{\Omega}^a \ (a=4,5,6,7), \quad \tilde{T}^8 = -\frac{N_c}{3},
\end{aligned}$$

and finds the rotational Hamiltonian

$$\mathcal{H}_{\rm rot} = \tilde{T}^a\tilde{\Omega}^a + \tilde{J}^i\tilde{\omega}^i - \mathcal{L}_{\rm rot} = \frac{1}{2I_1}\sum_{a=1}^{3}(\tilde{\mathbf{T}}-\mathbf{T}^{(1)})^2 + \frac{1}{2I_2}\sum_{a=4}^{7}(\tilde{T}^a)^2 \tag{21}$$

which needs to be quantized using the standard commutation relations for $\tilde{T}^a$ and for $\tilde{J}^i$ but with the constraint $\tilde{\mathbf{T}}+\tilde{\mathbf{J}} = \mathbf{K}$ and hypercharge $\tilde{Y} = -(2/\sqrt{3})\tilde{T}^8 = \frac{N_c}{3}$.

An important circumstance is that matrix elements of the one-quark operators $\mathbf{T}^{(1)}, \mathbf{J}^{(1)}$ are proportional to $\mathbf{K}$,

$$\mathbf{T}^{(1)} = \frac{K+1-(2K+1)c_K}{K(K+1)}\mathbf{K} = \frac{K+1-(2K+1)c_K}{K(K+1)}(\tilde{\mathbf{J}}+\tilde{\mathbf{T}}),$$

where c_K is related to the normalization of the radial functions of the K level,

$$c_K = \frac{\int dr\, r^2(h^2+j^2)}{\int dr\, r^2(f^2+g^2+h^2+j^2)},$$

and that $\tilde{\mathbf{J}}^2 = \mathbf{J}^2 = J(J+1)$ where J is the total angular momentum of the state. This allows to find the eigenvalues of the operator

$$(\tilde{\mathbf{T}}-\mathbf{T}^{(1)})^2 = a_K J(J+1) + (1-a_K)\tilde{T}(\tilde{T}+1) - a_K(1-a_K)K(K+1), \qquad a_K = \frac{1-c_K}{K} - \frac{c_K}{K+1}.$$

The second term in the Hamiltonian (21) can be written as

$$\sum_{a=4}^{7}(\tilde{T}^a)^2 = \sum_{a=1}^{8}(\tilde{T}^a)^2 - \sum_{a=1}^{3}(\tilde{T}^a)^2 - (\tilde{T}^8)^2 = C_2 - \tilde{T}(\tilde{T}+1) - \frac{3}{4}\tilde{Y}^2 \tag{22}$$

where $C_2 = \frac{1}{3}(p^2+pq+q^2+3p+3q)$ is the quadratic Casimir operator for an $SU(3)$ multiplet characterized by the Young tableau numbers p, q. For the large-N_c prototype of the octet one takes [2] $p = 1$, $q = \frac{N_c-1}{2}$ and $\tilde{T} = \frac{1}{2}$, $\tilde{Y} = \frac{N_c}{3}$, whereas for the prototype of the decuplet one takes $p = 3$, $q = \frac{Nc-3}{2}$, $\tilde{T} = \frac{3}{2}$, $\tilde{Y} = \frac{N_c}{3}$. In both cases the second term in the Hamiltonian (21) is a constant energy shift,

$$\frac{1}{2I_2}\sum_{a=4}^{7}(\tilde{T}^a)^2 = \frac{N_c}{4I_2} = \mathcal{O}(1).$$

To summarize, the masses of the centers of the octets and decuplets obtained as rotational excitations about a one-quark transition of u, d quarks from the ground state level 0^+ to an excited K^P level, are given by the equation

$$\mathcal{M} = \mathcal{M}_0 + \frac{N_c}{4I_2} + \Delta\mathcal{E}(0^+ \to K^P) + \frac{a_K J(J+1) + (1-a_K)\tilde{T}(\tilde{T}+1) - a_K(1-a_K)K(K+1)}{2I_1}. \quad (23)$$

One takes $\tilde{T} = \frac{1}{2}, \frac{3}{2}$ for the octet and decuplet, respectively, and the possible spins $J = \tilde{J}$ of the multiplets are found from the vector addition law $\tilde{\mathbf{J}} + \tilde{\mathbf{T}} = \mathbf{K}$. For example, if we take the one-quark excitation from the ground-state 0^+ to an excited $1^\pm$ level the corresponding rotational band consists of octets with spin $J = \frac{1}{2}$ and $J = \frac{3}{2}$, and decuplets with spin $J = \frac{1}{2}, \frac{3}{2}$ and $\frac{5}{2}$. The masses of the centers of those multiplets are given by Eq. (23), the coefficient $a_{K=1}$ being the same for all multiplets from the same band. In the leading N_c approximation, the moments of inertia $I_{1,2}$ are the same for all rotational bands about all one-quark excitations.

For the rotational excitations about the ground state 0^+ (or about the one-quark $0^+ \to 0^\pm$ transition) one has only two multiplets: the octet with spin $J = \tilde{T} = \frac{1}{2}$ and a decuplet with spin $J = \tilde{T} = \frac{3}{2}$, and Eq. (23) reduces to the well-known equation for the splitting between their centers:

$$\mathcal{M} = \mathcal{M}_0 + \frac{N_c}{4I_2} + \Delta\mathcal{E}(0^+ \to 0^\pm) + \frac{J(J+1)}{2I_1}. \quad (24)$$

6 Comparison with the experimental spectrum

We shall now look into the experimental spectrum of light baryon resonances up to around 2 GeV, trying to recognize among them the rotational bands about the one-quark excitations from the ground state to the $K^P = 0^\pm, 1^\pm, 2^\pm$ intrinsic quark levels. We shall treat the quantities entering Eq. (23) as free parameters to be fitted from the known masses, although in principle they are calculable if the (self-consistent) mean field is known. Still, there are much more resonances than free parameters, therefore the rotational bands are severely restricted by Eq. (23). As we shall see these restrictions are well satisfied by experimental masses, despite that in the real world N_c is only three.

Since at this time we do not take into account the splittings inside $SU(3)$ multiplets as due to the nonzero m_s, Eq. (23) should be compared with the *centers* of multiplets. For the octet, the center is defined as $\mathcal{M}_8 = (2m_N + 2m_\Xi + 3m_\Sigma + m_\Lambda)/8$, and for the decuplet it is $\mathcal{M}_{10} = (4m_\Delta + 3m_{\Sigma^*} + 2m_{\Xi^*} + m_\Omega)/10 \approx m_{\Sigma^*}$. We take the phenomenological values for $\mathcal{M}_8$, $\mathcal{M}_{10}$ from the paper by Guzey and Polyakov [13] who have recently analyzed baryon multiplets up to 2 GeV.

6.1 Spurious states

When comparing the mean-field predictions (valid at $N_c \to \infty$) with the data, it should be kept in mind that certain rotational states are in fact spurious, as they are artifacts of the mean-field approximation where the spatial wave function is a product of one-particle wave functions. When averaging over the center of mass is taken into account (which is an $\mathcal{O}(1/N_c)$ effect) the baryon wave functions depend only on the differences of quark coordinates, which for some states may contradict the Pauli principle. This effect has been long known both in nuclear physics [14] and in the non-relativistic quark model [15]. The simplest way to identify spurious states is to continuously deform the mean field to the non-relativistic oscillator potential where the wave functions are explicit. If some state is absent in that limit, it cannot appear from a continuous deformation. An independent way to check for spurious states is to deform the problem at hand to the exactly solvable (0+1)-dimensional four-fermion interaction model [16] where the large-N_c approximation is also possible and reveals extra states.

Specifically, in the parity-plus sector, the spurious state is $(\mathbf{10}, 1/2^+)$ arising from the rotational band about the $(0^+ \to 2^+)$ transition. Such state arises also from the $(0^+ \to 1^+)$ transition but then it is allowed.

In the parity-minus sector there are more spurious states: the multiplets $(\mathbf{10}, 5/2^-)$ and $(\mathbf{10}, 7/2^-)$ stemming from the $(0^+ \to 2^-)$ transition are spurious, two out of three multiplets $(\mathbf{10}, 3/2^-)$ arising from $(0^+ \to 0^-, 1^-, 2^-)$ transitions are spurious, and one out of two multiplets $(\mathbf{10}, 1/2^-)$ stemming from $(0^+ \to 1^-, 2^-)$ transitions is spurious, too. Spurious rotational states should be deleted when comparing with the data.

6.2 Parity-plus resonances

The two lowest multiplets $(\mathbf{8}, 1/2^+, 1149)$ and $(\mathbf{10}, 3/2^+, 1382)$ (the last number in the parentheses is the center of the multiplet) form the rotational band about the ground-state filling scheme shown in Fig. 2. Fitting these masses by Eq. (24) we find $\mathcal{M}_0 + \frac{3}{4I_2} = 1091\,\mathrm{MeV}$, $1/I_1 = 155\,\mathrm{MeV}$.

Apart from the two lowest multiplets, there is another low-lying pair with the same quantum numbers, $(\mathbf{8}, 1/2^+, 1605)$ and $(\mathbf{10}, 3/2^+, 1732)$. Other parity-plus multiplets are essentially higher. Therefore, one needs a $0^+ \to 0^+$ transition to explain this pair. From the fit to the masses one finds that the second $K^P = 0^+$ intrinsic quark level must be 483 MeV higher than the ground state 0^+ level, $\Delta\mathcal{E}(0^+ \to 0^+) = 483\,\mathrm{MeV}$. The moment of inertia appears to be considerably larger than for the ground-state multiplets, $1/I_1 = 85\,\mathrm{MeV}$. Although the difference is an $\mathcal{O}(1/N_c)$ effect, it may be enhanced if the radially excited 0^+ level has a much larger effective radius.

There is a group of five multiplets, $(\mathbf{8}, 3/2^+, 1865)$, $(\mathbf{8}, 5/2^+, 1873)$, $(\mathbf{10}, 3/2^+, 2087)$, $(\mathbf{10}, 5/2^+, 2031)$, $(\mathbf{10}, 7/2^+, 2038)$, that are good candidates for the rotational band about the $0^+ \to 2^+$ transition. Indeed, this is precisely the content of the rotational band for this transition (the spurious multiplet $(\mathbf{10}, 1/2^+)$ excluded), and a fit to the masses according to Eq. (23) gives a small $\sqrt{\chi^2} = 15\,\mathrm{MeV}$. It should be kept in mind, though, that not all members of all multiplets are well established [13], and those that are, have an experimental uncertainty in the masses. It means that the 'experimental' masses for the centers of multiplets are known at best to an accuracy of 20-40 MeV. We find from the fit $1/I_1 = 122\,\mathrm{MeV}$, $\Delta\mathcal{E}(0^+ \to 2^+) = 715\,\mathrm{MeV}$. Therefore, the intrinsic 2^+ level must be higher than the 0^+ one.

The only relatively well established multiplet that is left in the range below 2 GeV, is $(\mathbf{8}, 1/2^+, 1846)$. It prompts that it can arise from the rotational band about the $0^+ \to 1^+$ transition, however, other parts of the band are poorly known. If one looks into non-strange baryons that are left, one finds $N(1/2^+, 1710^{***})$, $N(1/2^+, 1900^{**})$, $\Delta(1/2^+, 1910^{***})$ and $\Delta(5/2^+, 2000^{**})$, with $\Delta(3/2^+)$ missing. The quantum numbers and the masses of these supposed resonances fit rather well the hypothesis that they arise as a rotational band about the $0^+ \to 1^+$ transition, however, their low status prevents a definite conclusion. The intrinsic 1^+ level must be approximately 70 MeV higher than the 2^+ quark level.

6.3 Parity-minus resonances

The situation here is similar to the parity-plus sector: one needs intrinsic quark levels with $K^P = 0^-, 1^-, 2^-$ to explain the resonances as belonging to rotational bands about these transitions. Given that several rotational states in the parity-minus sector are spurious, one expects to find the following multiplets stemming from these transitions: $(\mathbf{8},1/2^-)\times 2$, $(\mathbf{8},3/2^-)\times 2$, $(\mathbf{8},5/2^-)$, $(\mathbf{10},1/2^-)$, $(\mathbf{10},3/2^-)$: these are precisely the observed multiplets.

The multiplets $(\mathbf{8},1/2^-,1609)$ and $(\mathbf{10},3/2^-,1850)$ form the rotational band of the $0^+\to 0^-$ transition, with $1/I_1 = 161\,\mathrm{MeV}$, $\Delta\mathcal{E}(0^+\to 0^-) = 458\,\mathrm{MeV}$. Comparing it with the $0^=$ excitation energy 483 MeV, we see that the lowest intrinsic excitation is 0^- which is reasonable.

The multiplets $(\mathbf{8},1/2^-,1710)$, $(\mathbf{8},3/2^-,1895)$ and $(\mathbf{10},1/2^-,1758)$ form the rotational band of the $0^+\to 1^-$ transition, with $1/I_1 = 155\,\mathrm{MeV}$, $\Delta\mathcal{E}(0^+\to 0^-) = 586\,\mathrm{MeV}$.

Finally, the multiplets $(\mathbf{8},3/2^-,1673)$ and $(\mathbf{8},5/2^-,1801)$ are the only multiplets belonging to the $0^+\to 2^-$ rotational band. Assuming $1/I_1 = 155\,\mathrm{MeV}$ we find $\Delta\mathcal{E}(0^+\to 2^-) = 774\,\mathrm{MeV}$.

To summarize, all parity-plus and parity-minus baryons around 2 GeV and below can be accommodated by the scheme, assuming they all arise as rotational excitations about the $0^+\to 0^+,1^+,2^+$ and $0^+\to 0^-,1^-,2^-$ transitions, see Table 1. There are no unexplained resonances left, but there appears an extra state $\Delta(3/2^+,\sim 1945)$ stemming from the $0^+\to 1^+$ transition, which is so far unobserved, so this state is a prediction.

intrinsic quark levels	multiplets that are rotational states about this transition
$K^P = 0^+$ ground state	$(\mathbf{8},1/2^+,1149)$ $(\mathbf{10},3/2^+,1382)$
$0^+\to 0^+$ 483 MeV	$(\mathbf{8},1/2^+,1605)$ $(\mathbf{10},3/2^+,1732)$
$0^+\to 2^+$ 715 MeV	$(\mathbf{8},3/2^+,1865)$ $(\mathbf{8},5/2^+,1873)$ $(\mathbf{10},3/2^+,2087)$ $(\mathbf{10},5/2^+,2031)$ $(\mathbf{10},7/2^+,2038)$
$0^+\to 1^+$ $\sim 780\,\mathrm{MeV}$	$N(1/2^+,1710)$ $N(3/2^+,1900)$ $\Delta(1/2^+,1910)$ $\Delta(3/2^+,\sim 1945)$? $\Delta(5/2^+,2000)$
$0^+\to 0^-$ 458 MeV	$(\mathbf{8},1/2^-,1609)$ $(\mathbf{10},3/2^-,1850)$
$0^+\to 1^-$ 586 MeV	$(\mathbf{8},1/2^-,1710)$ $(\mathbf{8},3/2^-,1895)$ $(\mathbf{10},1/2^-,1758)$
$0^+\to 2^-$ 774 MeV	$(\mathbf{8},3/2^-,1673)$ $(\mathbf{8},5/2^-,1801)$

Table 1: Interpretation of all baryon resonances below 2 GeV, as rotational excitations on top of intrinsic quark states.

7 s quarks

As emphasized in Section 3, s quarks are in a completely different external field than u,d quarks, even in the chiral limit. Only the confining forces which we model by a linear rising scalar field are the same for all quarks. The two excited levels for s quarks are shown in Fig. 3: they are needed to explain the singlet $\Lambda(1/2^-,1405)$ and $\Lambda(3/2^-,1520)$ resonances. No more singlet Λ's are known below 2 GeV, therefore there should be no intrinsic s-quark levels either with positive or negative parity in this range.

The intriguing question is where is the highest *filled* level of s quarks? Presumably, it must be a level with quantum numbers $J^P = \frac{1}{2}^+$ as possessing maximal symmetry. There can be one-quark excitations from that level both to the s-quark excited levels $\frac{1}{2}^-$ and $\frac{3}{2}^-$, and to the

u, d excited levels $0^+, 0^-, \ldots$, see Fig. 4. Transitions of the first type generate a rotational band consisting of $(\mathbf{8}, 1/2^-)\times 2$, $(\mathbf{8}, 1/2^-)$, $(\mathbf{10}, 1/2^-)$, $(\mathbf{10}, 3/2^-)\times 2$ and $(\mathbf{10}, 5/2^-)$ [5]. Transitions of the second type called, in the terminology of nuclear physics, Gamov-Teller transitions, generate the exotic *antidecuplet* $(\overline{\mathbf{10}}, 1/2^+)$, etc. [4, 5].

It turns out that it is difficult, if not impossible, to move the highest filled level of s quarks $\frac{1}{2}^+$, which must satisfy Eq. (6) in a 'realistic' mean field, more than $\sim 700\,\mathrm{MeV}$ below the first excited level $\frac{1}{2}^-$. Therefore, the parity-minus resonances generated by the transition *1* in Fig. 4 must reveal themselves in the spectrum below 2 GeV. We note that such resonances will have a substantial 5-quark component $u(d)u(d)u(d)s\bar{s}$ since they require an s quark to be pulled out of the filled level and put onto an excited level. Probably, the real-world resonances are certain mixtures of these excitations with the u, d excitations described in the previous section. This is a welcome feature as, for example, the well-known resonance $N(1/2^-, 1535)$ has an otherwise un-explicable large coupling to the η meson [17]. The 'Gamov-Teller' transition *2* gives a natural explanation of the exotic Θ^+ resonance [18] exactly at the position where it has been claimed by a number of experiments [4, 5].

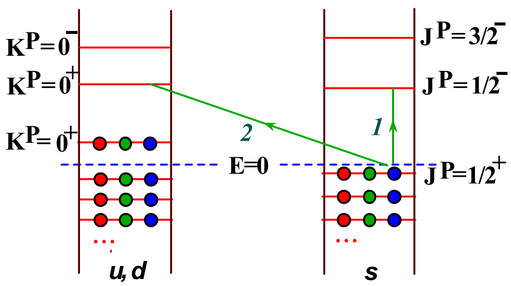

Figure 4: Possible transitions of the s quark from the highest filled s level to excited s levels (*1*), and to excited u, d levels (*2*).

8 Charmed and bottom baryons

If one of the light quarks in a light baryon is replaced by a heavy b or c quark, there are still $N_c - 1$ light quarks left. At large N_c, they form *the same* mean field as in light baryons, with the same sequence of Dirac levels, up to $1/N_c$ corrections. The heavy quark contributes to the mean $SU(3)_{\rm flav}$ symmetric field but it is a $1/N_c$ correction, too. It means that at large N_c one can *predict* the spectrum of the $Qq\ldots q$ (and $Qq\ldots qq\bar{q}$) baryons from the spectrum of light baryons. At $N_c = 3$ one does not expect qualitative difference with the $N_c \to \infty$ limit, although $1/N_c$ corrections should be kept in mind. We consider the heavy quark as a non-relativistic particle having spin $J_h = \frac{1}{2}$. $SU(4)_{\rm flav}$ symmetry is badly violated and is of no guidance.

The filling of Dirac levels for the ground-state c (or b) baryon is shown in Fig. 5, left: there is a hole in the 0^+ shell for u, d quarks as there are only $N_c - 1$ quarks there, in an antisymmetric state in colour. Adding the heavy quark makes the full state 'colourless'.

As in the case of light baryons, the filling scheme by itself does not tell us what are the quantum numbers of the state: they arise from quantizing the $SU(3)_{\rm flav}$ and $SO(3)_{\rm space}$ rotations of the given filling scheme. Let us do it for the ground-state heavy baryons.

First of all, we determine the hypercharge of the filling scheme: in this case it is $\tilde{Y} = \frac{1}{3}(N_c - 1)$ since there are $N_c - 1$ u, d quarks each having hypercharge one third. At $N_c = 3$ one has $\tilde{Y} = \frac{2}{3}$. There are two $SU(3)$ multiplets containing particles with hypercharge $\frac{2}{3}$: the anti-triplet $\overline{\mathbf{3}}$ $(p=0, q=1)$ and the sextet $\mathbf{6}$ $(p=2, q=0)$, therefore these are the allowed multiplets, see Fig. 5, right. What are their spins?

In the $\overline{\mathbf{3}}$ representation, there is one particle with $\tilde{Y} = \frac{2}{3}$ hence its isospin $\tilde{T} = 0$. The

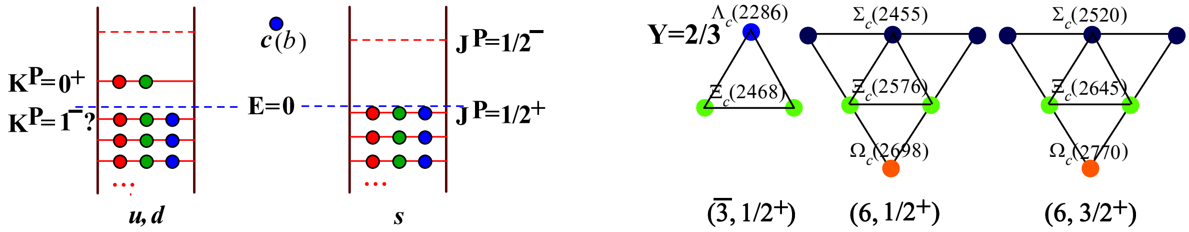

Figure 5: Filling u, d and s shells for the ground-state charmed baryons (left), and $SU(3)$ multiplets generated by this filling scheme (right): $(\overline{\mathbf{3}}, 1/2^+)$, $(\mathbf{6}, 1/2^+)$ and $(\mathbf{6}, 3/2^+)$.

possible spin of the multiplet is found from the vector addition law:

$$\mathbf{J} = \tilde{\mathbf{T}} + \mathbf{J_h}. \tag{25}$$

Therefore, the only possible spin of the anti-triplet is $\frac{1}{2}$, and parity plus. Its rotational energy is, according to Eq. (22),

$$E_{\rm rot}^{(\overline{\mathbf{3}})} = \frac{1}{2I_2} \tag{26}$$

(a more detailed derivation of the rotational energy in this case is given in Ref. [5]).

In the $\mathbf{6}$ representation, there are three particles with $\tilde{Y} = \frac{2}{3}$ hence their isospin $\tilde{T} = 1$. From Eq. (25) one finds then that there are *two* sextets, one with spin $\frac{1}{2}$ and another with spin $\frac{3}{2}$. They are degenerate in the leading order:

$$E_{\rm rot}^{(\mathbf{6})} = \frac{1}{2I_2} + \frac{1}{I_1}. \tag{27}$$

Thus the filling scheme in Fig. 5, left, implies in fact three $SU(3)$ multiplets: $(\overline{\mathbf{3}}, \frac{1}{2}^+)$, $(\mathbf{6}, \frac{1}{2}^+)$ and $(\mathbf{6}, \frac{3}{2}^+)$, see Fig. 5, right. The last two are degenerate (but the degeneracy is lifted in the next $1/N_c^2$ order and also from the $1/m_h$ corrections) whereas the center of the anti-triplet is separated from the center of the sextets by the rotational energy $\Delta E_{\rm rot} = \frac{1}{I_1}$. The splitting *inside* multiplets owing to the explicit violation of $SU(3)$ by the strange quark mass is $\mathcal{O}(m_s N_c)$. If m_s is treated as a small perturbation, $m_s = \mathcal{O}(1/N_c^2)$, as we claim it should [4], the splitting inside the sextet must be equidistant to a good accuracy. Let us confront these predictions with current data.

There are good candidates for the above ground-state multiplets: $\Lambda_c(2286)$ and $\Xi_c(2468)$ for $(\overline{\mathbf{3}}, 1/2^+)$; $\Sigma_c(2455)$, $\Xi_c(2576)$ and $\Omega_c(2698)$ for $(\mathbf{6}, 1/2^+)$; finally $\Sigma_c(2520)$, $\Xi_c(2645)$ and $\Omega_c(2770)$ presumably form $(\mathbf{6}, 3/2^+)$, see Fig. 5, right. Strictly speaking the J^P quantum numbers of most of these baryons are not measured directly but there is not much doubt they differ from the above assignments. Assuming they are correct, the observed parity-plus charmed baryons form precisely those multiplets that follow from the collective quantization.

The splittings inside the two sextets are equidistant to high accuracy, confirming that m_s can be treated as a small perturbation. Were m_s "not small", there would be substantial $\mathcal{O}(m_s^2)$ corrections to the masses, which would violate the equidistant character of the sextets spectrum.

The centers of the three multiplets are at

$$\begin{aligned}
m\left(\overline{\mathbf{3}}, 1/2^+\right) &= \frac{2287 + 2*2468}{3} = 2408 \text{ MeV}, \\
m\left(\mathbf{6}, 1/2^+\right) &= \frac{3*2455 + 2*2576 + 2698}{6} = 2536 \text{ MeV}, \\
m\left(\mathbf{6}, 3/2^+\right) &= \frac{3*2520 + 2*2645 + 2770}{6} = 2603 \text{ MeV}.
\end{aligned} \tag{28}$$

Although the two sextets are not exactly degenerate, their splitting 67 MeV (an unaccounted $1/N_c^2$ effect) is much less than the splitting between the anti-triplet and the mean mass of the sextets, which is

$$\frac{2536 + 2603}{2} - 2408 = 162 \text{ MeV} = E_{\rm rot}^{(\mathbf{6})} - E_{\rm rot}^{(\bar{\mathbf{3}})} = \frac{1}{I_1} = \mathcal{O}(1/N_c). \tag{29}$$

Furthermore, this number should be compared with the moment of inertia following from the splitting between *light* baryons, $(\mathbf{10}, \frac{3}{2}^+)$ and $(\mathbf{8}, \frac{1}{2}^+)$, yielding $1/I_1 = 155\,\text{MeV}$, see Section 6. The proximity of the two completely different determinations of the moment of inertia supports the basic idea that it is reasonable to view both light and heavy baryons from the same large-N_c perspective [2].

9 New exotic and stable charmed and bottom baryons?

As noticed in Ref. [5] the hole in the ground-state 0^+ shell (see Fig. 5) is provocative: It can be filled in either by exciting a u, d quark from lower lying shells, or by non-diagonal 'Gamov-Teller' transition from the highest filled s quark shell. In the second case the corresponding charmed baryon resonance will be a pentaquark exotic, see Fig. 6.

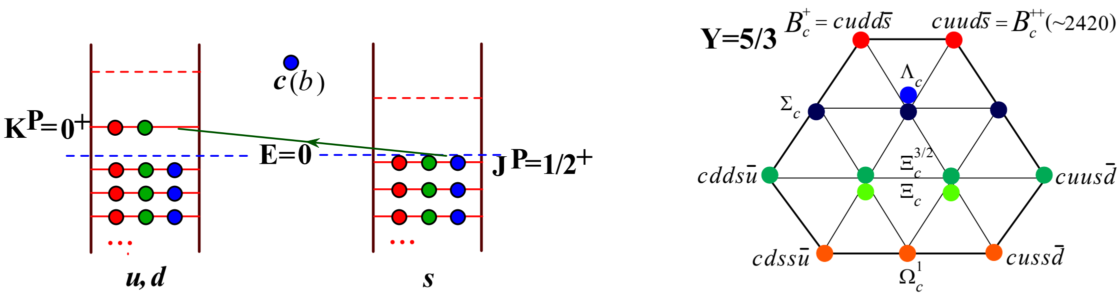

Figure 6: The arrow shows a possible Gamov–Teller excitation (left) leading to charmed 'Beta' pentaquarks $\mathcal{B}_c^{++} = cuud\bar{s}$, $\mathcal{B}_c^{+} = cudd\bar{s}$, in fact belonging to a parity-plus $SU(3)$ multiplet $\overline{\mathbf{15}}$ (right).

The masses of the "Beta baryons", $\mathcal{B}_c^{++} = cuud\bar{s}$, $\mathcal{B}_c^{+} = cudd\bar{s}$, and $\mathcal{B}_b^{+} = buud\bar{s}$, $\mathcal{B}_b^{0} = budd\bar{s}$, depend directly on the position of the highest filled level of s quarks, relative to the ground-state level for u, d quarks. It was argued in Ref. [5] that this energy difference may by quite small, 100-200 MeV, so that the exotic Beta baryons are light, $m(\mathcal{B}_c) \approx 2420\,\text{MeV}$, and hence stable against strong decays! See Ref. [5] for the discussion of the emerging antidecapentaplet of the exotic charmed (or bottom) pentaquarks and of the possibilities of their observation at LHC and b-factories.

10 Conclusions

If the number of colours N_c is treated as a free algebraic parameter, baryon resonances are classified in a simple way. At large N_c all baryon resonances are basically determined by the intrinsic quark spectrum which takes certain limiting shape at $N_c \to \infty$. This spectrum is the same in light baryons $q \ldots qq$ with N_c light quarks q, and in heavy baryons $q \ldots qQ$ with $N_c - 1$ light quarks and one heavy quark Q, since the difference is a $1/N_c$ effect.

One can excite quark levels in various ways called either one-particle or particle-hole excitations; in both cases the excitation energy is $\mathcal{O}(1)$. On top of each one-quark or quark-antiquark

[2] The relation $m(\mathbf{6}) - m(\bar{\mathbf{3}}) = \frac{2}{3}(m(\Delta) - m(N))$ has been first derived in Ref. [19] from the application of the Skyrme model to heavy baryons, an approach being similar in spirit to the present one.

excitation there is generically a band of $SU(3)$ multiplets of baryon resonances, that are rotational states of a baryon as a whole. Therefore, the splitting between multiplets is $\mathcal{O}(1/N_c)$. The rotational band is terminated when the rotational energy reaches $\mathcal{O}(1)$.

In reality N_c is only 3, and the above idealistic hierarchy of scales is somewhat blurred. Nevertheless, an inspection of the spectrum of baryon resonances reveals certain hierarchy schematically summarized as follows:

- Baryon mass: $\mathcal{O}(N_c)$, numerically 1200 MeV, the average mass of the ground-state octet
- One-quark and particle-hole excitations in the intrinsic spectrum: $\mathcal{O}(1)$, typically 400 MeV, for example the excitation of the Roper resonance
- Splitting between the centers of $SU(3)$ multiplets arising as rotational excitations of a given intrinsic state: $\mathcal{O}(1/N_c)$, typically 133 MeV
- Splitting between the centers of rotational multiplets differing by spin, that are degenerate in the leading order: $\mathcal{O}(1/N_c^2)$, typically 44 MeV
- Splitting inside a given multiplet owing to the nonzero strange quark mass: $\mathcal{O}(m_s N_c)$, typically 140 MeV.

In practical terms, we have shown that all baryon resonances up to 2 GeV made of light quarks can be understood as rotational excitations about certain transitions between intrinsic quark levels. The quantum numbers of the resonances and the splittings between multiplets belonging to the same rotational band are dictated by the quantum numbers of the intrinsic quark levels, and appear to be in good accordance with the data. The content and the splitting of the lowest charmed (and bottom) baryon multiplets are also in accordance with their interpretation as a rotational band about the ground-state filling scheme.

In this paper, we have concentrated on the algebraic aspect of the problem leaving aside the dynamical aspects. Dynamical models should answer the question why the intrinsic quark levels for u, d quarks with $K^P = 0^\pm, 1^\pm, 2^\pm$ and the s quark levels with $J^P = \frac{1}{2}^\pm, \frac{3}{2}^\pm$, *etc.*, have the particular energies summarized in Table 1. However, we feel that it is anyway a step forward: Instead of explaining two hundreds resonances one needs now to explain the positions of only a few intrinsic quark levels. Fig. 1 illustrates that approximately the needed intrinsic spectrum can be achieved from a reasonable set of mean fields.

The proposed scheme for understanding baryon resonances has numerous phenomenological consequences that can be investigated even before real dynamics is considered. Namely, the fact that certain groups of $SU(3)$ multiplets belong to the same rotational band related to one and the same one-quark transition implies relations between their couplings, form factors, splittings inside multiplets owing to the nonzero m_s, and so on.

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