The vicinity of the phase transition in the lattice Weinberg - Salam Model

M.A. Zubkov *

* *ITEP, B.Cheremushkinskaya 25, Moscow, 117259, Russia*

Abstract

We investigated the lattice Weinberg - Salam model without fermions for the Higgs mass around 300 GeV. On the phase diagram there exists the vicinity of the phase transition between the physical Higgs phase and the unphysical symmetric phase, where the fluctuations of the scalar field become strong while Nambu monopoles are dense. According to our numerical results (obtained on the lattices of sizes up to $20^3 \times 24$) the maximal value of the ultraviolet cutoff in the model cannot exceed the value around 1.4 TeV.

1 Introduction

Nambu monopoles are not described by means of a perturbation expansion around the trivial vacuum background. Therefore, nonperturbative methods should be used in order to investigate their physics. However, their mass is estimated at the Tev scale. That's why at the energies much less than 1 Tev their effect on physical observables is negligible. However, when energy of the processes approaches 1 Tev we expect these objects influence the dynamics.

The phase diagram of lattice Weinberg - Salam model contains physical Higgs phase, where scalar field is condensed and gauge bosons Z and W acquire their masses. This physical phase is bounded by the phase transition surface. Crossing this surface one leaves the physical phase and enters the phase of the lattice theory that has nothing to do with the conventional continuum Electroweak theory. In the physical phase of the theory the Electroweak symmetry is broken spontaneously while in the unphysical phase the Electroweak symmetry is not broken.

Moving along the line of constant physics in the direction of the increase of the ultraviolet cutoff $\Lambda = \pi/a$ (a is the lattice spacing) we approach the transition surface[1]. We find the indications that there exists the maximal possible ultraviolet cutoff Λ_c within the physical phase. Our estimate (for the Higgs mass $M_H \sim 300$ Gev) is $\Lambda_c \sim 1.4$ TeV. It is important to compare this result with the limitations on the Ultraviolet Cutoff, that come from the perturbation theory. The latter appear as a consequence of the triviality problem, which is related to Landau pole in scalar field self coupling λ. Due to the Landau pole the renormalized λ is zero, and the only way to keep it equal to its measured value is to impose the limitation on the cutoff. That's why the Electroweak theory is usually thought of as a finite cutoff theory. For small Higgs masses (less than about 350 Gev) the correspondent energy scale calculated within the perturbation theory is much larger, than 1 Tev. In particular, for $M_H \sim 300$ Gev this value is about 10^3 TeV.

On the tree level the W-boson (Z- boson) mass in lattice units vanishes on the transition line at small enough λ. This means that the tree level estimate predicts the appearance of an infinite ultraviolet cutoff at the transition point for small λ. At infinite λ the tree level estimate gives nonzero values of lattice M_W, M_Z at the transition point. Our numerical investigation of

*zubkov@itep.ru

[1] The line of constant physics on the phase diagram of lattice model is the line along which physical observables are constant while the ultraviolet cutoff is not.

$SU(2) \otimes U(1)$ model (at $\lambda = 0.009$) and previous calculations in the $SU(2)$ Gauge Higgs model (both at finite λ and at $\lambda = \infty$) show that for the considered lattice sizes renormalized masses do not vanish and the transition is either of the first order or a crossover. (Actually, the situation, when the cutoff tends to infinity at the position of the transition point means that there is a second order phase transition.)

In table 1 of [1] the data on the ultraviolet cutoff achieved in selected lattice studies of the $SU(2)$ Gauge Higgs model are presented. Everywhere β is around $\beta \sim 8$ and the renormalized fine structure constant is around $\alpha \sim 1/110$. This table shows that the maximal value of the cutoff $\Lambda = \frac{\pi}{a}$ ever achieved in these studies is around 1.4 Tev.

Possible explanation of the mentioned discrepancy between lattice results and the results given by the perturbation theory is that in some vicinity of the transition the perturbation theory does not work. Indeed we find that there exists the vicinity of the phase transition between the Higgs phase and the symmetric phase in the Weinberg - Salam model, where the fluctuations of the scalar field become strong and the perturbation expansion around trivial vacuum cannot be applied. As it was mentioned above, the continuum theory is to be approached within the vicinity of the phase transition, i.e. the cutoff is increased along the line of constant physics when one approaches the point of the transition. That's why the conventional prediction on the value of the cutoff admitted in the Standard Model based on the perturbation theory may be incorrect.

The nature of the fluctuational region is illustrated by the behavior of quantum Nambu monopoles [2, 7]. We show that their lattice density increases when the phase transition point is approached. Within the FR these objects are so dense that it is not possible at all to speak of them as of single monopoles. Namely, within this region the average distance between the Nambu monopoles is of the order of their size. Such complicated configurations obviously have nothing to do with the conventional vacuum used in the continuum perturbation theory.

2 The lattice model under investigation

The lattice Weinberg - Salam Model without fermions contains gauge field $\mathcal{U} = (U, \theta)$ (where $U \in SU(2)$, $e^{i\theta} \in U(1)$ are realized as link variables), and the scalar doublet Φ_α, $(\alpha = 1, 2)$ defined on sites.

The action is taken in the form

$$\begin{aligned} S &= \beta \sum_{\text{plaquettes}} ((1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_p) + \frac{1}{\mathrm{tg}^2\theta_W}(1 - \cos\theta_p)) + \\ &\quad -\gamma \sum_{xy} Re(\Phi^+ U_{xy} e^{i\theta_{xy}} \Phi) + \sum_x (|\Phi_x|^2 + \lambda(|\Phi_x|^2 - 1)^2), \end{aligned} \tag{1}$$

where the plaquette variables are defined as $U_p = U_{xy} U_{yz} U^*_{wz} U^*_{xw}$, and $\theta_p = \theta_{xy} + \theta_{yz} - \theta_{wz} - \theta_{xw}$ for the plaquette composed of the vertices x, y, z, w. Here λ is the scalar self coupling, and $\gamma = 2\kappa$, where κ corresponds to the constant used in the investigations of the $SU(2)$ gauge Higgs model. θ_W is the Weinberg angle. Bare fine structure constant α is expressed through β and θ_W as $\alpha = \frac{\mathrm{tg}^2\theta_W}{\pi\beta(1+\mathrm{tg}^2\theta_W)}$. We consider the region of the phase diagram with $\beta \sim 12$ and $\theta_W \sim \pi/6$. Therefore, bare couplings are $\sin^2\theta_W \sim 0.25$; $\alpha \sim \frac{1}{150}$. These values are to be compared with the experimental ones $\sin^2\theta_W(100\mathrm{Gev}) \sim 0.23$; $\alpha(100\mathrm{Gev}) \sim \frac{1}{128}$. All simulations were performed on lattices of sizes $8^3 \times 16$, $12^3 \times 16$, and 16^4. The transition point was checked using the larger lattice ($20^3 \times 24$).

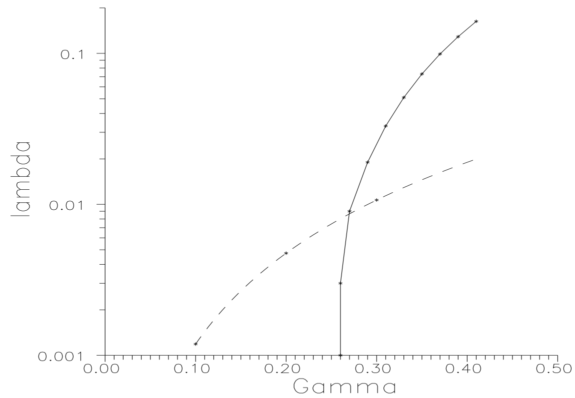

Figure 1: The phase diagram of the model in the (γ, λ)-plane at $\beta = 12$. The dashed line is the tree - level estimate for the line of constant physics correspondent to bare $M_H^0 = 270$ Gev. The continuous line is the line of phase transition between the physical Higgs phase and the unphysical symmetric phase (statistical errors for the values of γ on this line are about 0.005).

3 Nambu monopoles

Nambu monopoles are defined as the endpoints of the Z-string [2]. The Z-string is the classical field configuration that represents the object, which is characterized by the magnetic flux extracted from the Z-boson field. The size of Nambu monopoles was estimated [2] to be of the order of the inverse Higgs mass, while its mass should be of the order of a few TeV. According to [2] Nambu monopoles may appear only in the form of a bound state of a monopole-antimonopole pair. In lattice theory the following variables are considered as creating the Z boson: $Z_{xy} = Z^{\mu}_x \ = -\sin\,[\mathrm{Arg}(\Phi_x^+ U_{xy} e^{i\theta_{xy}} \Phi_y)]$, and: $Z'_{xy} = Z^{\mu}_x \ = -\,[\mathrm{Arg}(\Phi_x^+ U_{xy} e^{i\theta_{xy}} \Phi_y)]$.

The classical solution corresponding to a Z-string should be formed around the 2-dimensional topological defect which is represented by the integer-valued field defined on the dual lattice $\Sigma = \frac{1}{2\pi}{}^*([dZ']_{\mathrm{mod}2\pi} - dZ')$. (Here we used the notations of differential forms on the lattice. For a definition of those notations see, for example, [10].) Therefore, Σ can be treated as the worldsheet of a *quantum* Z-string[7]. Then, the worldlines of quantum Nambu monopoles appear as the boundary of the Z-string worldsheet: $j_Z = \delta\Sigma$.

For historical reasons in lattice simulations we fix unitary gauge $\Phi_2 = 0$; $\Phi_1 \in \mathcal{R}$; $\Phi_1 \geq 0$ (instead of the usual $\Phi_1 = 0$; $\Phi_2 \in \mathcal{R}$), and the lattice Electroweak theory becomes a lattice $U(1)$ gauge theory with the $U(1)$ gauge field $A_{xy} = A^{\mu}_x \ = [Z' + 2\theta_{xy}] \,\mathrm{mod}\, 2\pi$, (The usual lattice Electromagnetic field is related to A as $A_{\mathrm{EM}} = A - Z' + 2\sin^2\theta_W Z'$.) One may try to extract monopole trajectories directly from A. The monopole current is given by $j_A = \frac{1}{2\pi}{}^*d([dA]\mathrm{mod}2\pi)$. Both j_Z, and j_A carry magnetic charges. That's why it is important to find the correspondence between them. In continuum notations we have $A^{\mu} = Z^{\mu} + 2B^{\mu}$, where B is the hypercharge field. Its strength is divergenceless. As a result in continuum theory the net Z flux emanating from the center of the monopole is equal to the net A flux. (Both A and Z are undefined inside the monopole.) This means that in the continuum limit the position of the Nambu monopole must coincide with the position of the monopole extracted from the field A. Therefore, one can consider j_A as another definition of a quantum Nambu monopole [5]. Actually, in our numerical simulations we use this definition.

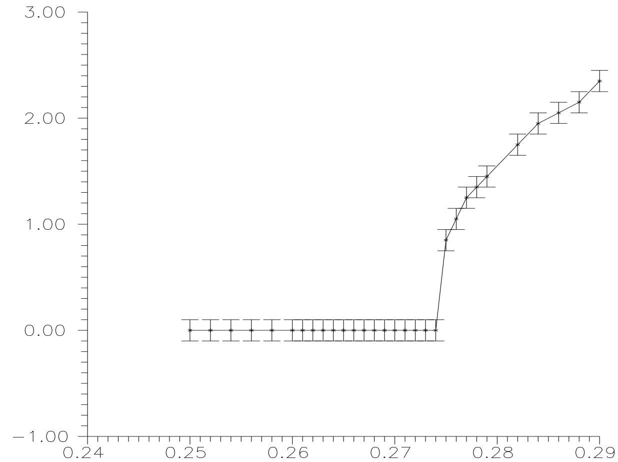

Figure 2: ϕ_m as a function of γ at $\lambda = 0.009$ and $\beta = 12$.

4 Phase diagram

In the three - dimensional (β, γ, λ) phase diagram the transition surfaces are two - dimensional. The lines of constant physics on the tree level are the lines ($\frac{\lambda}{\gamma^2} = \frac{1}{8\beta}\frac{M^2_H}{M^2_W} = \text{const}$; $\beta = \frac{1}{4\pi\alpha} = \text{const}$). We suppose that in the vicinity of the transition the deviation of the lines of constant physics from the tree level estimate may be significant. However, qualitatively their behavior is the same. Namely, the cutoff is increased along the line of constant physics when γ is decreased and the maximal value of the cutoff is achieved at the transition point. Nambu monopole density in lattice units is also increased when the ultraviolet cutoff is increased.

At $\beta = 12$ (corresponds to bare $\alpha \sim 1/150$) the phase diagram is represented on Fig. 1. The physical Higgs phase is situated right to the transition line. The position of the transition is localized at the point where the susceptibility extracted from the Higgs field creation operator achieves its maximum. We use the susceptibility $\chi = \langle H^2 \rangle - \langle H \rangle^2$ extracted from $H = \sum_y Z^2_{xy}$ (see, for example, Fig. 4). We observe no difference between the values of the susceptibility calculated using the lattices of the sizes $8^3 \times 16$, $12^3 \times 16$, and 16^4. This indicates that the transition may be a crossover.

It is worth mentioning that the value of the renormalized Higgs boson mass does not deviate significantly from its bare value. For example, for λ around 0.009 and γ in the vicinity of the phase transition bare value of the Higgs mass is around 270 Gev while the observed renormalized value is 300 ± 70 Gev.

5 Effective constraint potential

We have calculated the constraint effective potential for $|\Phi|$ using the histogram method. The calculations have been performed on the lattice $8^3 \times 16$. The probability $h(\phi)$ to find the value of $|\Phi|$ within the interval $[\phi - 0.05; \phi + 0.05)$ has been calculated for $\phi = 0.05 + N * 0.1$, $N = 0, 1, 2, ...$ This probability is related to the effective potential as $h(\phi) = \phi^3 e^{-V(\phi)}$. That's why we extract the potential from $h(\phi)$ as

$$V(\phi) = -\log h(\phi) + 3 \log \phi \tag{2}$$

Next, we introduce the useful quantity $H = V(0) - V(\phi_m)$, which is called the potential barrier hight (here ϕ_m is the point, where V achieves its minimum).

On Fig. 2 we represent the values of ϕ_m for $\lambda = 0.009$, $\beta = 12$. On Fig. 3 we represent the values of H for $\lambda = 0.009$, $\beta = 12$. One can see that the values of ϕ_m and H increase

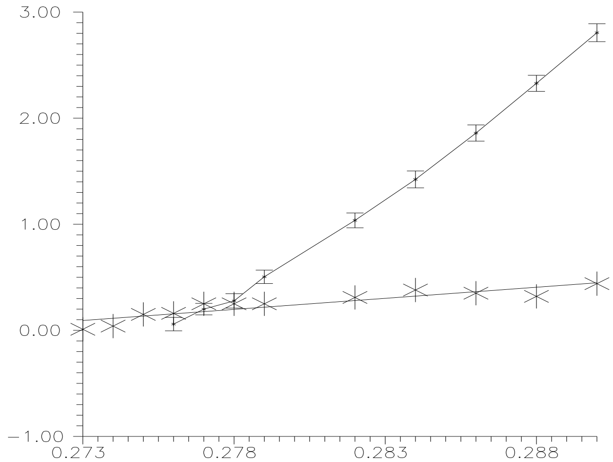

Figure 3: H (points) vs. H_{fluct} (stars) as a function of γ at $\lambda = 0.009$ and $\beta = 12$.

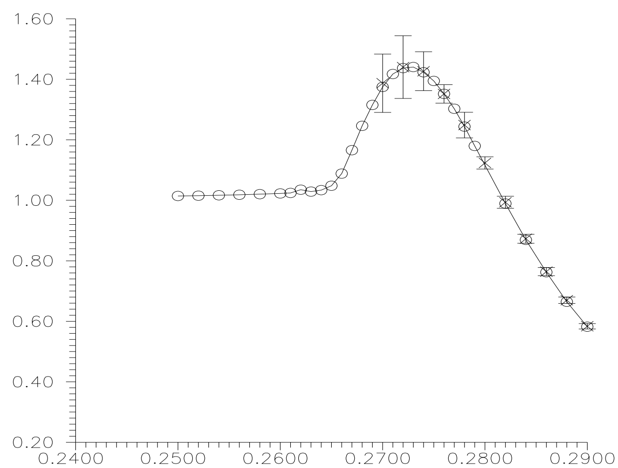

Figure 4: Susceptibility $\langle H^2\rangle - \langle H\rangle^2$ (for $H_x = \sum_y Z^2_{xy}$) as a function of γ at $\lambda = 0.009$ and $\beta = 12$. Circles correspond to the lattice $8^3 \times 16$. Crosses correspond to the lattice $12^3 \times 16$.

when γ is increased. At $\gamma = 0.274$, $\lambda = 0.009$ the minimum of the potential is at $\phi = 0$. This point corresponds to the maximum of the susceptibility constructed of the Higgs field creation operator (see Fig. 4). At $\gamma = 0.275$, $\lambda = 0.009$ minimum of the potential is observed at nonzero ϕ_m. That's why we localize the position of the transition point at $\gamma = 0.273 \pm 0.002$.

It is important to understand which value of barrier hight can be considered as small and which value can be considered as large. Our suggestion is to compare $H = V(0) - V(\phi_m)$ with $H_{\rm fluct} = V(\phi_m + \delta\phi) - V(\phi_m)$, where $\delta\phi$ is the fluctuation of $|\Phi|$. From Fig. 3 it is clear that there exists the value of γ (we denote it γ_{c2}) such that at $\gamma_c < \gamma < \gamma_{c2}$ the barrier hight H is of the order of $H_{\rm fluct}$ while for $\gamma_{c2} << \gamma$ the barrier hight is essentially larger than $H_{\rm fluct}$. The rough estimate for this pseudocritical value is $\gamma_{c2} \sim 0.278$. We estimate the fluctuations of $|\Phi|$ to be around $\delta\phi \sim 0.6$ for all considered values of γ at $\lambda = 0.009$, $\beta = 12$. It follows from our data that $\phi_m, \langle|\phi|\rangle >> \delta\phi$ at $\gamma_{c2} << \gamma$ while $\phi_m \sim \delta\phi$ at $\gamma_{c2} > \gamma$. Basing on these observations we expect that in the region $\gamma_{c2} << \gamma$ the usual perturbation expansion around trivial vacuum of spontaneously broken theory can be applied to the lattice Weinberg - Salam model while in the FR $\gamma_c < \gamma < \gamma_{c2}$ it cannot be applied. At the value of γ equal to γ_{c2} the calculated value of the cutoff is 1.0 ± 0.1 Tev.

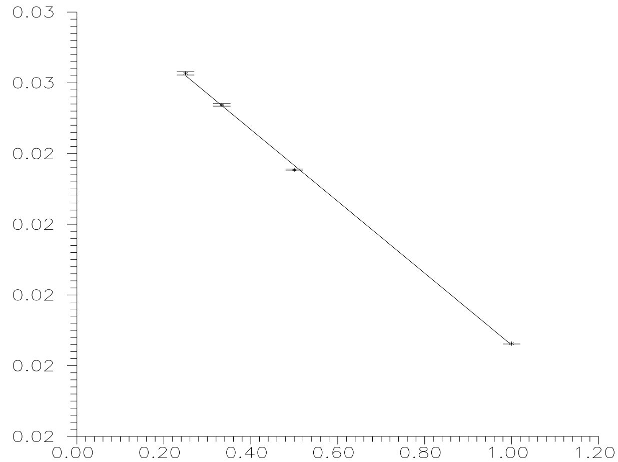

Figure 5: The potential for the right - handed leptons vs. $1/R$ at $\gamma = 0.277$, $\lambda = 0.009$, and $\beta = 12$ (lattice $8^3 \times 16$).

6 The renormalized coupling

In order to calculate the renormalized fine structure constant $\alpha_R = e^2/4\pi$ (where e is the electric charge) we use the potential for infinitely heavy external fermions. We consider Wilson loops for the right-handed external leptons:

$$\mathcal{W}^{\rm R}_{\rm lept}(l) = \langle {\rm Re}\, \Pi_{(xy)\in l} e^{2i\theta_{xy}} \rangle. \tag{3}$$

Here l denotes a closed contour on the lattice. We consider the following quantity constructed from the rectangular Wilson loop of size $r \times t$:

$$\mathcal{V}(r) = \log \lim_{t\to\infty} \frac{\mathcal{W}(r\times t)}{\mathcal{W}(r\times (t+1))}. \tag{4}$$

Due to exchange by virtual photons at large enough distances we expect the appearance of the Coulomb interaction

$$\mathcal{V}(r) = -\frac{\alpha_R}{r} + const. \tag{5}$$

On Fig. 5 we represent as an example the dependence of the potential on $1/R$ for $\gamma = 0.277$, $\lambda = 0.009$, and $\beta = 12$. In the vicinity of the transition the fit (5) gives values of renormalized fine structure constant around 1/100. The calculated values are to be compared with bare constant $\alpha_0 = 1/(4\pi\beta) \sim 1/150$ at $\beta = 12$. However, for $\gamma >> \gamma_{c2}$ the tree level estimate is approached. This is in correspondence with our supposition that the perturbation theory cannot be valid within the FR while it works well far from the FR.

7 Masses and the lattice spacing

From the very beginning we fix the unitary gauge $\Phi_1 = const.$, $\Phi_2 = 0$. The following variables are considered as creating a Z boson and a W boson, respectively:

$$\begin{aligned} Z_{xy} &= Z^{\mu}_{x} = -{\rm sin}\,[{\rm Arg} U^{11}_{xy} + \theta_{xy}], \\ W_{xy} &= W^{\mu}_{x} = U^{12}_{xy} e^{-i\theta_{xy}}. \end{aligned} \tag{6}$$

Here, μ represents the direction (xy).

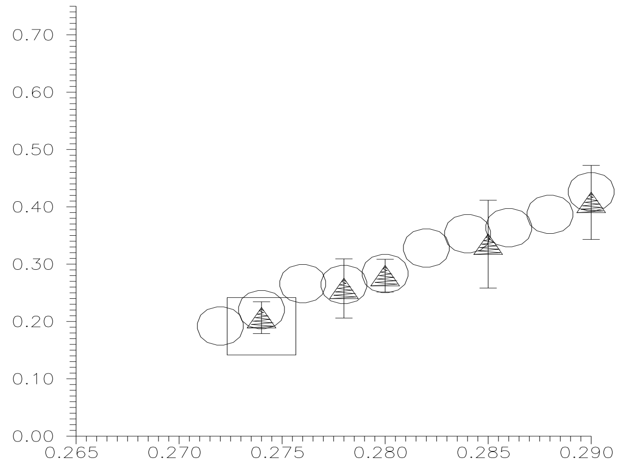

Figure 6: Z - boson mass in lattice units at $\lambda = 0.009$ and $\beta = 12$ as a function of γ. Circles correspond to lattice $12^3 \times 16$. Triangles correspond to lattice 16^4. Squares correspond to lattice $20^3 \times 24$ (the error bars are about of the same size as the symbols used).

After fixing the unitary gauge the electromagnetic $U(1)$ symmetry remains:

$$\begin{aligned} U_{xy} &\rightarrow g^{\dagger}_x U_{xy} g_y, \\ \theta_{xy} &\rightarrow \theta_{xy} - \alpha_y/2 + \alpha_x/2, \end{aligned} \tag{7}$$

where $g_x = \mathrm{diag}(e^{i\alpha_x/2}, e^{-i\alpha_x/2})$. There exists a $U(1)$ lattice gauge field, which is defined as

$$A_{xy} = A^{\mu}_x = [-\mathrm{Arg}U^{11}_{xy} + \theta_{xy}] \bmod 2\pi \tag{8}$$

that transforms as $A_{xy} \rightarrow A_{xy} - \alpha_y + \alpha_x$. The field W transforms as $W_{xy} \rightarrow W_{xy} e^{-i\alpha_x}$.

In order to evaluate the masses of the Z-boson and the Higgs boson we use the correlators:

$$\frac{1}{N^6} \sum_{\bar{x},\bar{y}} \langle \sum_{\mu} Z^{\mu}_x Z^{\mu}_y \rangle \sim e^{-M_Z|x_0-y_0|} + e^{-M_Z(L-|x_0-y_0|)} \tag{9}$$

and

$$\frac{1}{N^6} \sum_{\bar{x},\bar{y}} (\langle H_x H_y \rangle - \langle H \rangle^2) \sim e^{-M_H|x_0-y_0|} + e^{-M_H(L-|x_0-y_0|)}, \tag{10}$$

Here the summation $\sum_{\bar{x},\bar{y}}$ is over the three "space" components of the four - vectors x and y while x_0, y_0 denote their "time" components. N is the lattice length in "space" direction. L is the lattice length in the "time" direction. In lattice calculations we used two different operators that create Higgs bosons: $H_x = |\Phi|$ and $H_x = \sum_y Z^2_{xy}$. In both cases H_x is defined at the site x, the sum $\sum_y$ is over its neighboring sites y.

The physical scale is given in our lattice theory by the value of the Z-boson mass $M^{phys}_Z \sim 91$ GeV. Therefore the lattice spacing is evaluated to be $a \sim [91\mathrm{GeV}]^{-1} M_Z$, where M_Z is the Z boson mass in lattice units. It has been found that the W - boson mass contains an artificial dependence on the lattice size. We suppose, that this dependence is due to the photon cloud surrounding the W - boson. The energy of this cloud is related to the renormalization of the fine structure constant. Therefore the Z - boson mass was used in order to fix the scale.

Our data show that $\Lambda = \frac{\pi}{a} = (\pi \times 91 \text{ GeV})/M_Z$ is increased slowly with the decrease of γ at any fixed λ. We investigated carefully the vicinity of the transition point at fixed $\lambda = 0.009$ and $\beta = 12$. It has been found that at the transition point the value of Λ is equal to 1.4 ± 0.2 Tev. The check of the dependence on the lattice size ($8^3 \times 16$, $12^3 \times 16$, 16^4, $20^3 \times 24$) does not show an essential increase of this value (see Fig. 6).

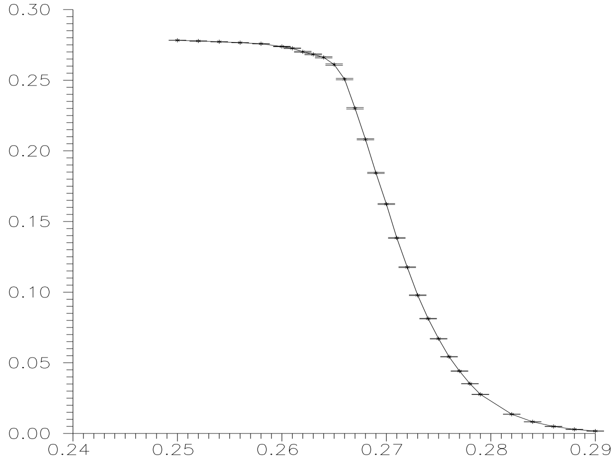

Figure 7: Nambu monopole density as a function of γ at $\lambda = 0.009$ and $\beta = 12$.

In the Higgs channel the situation is difficult. First, due to the lack of statistics we cannot estimate the masses in this channel using the correlators (10) at all considered values of γ. At the present moment at $\lambda = 0.009$ we can represent the data at the two points on the lattice $8^3 \times 16$: ($\gamma = 0.274$, $\lambda = 0.009$, $\beta = 12$) and ($\gamma = 0.290$, $\lambda = 0.009$, $\beta = 12$). The first point roughly corresponds to the position of the transition while the second point is situated deep within the Higgs phase. At the point ($\gamma = 0.274$, $\lambda = 0.009$, $\beta = 12$) we have collected enough statistics to calculate correlator (10) up to the "time" separation $|x_0 - y_0| = 4$. The value $\gamma = 0.274$ corresponds roughly to the position of the phase transition. The mass found in this channel in lattice units is $M_H^L = 0.75 \pm 0.1$ while bare value of M_H is $M_H^0 \sim 270$ Gev. At the same time $M_Z^L = 0.23 \pm 0.007$. Thus we estimate at this point $M_H = 300 \pm 40$ Gev. At the point ($\gamma = 0.29$, $\lambda = 0.009$, $\beta = 12$) we calculate the correlator with reasonable accuracy up to $|x_0 - y_0| = 3$. At this point bare value of M_H is $M_H^0 \sim 260$ Gev while the renormalized Higgs mass in lattice units is $M_H^L = 1.2 \pm 0.3$. At the same time $M_Z^L = 0.41 \pm 0.01$. Thus we estimate at this point $M_H = 265 \pm 70$ Gev.

8 Nambu monopole density

The monopole density is defined as $\rho = \left\langle \frac{\sum_{\text{links}} |j_{\text{link}}|}{4V^L} \right\rangle$, where V^L is the lattice volume. On Fig 7 we represent Nambu monopole density as a function of γ at $\lambda = 0.009$, $\beta = 12$. The value of monopole density at γ_c is around 0.1. At this point the value of the cutoff is $\Lambda \sim 1.4 \pm 0.2$ Tev.

According to the classical picture the Nambu monopole size is of the order of M_H^{-1}. Therefore, for example, for $a^{-1} \sim 430$ Gev and $M_H \sim 300, 150, 100$ Gev the expected size of the monopole is about a lattice spacing. The monopole density around 0.1 means that among 10 sites there exist 4 sites that are occupied by the monopole. Average distance between the two monopoles is, therefore, less than 1 lattice spacing and it is not possible at all to speak of the given configurations as of representing the physical Nambu monopole. At $\gamma = \gamma_{c2}$ the Nambu monopole density is around 0.03. This means that among 7 sites there exists one site that is occupied by the monopole. Average distance between the two monopoles is, therefore, approximately 2 lattice spacings or $\sim \frac{1}{160\,\text{Gev}}$. Thus, the Nambu monopole density in physical units is around $[160\,\text{GeV}]^3$. We see that at this value of γ the average distance between Nambu monopoles is of the order of their size.

We summarize the above observations as follows. Within the fluctuational region the configurations under consideration do not represent single Nambu monopoles. Instead these configurations can be considered as the collection of monopole - like objects that is so dense that the average distance between the objects is of the order of their size. On the other hand, at $\gamma >> \gamma_{c2}$ the considered configurations do represent single Nambu monopoles and the average distance between them is much larger than their size. In other words out of the FR vacuum can be treated as a gas of Nambu monopoles while within the FR vacuum can be treated as a liquid composed of monopole - like objects.

It is worth mentioning that somewhere inside the Z string connecting the classical Nambu monopoles the Higgs field is zero: $|\Phi| = 0$. This means that the Z string with the Nambu monopoles at its ends can be considered as an embryo of the symmetric phase within the Higgs phase. We observe that the density of these embryos is increased when the phase transition is approached. Within the fluctuational region the two phases are mixed, which is related to the large value of Nambu monopole density. That's why we come to the conclusion that vacuum of lattice Weinberg - Salam model within the FR has nothing to do with the continuum perturbation theory. This means that the usual perturbation expansion around trivial vacuum (gauge field equal to zero, the scalar field equal to $(\phi_m, 0)^T$) cannot be valid within the FR. This might explain why we do not observe in our numerical simulations the large values of Λ predicted by the conventional perturbation theory.

9 Conclusions

The continuum physics is to be approached in the vicinity of the phase transition between the physical Higgs phase and the unphysical symmetric phase of the model. The ultraviolet cutoff is increased when the transition point is approached along the line of constant physics. There exists the so - called fluctuational region (FR) on the phase diagram of the lattice Weinberg - Salam model. This region is situated in the vicinity of the phase transition. We calculate the effective constraint potential $V(\phi)$ for the Higgs field. It has a minimum at the nonzero value ϕ_m in the physical Higgs phase. Within the FR the fluctuations of the scalar field become of the order of ϕ_m. Moreover, the "barrier hight" $H = V(0) - V(\phi_m)$ is of the order of $V(\phi_m + \delta\phi) - V(\phi_m)$, where $\delta\phi$ is the fluctuation of $|\Phi|$.

The scalar field must be equal to zero somewhere within the classical Nambu monopole. That's why this object can be considered as an embryo of the unphysical symmetric phase within the physical Higgs phase of the model. We investigate properties of the quantum Nambu monopoles. Within the FR they are so dense that the average distance between them becomes of the order of their size. This means that the two phases are mixed within the FR. All these results show that the vacuum of lattice Weinberg - Salam model in the FR is essentially different from the trivial vacuum used in the conventional perturbation theory. As a result the use of the perturbation theory in this region is limited.

Our numerical results show that at M_H around 300 GeV the maximal value of the cutoff admitted out of the FR for the considered lattice sizes cannot exceed the value around 1.0 ± 0.1 Tev. Within the FR the larger values of the cutoff can be achieved. The absolute maximum for the value of the cutoff within the Higgs phase of the lattice model is achieved at the point of the phase transition. Our estimate for this value is 1.4 ± 0.2 Tev for the lattice sizes up to $20^3 \times 24$.

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References

[1] B.L.G. Bakker, A.I. Veselov, M.A. Zubkov. J.Phys.G36:075008,2009.

[2] Y. Nambu, Nucl.Phys. B **130**, 505 (1977);
Ana Achucarro and Tanmay Vachaspati, Phys. Rept. **327**, 347 (2000); Phys. Rept. **327**, 427 (2000).

[3] B.L.G. Bakker, A.I. Veselov, M.A. Zubkov. Phys.Lett.B620:156-163,2005.

[4] B.L.G. Bakker, A.I. Veselov, M.A. Zubkov. PoS LAT2007:337,2007,. [arXiv:0708.2864]

[5] A.I. Veselov, M.A. Zubkov, JHEP 0812:109,2008.

[6] M.A. Zubkov. Phys.Lett.B684:141-146,2010.

[7] M.N. Chernodub, JETP Lett. **66**, 605 (1997)

[8] I. Montvay, Nucl. Phys. B **269**, 170 (1986).

[9] W.Langguth, I.Montvay, P.Weisz Nucl.Phys.B277:11,1986

[10] M.I. Polikarpov, U.J. Wiese, and M.A. Zubkov, Phys. Lett. B **309**, 133 (1993).

[11] M.N. Chernodub, Phys.Rev.Lett. 95 (2005) 252002