Scalar Leptoquark Contributions into $l_i \to l_j\gamma$ Processes

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Abstract

The contributions of scalar leptoquarks in lepton flavor violating (LFV) processes of type $l_i \to l_j\gamma$ are investigated in frame of the minimal model with the four color symmetry and Higgs mechanism of quark and lepton mass generation. It is shown that experimental data on the decays $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$ allow the existence of light scalar leptoquarks of type under consideration, with masses of order 1 TeV or below.

One of the possible variant of new physics beyond the SM can be the variant induced by the four color symmetry between quarks and leptons of Pati-Salam type [1]. The minimal realization this symmetry MQLS model [2] is predicted the existence doublets scalar leptoquarks, which appear to be some kind of partner of the standard Higgs doublet. Current limits on masses from direct search of scalar leptoquarks are small $M_{LQ} \sim 200 - 300$ GeV [3], Indirect limits from $K_L^0 \to \mu^\pm e^\mp$ [4], S, T, U parameters [5], $g-2$ [6] and others are close to direct limits. Other source limit on the masses scalar leptoquark can be LFV processes. Exist strong experimental limits on LFV processes

$$Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11} \qquad [7],$$
$$Br(\tau \to \mu\gamma) < 4.5 \cdot 10^{-8} \qquad [8],$$
$$Br(\tau \to e\gamma) < 3.3 \cdot 10^{-8} \qquad [9].$$

The topic of my talk is investigated the contributions new physics into processes with lepton flavor violation in framework the minimal four color symmetry model.

MQLS model is based on the group

$$G = SU_V(4) \times SU_L(2) \times U_R(1).$$

In the MQLS model the basic left- (L) and right- (R) handed quarks $Q'^{L,R}_{ia\alpha}$ and leptons $l'^{L,R}_{ia}$ form the fundamental quartets of $SU(4)$ color group, and can be written, in general, as superpositions of the quark and lepton mass eigenstates $Q^{L,R}_{ia\alpha}$ and $l^{L,R}_{ia}$

$$\psi^{L,R}_{ia\alpha} = Q'^{L,R}_{ia\alpha} = \sum_j \left(A^{L,R}_{Q_a}\right)_{ij} Q^{L,R}_{ja\alpha},\ \psi^{L,R}_{ia4} = l'^{L,R}_{ia} = \sum_j \left(A^{L,R}_{l_a}\right)_{ij} l^{L,R}_{ja},$$

where i=1,2,3 are the generation indices $a = 1, 2$ are the $SU_L(2)$ indices and $A = \alpha, 4$- $SU_V(4)$ indices $\alpha = 1, 2, 3$ are the $SU_c(3)$ color indices. The unitary matrices $A^{L,R}_{Q_a}$ and $A^{L,R}_{l_a}$ describe the fermion mixing and diagonalize the mass matrices of quarks and leptons. This matrices combined $C_Q = (A^L_{Q_1})^+ A^L_{Q_2}$ the Cabibbo-Kobayashi-Maskawa matrix, which is know to be due

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to the distinction between the mixing matrices $A^L_{Q_1}$ and $A^L_{Q_2}$ in(1) for up and down left-handed quarks, $C_l = (A^L_{l_1})^+ A^L_{l_2}$ the matrix that is analog its in the lepton sector and is not diagonal, this evident from neutrino oscillation, which is due to the possible distinction between the mixing matrices $A^L_{l_1}$ and $A^L_{l_2}$ and $K^{L,R}_a = (A^{L,R}_{Q_a})^+ A^{L,R}_{l_a}$ unitary matrices additional fermion mixing in model. (that are due to the possible distinctions between the quarks and leptons mixing matrices $A^{L,R}_{Q_a}$ and $A^{L,R}_{l_a}$).

In gauge sector model predicted of two vector leptoquarks $V^{\pm}_{\alpha\mu}(\alpha = 1, 2, 3)$ and of additional neutral Z' boson.

The scalar sector of the model, contains four multiplets, which transformed according to $(4, 1, 1)$, $(1, 2, 1)$, $(15, 2, 1)$, $(15, 1, 0)$ representations with respectively η_1, η_2, η_3, η_4 – VEV.

The MQLS model is based on the Higgs mechanism of splitting of the quarks and leptons masses and predicted in addition to vector leptoquarks, the existence of the doublets of scalar leptoquarks. In this approach, the SM Higgs doublet $\Phi^{(SM)}$ appearing to be superposition of the doublets $\Phi^{(2)}_a$ and $\Phi^{(3)}_{15,a}$ representation (1.2.1) and (15.2.1) with VEV η_2 and η_3, corresponding. and colorless doublets $\Phi^{(3)}_{15}$, that mixing with the doublet $\Phi^{(2)}_a$ from the representation (1,2,1) give Higgs doublets SM and additional doublets Φ'. Here $\eta = \sqrt{\eta_2^2 + \eta_3^2}$ is the SM VEV.

In particular, the representation (15,2,1) is kept two doublets SLQs

$$(15.2.1) \begin{pmatrix} S^{(+)}_{1\alpha} \\ S^{(+)}_{2\alpha} \end{pmatrix}; \begin{pmatrix} S^{(-)}_{1\alpha} \\ S^{(-)}_{2\alpha} \end{pmatrix},$$

with electric charges of the component scalar doublets

$$Q_{em} \quad \begin{pmatrix} 5/3 \\ 2/3 \end{pmatrix}; \begin{pmatrix} 1/3 \\ -2/3 \end{pmatrix}.$$

The scalar leptoquarks with electric charge 2/3 are superpositions three physical scalar leptoquarks S_1, S_2, S_3 and Goldstone mode S_0,

$$S^{(+)}_2 = \sum_m C^{(+)}_m S_m, \ \overset{*}{S}{}^{(-)}_2 = \sum_m C^{(-)}_m S_m.$$

where $C^{(\pm)}_m$ are the elements of the complex unitary matrices the mixing of scalar leptoquarks of electric charge 2/3.

In the unitary gauge the physical leptoquarks fields are as follows: two of up component doublets leptoquark $S^{(+)}_1$ and $S^{(-)}_1$ of electric charge 5/3 and -1/3, respectively, and three scalar leptoquarks $S_m(m = 1, 2, 3)$ of electric charge 2/3.

The lagrangian interaction scalar leptoquarks with quarks and charge leptons can be written in the following form [10],

$$\begin{aligned} L_{ulS^{(+)}_1} &= \bar{u}_{i\alpha}\Big[(h^L_+)_{ij} P_L + (h^R_+)_{ij} P_R\Big] l_j S^{(+)}_{1\alpha} + \text{h.c.}, \\ L_{dlS_m} &= \bar{d}_{i\alpha}\Big[(h^L_{2m})_{ij} P_L + (h^R_{2m})_{ij} P_R\Big] l_j S_{m\alpha} + \text{h.c.} \end{aligned}$$

here, u, d, l, are, relatively, up- and down- quarks, charge leptons of the generation i, $P_{L,R}$ are the left and right projection operators, $h^{L,R}$ are the coupling constant matrices for generations.

The expression for coupling constant because of Higgs origin are proportional to the ratios of fermion masses to the SM VEV. This ratios are quite small for first and second generation fermion $m_u/\eta, m_d/\eta, m_s/\eta \sim 10^{-5}$ and $m_c/\eta, m_b/\eta \sim 10^{-2}$, but the ratio of the t-quark is not small $m_t/\eta \sim 0.7$. Therefore general contributions into couplings constants give ratio of the t-quark mass to SM VEV. We neglected in coupling constant all fermionic masses, except mass t-quark.

The dominant contribution in the coupling constant can be written as

$$
\begin{aligned}
(h_+^L)_{3j} &= \frac{\sqrt{6}\, m_t}{2\eta \sin\beta}\, (K_1^L C_l)_{3j} \\
(h_+^R)_{3j} &= -\frac{\sqrt{6}\, m_b}{2\eta \sin\beta}\, (C_Q)_{33} (K_2^R)_{3j} \\
(h_{2m}^{L,R})_{3j} &= -\frac{\sqrt{6}\, m_b}{2\eta \sin\beta} (K_2^{L,R})_{3j}\, c_m^{(\mp)}.
\end{aligned}
$$

Take notice, that SLQ of MQLS model are like LQ of 3-d generation [11].

The contribution of scalar leptoquarks into the $l_i \to l_j \gamma$ Processes: is described by the two diagrams in figure 1. Contributions of S_m and b-quark in the amplitude $l_i \to l_j \gamma$ suppress by m_b^2/m_t^2 comparing with ones of $S_1^{(+)}$ and t-quark.

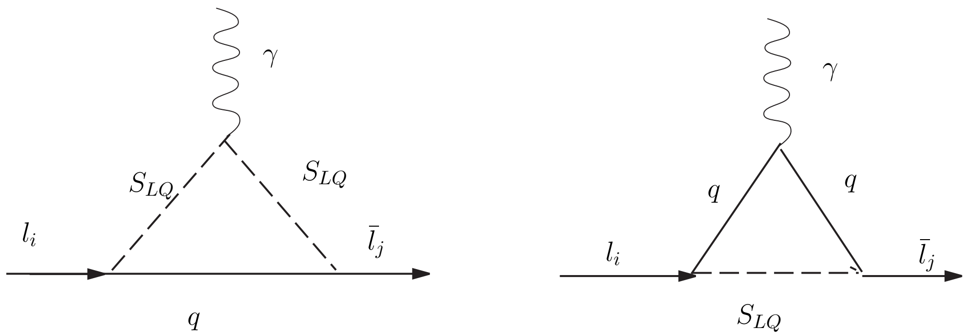

Figure 1: Diagrams representing the contribution of scalar leptoquarks (SLQ) into the $l_i \to l_j \gamma$ Processes: $q = u_i(d_i)$ is the up(down) quark of the ith generation and $S_{LQ} = S_1^{(+)}(S_m)$ is the SLQ corresponding to the above quarks.

In general, the total one-loop contribution from this diagrams can be written as

$$
\begin{aligned}
M &= -\frac{|e|}{64\pi^2 m_{LQ}^2} \bar{l}_j \sigma^{\mu\nu} q^\nu \Bigg[m_i \Big(Q_k F_4(x) - Q_s F_2(x) \Big) \Big((|h^L|^2)_{ji} P_R + (|h^R|^2)_{ji} P_L \Big) \\
&+ 2m_k \Big(Q_k F_3(x) - Q_s F_3(x) \Big) \Big((h^{+L} h^R)_{ji} P_R + (h^{+R} h^L)_{ji} P_L \Big) \Bigg] l_i \epsilon^\mu,
\end{aligned}
$$

where Q_k, Q_S are electric charge of q_k-quark and LQ, and $x = m_k^2/m_{LQ}^2$,

$$
\begin{aligned}
F_2(x) &= \frac{1}{6(1-x)^4}(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x), \\
F_3(x) &= \frac{1}{(1-x)^3}(1 - x^2 + 2x \ln x), \\
F_5(x) &= \frac{1}{6(1-x)^4}(2 + 3x - 6x^2 + x^3 - 6x \ln x), \\
F_6(x) &= \frac{1}{(1-x)^3}(-3 + 4x - x^2 - 2 \ln x).
\end{aligned}
$$

As can be seen from amplitude the first term is proportional to the initial lepton mass, (whereas the second term is proportional to the quark mass.)

The probability this processes can be written as

$$
\begin{aligned}
W(l_i \to l_j\gamma) &= \frac{9\alpha m_i}{256(4\pi)^4}(\frac{m_i}{\eta})^4 x^2 \Bigg(B_1^2(x) k_{ij}^{(1)} + 4(\frac{m_b}{m_i})^2 B_2^2(x) k_{ij}^{(2)} \\
&- 2\frac{m_b}{m_i} B_1(x) B_2(x) Re(k_{ij}^{(12)}) \Bigg), \\
B_1(x) &= Q_k F_4(x) - Q_S F_2(x), \qquad B_2(x) = Q_k F_3(x) - Q_S F_1(x),
\end{aligned}
$$

here we proposes that there are t-quark in loop, and k_{ij} is the matrices of mixing parameter in the model

$$
\begin{aligned}
k_{ij}^{(1)} &= \frac{|(K_1^L C_l)_{3j}|^2 |(K_1^L C_l)_{3i}|^2}{\sin^4\beta}, \\
k_{ij}^{(2)} &= \frac{1}{\sin^4\beta}\Bigg(|(K_1^L C_l)_{3j}|^2 |(K_2^R)_{3i}|^2 + (i \leftrightarrow j) \Bigg), \\
k_{ij}^{(12)} &= \frac{1}{\sin^4\beta}\Bigg((|(K_1^L C_l)_{3j}|^2 + |(K_2^R)_{3j}|^2) \times \\
&\times \ ((K_1^L C_l)_{3i}^* (K_2^R)_{3i} + (K_2^R)_{3i}^* (K_1^L C_l)_{3i}) + (i \leftrightarrow j) \Bigg).
\end{aligned}
$$

In probability decay second term are proportional ratios of b-quark mass to the initial lepton mass (therefore this term give large contribution in numerical calculations).

Substituting the numerical values into eqnarray we obtain following expressions

$$
\begin{aligned}
Br(\mu \to e\gamma) &= 1.1 \times 10^{-4} x^2 \Bigg(B_1^2(x) k_{\mu e}^{(1)} + \\
&+ 7056 B_2^2(x) k_{\mu e}^{(2)} - 84 B_1(x) B_2(x) Re(k_{\mu e}^{(12)}) \Bigg), \qquad (1) \\
Br(\tau \to e\gamma) &= 2.2 \times 10^{-5} x^2 \Bigg(B_1^2(x) k_{\tau e}^{(1)} + \\
&+ 20 B_2^2(x) k_{\tau e}^{(2)} - 4.8 B_1(x) B_2(x) Re(k_{\tau e}^{(12)}) \Bigg), \\
Br(\tau \to \mu\gamma) &= 2.2 \times 10^{-5} x^2 \Bigg(B_1^2(x) k_{\tau \mu}^{(1)} + \\
&+ 20 B_2^2(x) k_{\tau \mu}^{(2)} - 4.8 B_1(x) B_2(x) Re(k_{\tau \mu}^{(12)}) \Bigg).
\end{aligned}
$$

By virtue of the fact that experimental restrictions on the $\tau-$ decays are weaker than on the $\mu-$ decay, as results of the corresponding limitation on the masses SLQ are small. Thus we examine in the beginning $\mu-$ decay.

I) The General Contribution: we retain only the second term in eqnarray (1) in our numerical calculations, then this expression can be written as

$$
\begin{aligned}
Br(\mu \to e\gamma) &= 0.7 x^2 B_2^2(x) k_{\mu e}^{(2)}, \\
k_{\mu e}^{(2)} &= \frac{1}{\sin^4\beta}\Bigg(|(K_1^L C_l)_{3e}|^2 |(K_2^R)_{3\mu}|^2 + (\mu \leftrightarrow e) \Bigg).
\end{aligned}
$$

The expression are simple for analysis. The lower limit on masses scalar leptoquark $S_1^{(+)}$ from decay $\mu \to e\gamma$ for different value parameter $k_{\mu e}^{(2)}$ shown on picture 2. As show mass scalar leptoquark $S_1^{(+)}$ can be below 1 TeV if matrix element $(K_2^R)_{13}, (K_1^L C_l)_{13} \sim 10^{-3}$ and $(K_2^R)_{23}, (K_1^L C_l)_{23} \sim 10^{-2}$.

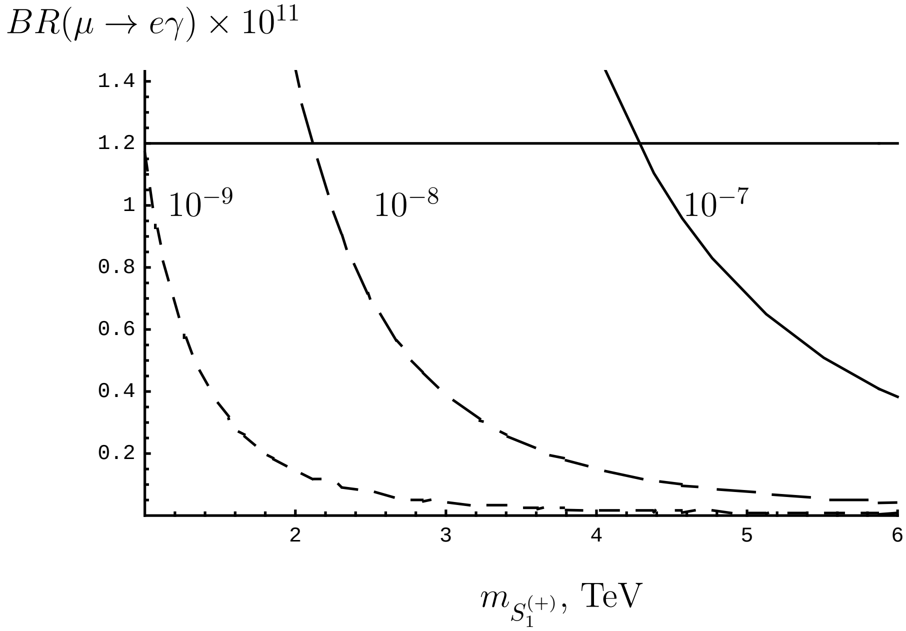

Figure 2: The lower limit on mass scalar leptoquark $S_1^{(+)}$ resulting from $BR(\mu \to e\gamma)$ with value of parameter $k_{\mu e}^{(2)} = 10^{-9}, 10^{-8}, 10^{-7}$ (Horizontal line corresponds experimental limit $Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$)

II) In particular case, matrix $K_2^R = I$, we retain only the first term in eqnarray (1):

$$Br(\mu \to e\gamma) = 1.1 \times 10^{-4} x^2 B_1^2(x) k_{\mu e}^{(1)},$$

$$k_{\mu e}^{(1)} = \frac{|(K_1^L C_l)_{3e}|^2 |(K_1^L C_l)_{3\mu}|^2}{\sin^4 \beta}.$$

In figure 3 shown, the region possible parameters $m_{S_1^{(+)}}$ and $k_{\mu e}^{(1)}$ resulting from $BR(\mu \to e\gamma)$ with $K_2^R = I$. As show masses LQ can be below 1 TeV if matrix element $(K_1^L C_l)_{13} \sim 10^{-2}$, $(K_1^L C_l)_{23} \sim 0.1$, this restriction is weak that in previous Variant.

IIa) In particular case $K_2^L = K_2^R = I$ mixing parameter $k_{\mu e}^{(1)}$ can be written in the form

$$k_{\mu e}^{(1)} = \frac{|(U)_{13}|^2 |(U)_{23}|^2}{\sin^4 \beta}.$$

where $\overset{+}{C_l} = U_{PMNS} \equiv U$ is Pontecorvo-Maki-Nakagawa-Sakata matrix and U_{13} is its unknown element.

$m_{S_1^{(+)}}$ TeV	0.55	1.3	9.3
$\sin\beta = 0.2$	6×10^{-5}	2×10^{-4}	6×10^{-3}
$\sin\beta = 1$	1×10^{-3}	5×10^{-3}	0.14

Table 1: Upper limit on the matrix element U_{13} resulting from process $\mu \to e\gamma$ with $K^{R,L} = I$ in dependence on $\sin\beta$ and $S_1^{(+)}$ mass.

In table 1 for example given up limit value matrix element U_{13} for different mass scalar leptoquarks and model parameter $\sin\beta = \eta_3/\eta$. As can be seen value in model lower than

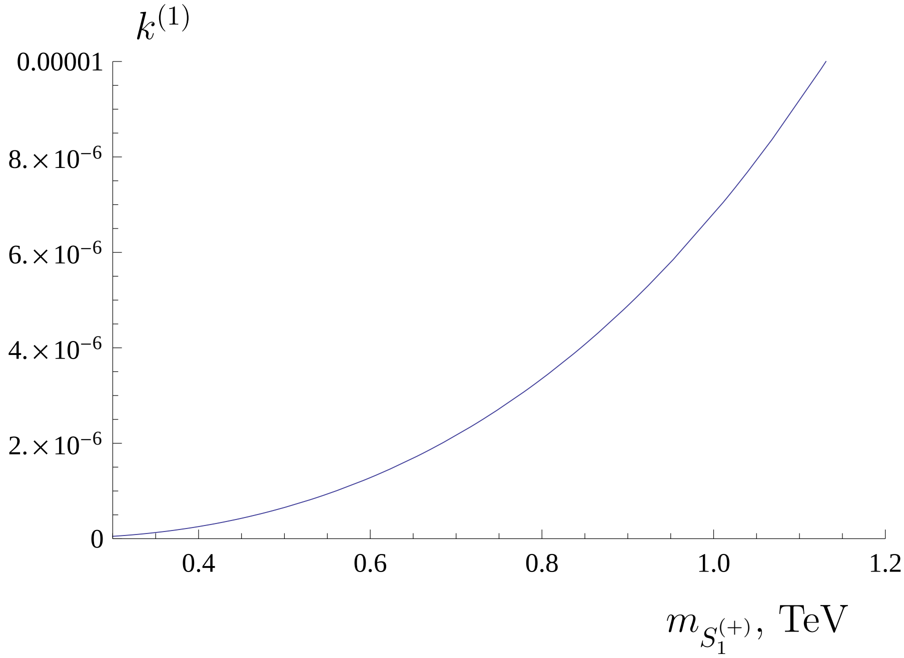

Figure 3: The allowed region of parameters $m_{S_1^{(+)}}$ and $k_{\mu e}^{(1)}$ resulting from $BR(\mu \to e\gamma)$ with $K^R = I$(below the curve) .

experimental restriction $U_{13} < 0.16$ In figure 4 shown the up limit matrix element $|U_{13}|^2$ as function mass scalar leptoquark. The current experimental limit are $|U_{13}|^2 < 0.032$ [12]. As can be seen scalar leptoquarks with arbitrary masses not restrict from $|U_{13}|^2$.

III) Variant where matrix elements $(K_1^L C_l)_{13}$, $(K_2^R)_{13}$ are zero. When only processes $\tau \to \mu\gamma$ exist and from its experimental restriction $Br(\tau \to \mu\gamma) < 4.5 \cdot 10^{-8}$ we obtain corresponding limits. Lower limit on the mass scalar leptoquark $S_1^{(+)}$ resulting from $\tau \to \mu\gamma$ in dependence on different parameter $k_{\tau\mu}^{(a)}, a = 1, 2, 12$ shown in table 2. As shown this restriction is small.

$k_{\tau\mu}^{(1)} = k_{\tau\mu}^{(2)} = k_{\tau\mu}^{(12)}$	10^{-4}	10^{-3}	10^{-2}
$m_{S_1^{(+)}}$ TeV	0.3	0.7	1.4

Table 2: Lower Limit on the $S_1^{(+)}$ Mass resulting from $\tau \to \mu\gamma$ in dependence on $k_{\tau\mu}^{(1)} = k_{\tau\mu}^{(2)} = k_{\tau\mu}^{(12)}$.

IV) Interaction of scalar leptoquark S_m $(Q = 2/3)$ and b-quark gives the lower limit on m_{S_m} from $\mu \to e\gamma$ weaker than the current experimental ones.

V) The Case of the chiral interaction of scalar leptoquarks $S_1^{(+)}$ with fermions gives the limits which coincide with those of the [Variant II].

Conclusion

The contributions of scalar leptoquarks $S_1^{(+)}$, S_m from the MQLS model in $l_i \to l_j\gamma$ decays are analyzed in comparison with experimental data on $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$ decays.

It is shown that in the appropriate region of the mixing parameters relatively light scalar leptoquarks (with masses of order 1 TeV or below) do not contradict current experimental restrictions on LFV processes.

Acknowledgements

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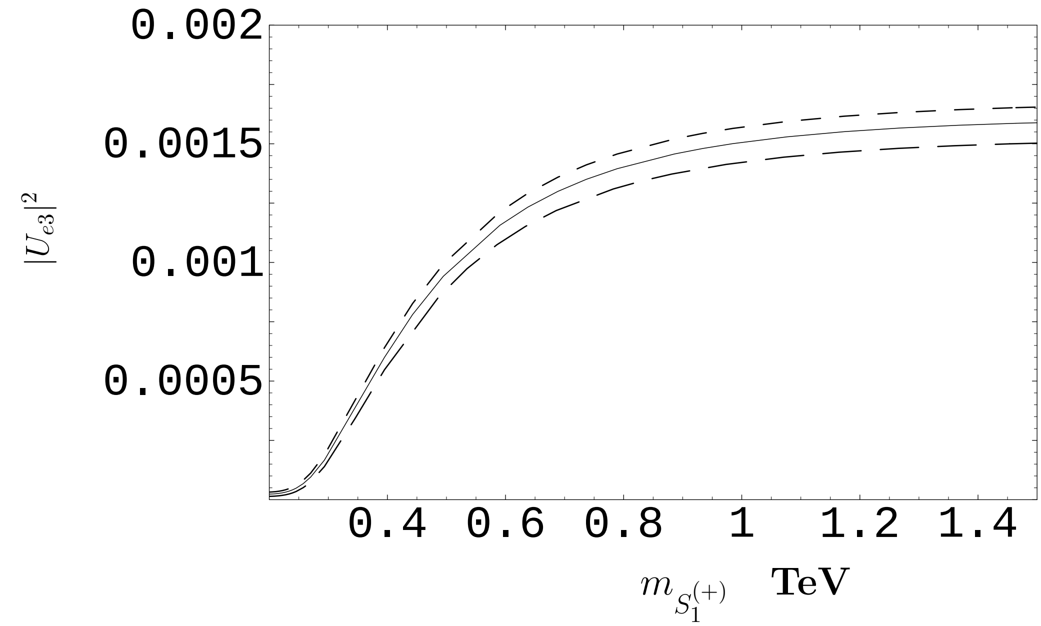

Figure 4: Upper limit on the matrix element U_{13} resulting from $\mu \to e\gamma$ with $K^{R,L} = I$ as function of $S_1^{(+)}$ mass at $U_{23} = 0.7^{+0.12}_{-0.16}$ [13] and $\sin\beta = 1$. Current experimental limit is $|U_{13}|^2 < 0.032$ [12] .

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