$B^0 - \bar{B}^0$ at NLO of $1/m_b$ expansion

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Abstract

I present results of the calculation of matrix elements of subleading operators for $B^0 - \bar{B}^0$ mixing at the Next-to-Leading Order of $1/m_b$ expansion as obained in ref. [1].

1 Outline

The outline of my talk is as follows:

- Flavordynamics in Standard Model
- Phenomenological description of $B^0 - \bar{B}^0$ system:
  observables
- $B^0 - \bar{B}^0$ mixing in SM:
  operators, factorization, bag parameters B_B
- Matrix elements in sum rules approach:
  - reproducing factorization results in sum rules
  - nonPT (condensates) corrections to factorization
  - PT corrections: pQCD analysis of three-point correlator at three loops
- Conclusion

2 Flavors in SM

As an introduction I briefly remind of the flavor structure of the Standard Model. Three generations of matter fields (fermions)

$$\text{quarks}: \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} + \text{leptons}: \begin{pmatrix} \nu \\ e \end{pmatrix}, \dots$$

$u, d, \dots$ – flavors (of quarks) are originally massless and form doublets and singlets with respect of the electroweak gauge symmetry group. The masses of the fermion matter fields are generated by the Yukawa interaction with the Higgs boson field H after the electroweak phase transition

$$\lambda_f H \bar{f} f \to m_f \bar{f} f, \quad m_f = \lambda_f \langle H \rangle, \quad \langle H \rangle \neq 0.$$

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Thus, the masses are proportional to the vacuum expectation value of the Higgs boson field $\langle H\rangle$ and the Yukawa coupling constants λ_f. The electroweak gauge interaction of up $U_i = (u, c, t)$ and down $D_j = (d, s, b)$ fields with W-bosons includes CKM matrix V_{ij}

$$V_{ij} \cdot \bar{U}_i \gamma_\mu D_j W^\mu.$$

The nonvanishing off-diagonal elements of CKM matrix V_{ij} initiate transitions between different flavors, for example,

$$b \to c, \quad b \leftarrow s, \quad s \to u, \quad c \leftrightarrow b, \quad \ldots$$

The CKM parameters V_{ij} and quark masses m_f $(= \lambda_f)$ are therefore the coupling constants of the standard model lagrangian to be extracted from experimental data.

Strength of flavor changing transitions is governed by the CKM parameters V_{ij} with the quantitative hierarchy given by the value of V_{us} such that for $\lambda = V_{us} = \sin\theta_C \approx 0.22$ one finds the following structure of CKM matrix (Wolfenstein parameterization)

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3 (\rho - i\eta) \\ -\lambda(1 + iA^2\lambda^4\eta) & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

The pattern of mass (or Yukawa couplings) hierarchy is not given by any formula but the numerical values of the masses are well known

$$m_{u,d} \sim 5 \times 10^{-3} \text{ GeV}, \quad m_s = 0.13 \text{ GeV},$$
$$m_c = 1.3 \text{ GeV}, \; m_b = 4.2 \text{ GeV}, \; m_t = 175 \text{ GeV}.$$

Because of large differences in the masses many flavor changing decays are kinematically allowed.

In contrast to leptons (τ, μ decays, neutrino mixing) no "free" quarks detected in experiments. Experimantally, the flavor changing transitions are between flavored hadrons: $b \to s$ means $B \to K$ or $B \to X_s$. Here QCD enters the game: this is the most difficult and interesting from the point of view of dynamics part of the analysis of EW flavor structure of quark sector in SM.

Of some special interest are $\Delta F = 2$ transitions that are also known as the mixing of different flavor neutral mesons

$$sd: \; K^0 - \bar{K}^0; \quad cu: \; D^0 - \bar{D}^0; \quad bd, bs: \; B^0 - \bar{B}^0$$

These processes are important as a primary source of CP violation studies in the standard model and the place to search for new physics beyond SM.

3 Phenomenological description of $B^0 - \bar{B}^0$ system

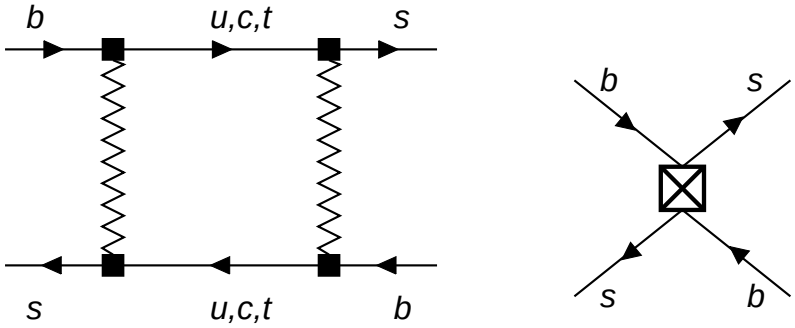
The time evolution of $(B^0, \bar{B}^0)$ system is given by the equation

$$i\frac{d}{dt}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = H_{eff}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

where H_{eff} is just a 2×2 mass operator of a decaying particle

$$H_{eff} = (M - i\Gamma/2)_{ij}, \quad i, j = 1, 2.$$

The non-diagonal elements M_{12} and Γ_{12} are effective $\Delta B = 2$ interactions which are not present in the SM fundamental Lagrangian but theoretically calculable. The observables of



$B^0 - \bar{B}^0$ system are:
1) Mass difference: $\Delta m = M_{heavy} - M_{light} \approx 2\,|M_{12}|$,
2) Decay rate difference: $\Delta\Gamma = \Gamma_L - \Gamma_H \approx -2\,|\Gamma_{12}|\cos\Phi$ with $\Phi = \arg(-M_{12}/\Gamma_{12})$.

Experimentally measured are the quantities for B_d system $\Delta m_d = 0.508 \pm 0.004\ ps^{-1}$ while $(\Delta\Gamma/\Gamma)_d = (9 \pm 37)\cdot 10^{-3}$. It is expected (theory) that $(\Delta\Gamma/\Gamma)_s = 0.158 \pm 0.050$.

These observables can be used to extract the CKM parameters from data provided that the relevant theoretical formulae are accurate. What are theoretical formulae then?

4 $B^0 - \bar{B}^0$ mixing in SM: operators, factorization, and B_B parameters

In theory, at the quark level in SM the $\Delta B = 2$ processes go through a box diagram that produces a complicated, non-local transition operator (an effective Hamiltonian). However, this non-local transition operator simplifies because of mass and CKM hierarchies: it shrinks to a point reducing the effective Hamiltonian to a set of local operators. The mechanisms are different for Δm and $\Delta\Gamma$ though. For the Δm case: because $V_{tb} \gg V_{cb}, V_{ub}$ the top quark saturates the loop, and because $m_t \gg m_b$ one can localize the loop with the NLO QCD result

$$M_{12} = \frac{G_F^2 M_W^2}{4\pi^2}\,(V_{tb}{}^* V_{td})^2\,\eta_B S_0(x_t)\left[\alpha_s^{(5)}(\mu)\right]^{-6/23}\left[1 + \frac{\alpha_s^{(5)}(\mu)}{4\pi} J_5\right]\langle \bar{B}^0|Q(\mu)|B^0\rangle$$

Here $\eta_B = 0.55 \pm 0.1$, $J_5 = 1.627$ in the NDR scheme, $S_0(x_t)$ is the short distance function, $x_t = m_t^2/m_W^2$, $Q(\mu) = (\bar{b}_L\gamma_\sigma d_L)(\bar{b}_L\gamma_\sigma d_L)(\mu)$ is the local four-quark operator that represents a genuine QCD quantity with a "current⊗current" $(J \otimes J)$ structure.

The width difference (for B_s) is given by

$$\Delta\Gamma \sim \Gamma_{12} = \langle \bar{B}_s|\mathcal{T}|B_s\rangle/2M_{B_s}$$

The final states in decays are (c, u) "quarks", $m_b \gg m_c, m_u$ and Heavy Quark Expansion in $1/m_b$ can be used in the form

$$\langle \bar{B}_s|\mathcal{T}|B_s\rangle = \sum_n \frac{C_n}{m_b^n}\langle \bar{B}_s|\mathcal{O}_n^{\Delta B=2}|B_s\rangle$$

where the coefficients C_n are calculable in PT as series in the strong coupling constant. The nonPT (long distance or infrared sensitive) physics is contained in matrix elements (ME) of the local operators $\mathcal{O}_n^{\Delta B=2}$. At LO in $1/m_b$ the two relevant four-quark operators in $\mathcal{T}$ are

$$Q = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \quad Q_S = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}.$$

At NLO in $1/m_b$ there are in total five operators. The two most important ones are

$$\begin{aligned} R_2 &= \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\mu D^\mu s_i)_{V-A}(\bar{b}_i s_i)_{V-A}, \\ R_3 &= \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\mu D^\mu s_i)_{S-P}(\bar{b}_i s_i)_{S-P}. \end{aligned}$$

Thus M_{12} and Γ_{12} reduce to evaluation of $\langle \bar{B}|Q_i|B\rangle$ in QCD that is a genuine nonPT task. No direct techniques are available for this evaluation at present.

In phenomenological applications the most popular approximation for the evaluation of these ME is factorization which is intuitive for four-quark operators. Indeed, since $Q_i \sim J \cdot J$ with $J \sim \bar{s}b$ and $\langle \bar{B}|'' ='' s\bar{b}$ it is tempting "to factorize" the matrix element according to the following rule

$$\langle \bar{B}|Q_i|B\rangle = \langle \bar{B}|J \cdot J|B\rangle = C_{\rm comb}\langle \bar{B}|J|0\rangle\langle 0|J|B\rangle.$$

For the current $J = \bar{b}_L\gamma_\mu d_L$ one has $\langle 0|\bar{b}_L\gamma_\mu d_L|B^0(\mathbf{p})\rangle = ip_\mu f_B/2$ and the complicated ME is expressed through a leptonic decay constant f_B. The main problem for such an approximaton is an accuracy that is not under any control. Writting $\langle \bar{B}_s|\mathcal{O}_i|B_s\rangle = B_i\langle \bar{B}_s|\mathcal{O}_i|B_s\rangle^{fac}$ one introduces bag parameters B_i – genuine dynamical QCD quantities controlling the accuracy of the factorization, with normalization $B_i = 1$ in factorization approxmation.

For relevant operators one writes the expressions

$$\begin{aligned} \langle \bar{B}|Q|B\rangle &= f_B^2 M_B^2 2\left(1+\frac{1}{N_c}\right)B \\ \langle \bar{B}|Q_S|B\rangle &= -f_B^2 M_B^2 \frac{M_B^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right)B_S \\ \langle \bar{B}|R_2|B\rangle &= -f_B^2 M_B^2 \left(\frac{M_B^2}{m_b^2}-1\right)\left(1-\frac{1}{N_c}\right)B_2 \\ \langle \bar{B}|R_3|B\rangle &= f_B^2 M_B^2 \left(\frac{M_B^2}{m_b^2}-1\right)\left(1+\frac{1}{2N_c}\right)B_3, \end{aligned}$$

The main theoretical uncertainties for the analysis of mixing are related to the ME of the local operators $\mathcal{O}_i \in \{Q, Q_S, R_2, R_3\}$, or equivalently, the bag parameters B_i.

The evaluation of bag B_B parameter (and the analogous parameter B_K of $K^0 - \bar{K}^0$ mixing) has long history (factorization, quark models, phenomenological unitarity, ChPT, large N_c, lattice,...). Recently a rigouros approach to evaluation of ME emerged. QCD factorization for $B \to \pi\pi$ and $B \to D\pi$ (BBNS-Beneke,Buchalla,Neubert,Sachrajda). It allows for an expansion in $\Lambda_{\rm QCD}/m_b$ as small parameter ($m_b = 4.2$ GeV, $\Lambda_{\rm QCD} = 0.5$ GeV) and account for pQCD corrections. A proper theoretical tool is the Soft-Collinear Effective Theory (SCET).

In my talk I consider the calculation based on OPE and QCD sum rules:

- very close in spirit to lattice computations, which is a model-independent, first-principles method. QCD sum rule approach relies on asymptotic expansions of a Green's function (analytically in a small parameter) while on the lattice the whole function can be found (numerically)

- sum rule techniques provide a consistent way of treating perturbative corrections to matrix elements which is needed to retain RG invariance of physical observables usually violated in other approximations (factorization)

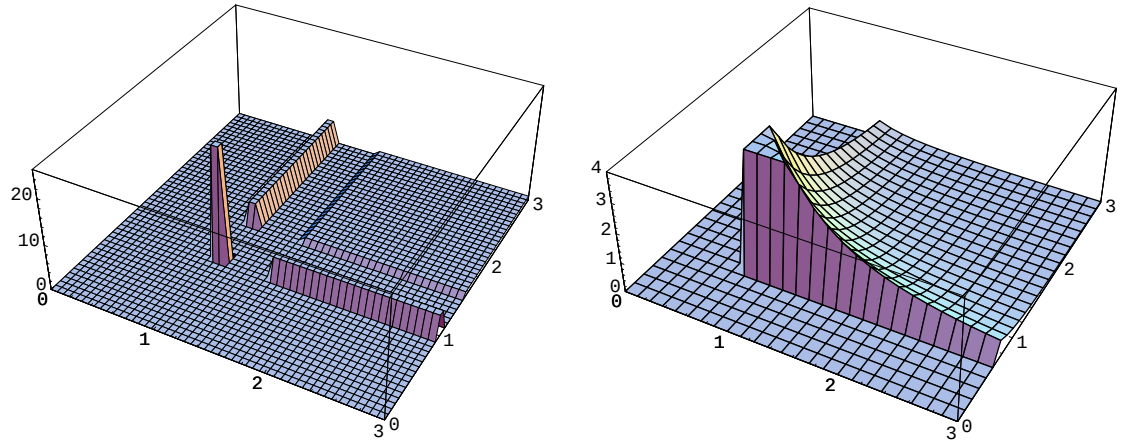


Figure 1: One-resonance hadronic spectrum and OPE approximation

5 Sum rules analysis

The starting point of the analysis is the three-point correlator

$$T(p_1,p_2) = i^2 \int d^4x d^4y e^{ip_1x - ip_2y} \langle T j(x) \mathcal{O}(0) j(y) \rangle$$

where $\mathcal{O} \in \{Q, Q_S, R_2, R_3\}$ is a generic four-quark operator and j can be either AV or PS current

$$j_5^\mu = \bar{s}\gamma^\mu\gamma_5 b \quad \text{(AV)}, \qquad j_5 = \bar{s} i \gamma_5 b \quad \text{(PS)}.$$

The overlap is given by the matrix elements

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 b(0)|\bar{B}(p)\rangle = i f_B p_\mu, \quad \langle 0|\bar{s} i\gamma_5 b(0)|\bar{B}(p)\rangle = \frac{f_B M_B^2}{m_b + m_s}.$$

For the AV current the correlator is a tensor, and one takes the tensor structure $p_1^\mu p_2^\nu$

$$T^{\mu\nu}(p_1,p_2) = p_1^\mu p_2^\nu T(p_1,p_2) + \ldots$$

The dispersion relation

$$T(p_1,p_2) = \int ds_1 ds_2 \frac{\rho(s_1,s_2,q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

determines the spectral density $\rho(s_1,s_2,q^2)$ that containes physics. To derive the sum rules the spectral density is evaluated in two ways.

1. In hadronic picture: B-meson pole plus continuum

$$\rho_{\rm AV}^{\rm had}(s_1,s_2) = f_B^2 \delta(s_1 - M_B^2)\delta(s_2 - M_B^2)\langle \bar{B}|\mathcal{O}|B\rangle + \rho_{\rm AV}^{\rm cont}$$

2. With QCD using OPE: $\rho_i^{\rm OPE}$ are the sum of a PT and a nonPT involving condensates. The idea of QCD sum rules is to use duality

$$\int ds_1 ds_2\, \rho_i^{\rm had}(s_1,s_2) = \int ds_1 ds_2\, \rho_i^{\rm OPE}(s_1,s_2).$$

We illustrate the physical spectrum and the spectrum obtained in some approximation of the OPE in QCD by the pictures above.

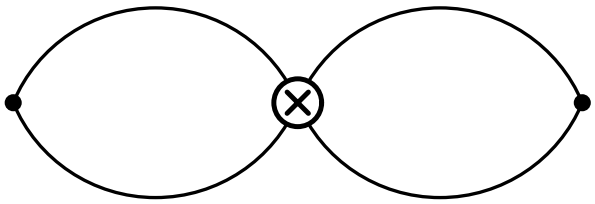


Figure 2: PT diagram at LO

In practice one is using two popular techniques for the analysis within sum rules:

1. Finite Energy sum rules with the duality region Δ being the square $m_b^2 < s_i < s_0$ in the (s_1, s_2) plane

$$f_B^2 \langle \bar{B}|\mathcal{O}|B\rangle = \int_\Delta ds_1 ds_2\, \rho_{\rm AV}^{\rm OPE}(s_1, s_2),$$

2. Borel sum rules with modeling the hadronic continuum with the OPE prediction and using Borel transform

$$f_B^2 \langle \bar{B}|\mathcal{O}|B\rangle e^{-\frac{M_B^2}{M_1^2}-\frac{M_B^2}{M_2^2}} = \int_\Delta ds_1 ds_2\, e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}}\, \rho_{\rm AV}^{\rm OPE}(s_1, s_2).$$

5.1 Factorization in sum rules

It is instructive to see how factorization is implemented within the sum rules analysis. It turns out that the OPE diagrams show that one can split the three-point correlator into two pieces

$$T(p_1, p_2) = T_{\rm fac}(p_1, p_2) + \Delta T(p_1, p_2).$$

The factorized part has an explicit form

$$T_{fac}(p_1, p_2) = \text{const} \times \Pi(p_1)\Pi(p_2)$$

with the "const" and the $\Pi(p_i)$ specific to the operator involved. For the operators of V-A structure one finds

$$T_{fac}^{\rm AV}(p_1, p_2) = 2\left(1 + \frac{1}{N_c}\right)\Pi^V(p_1)\Pi^V(p_2)$$

with the definition $p^\alpha \Pi^V(p) = i\int dx e^{ipx}\langle T j(x)\bar{b}\gamma^\alpha(1-\gamma_5)s(0)\rangle$. The sum rule for the factorized piece $T_{\rm fac}(p_1, p_2)$ yields $B = 1$ by construction. Then one finds a sum rule for $\Delta B = B - 1$ directly

$$f_B^2 \Delta B e^{-\frac{M_B^2}{M_1^2}-\frac{M_B^2}{M_2^2}} = \int ds_1 ds_2 \Delta\rho_{\rm AV}^{\rm OPE}(s_1, s_2) e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}}$$

for the Borel sum rule with an AV current and analogously for the other cases.

At LO in pQCD the three-point function factorizes

$$T(p_1, p_2) = T_{\rm fac}(p_1, p_2), \quad \Delta T(p_1, p_2) = 0$$

and

$$T^{\rm LO}(p_1, p_2) = T_{fac}^{\rm LO}(p_1, p_2) = \text{const} \times \Pi^{\rm LO}(p_1)\Pi^{\rm LO}(p_2)$$

Then we have $B = 1$. But this is only LO analysis.

One can do better as higher order diagrams build up the full function $\Pi^{\rm LO}(p_1) \to \Pi(p_1)$ and also

$$T_{fac}^{\rm LO}(p_1, p_2) \to T_{fac}(p_1, p_2) = \text{const} \times \Pi(p_1)\Pi(p_2)$$

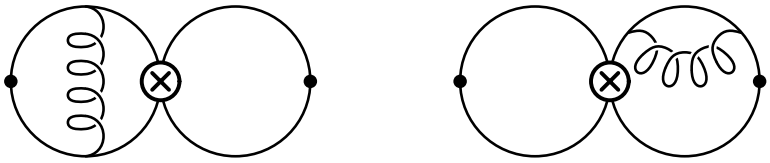


Figure 3: Factorizable diagrams at NLO

Indeed, NLO pQCD gives the contribution to the factorizable part of the correlator shown in Fig. 3. Thus the NLO factorizable contributions are given by the product of two-point correlation functions

$$\Pi^f_{\rm NLO} = \frac{8}{3}(p_1.p_2)\{\Pi_{\rm LO}(p_1^2)\Pi_{\rm NLO}(p_2^2) + {\rm symm}(p_1,p_2)\}.$$

Note that the spectral density of $\Pi_{\rm NLO}(p^2)$ is known analytically: this solves the problem of the NLO analysis in the approximation of factorization.

As an example of the nonPT (condensate) factorizable contributions we give the contribution of the gluon condensate as shown in Fig. 4.

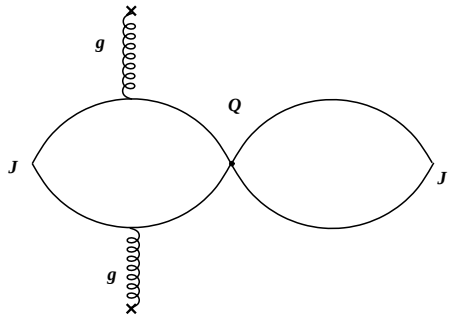


Figure 4: A factorizable nonPT GG diagram

Factorizable diagrams form an important subset of all contributions, as they are independently gauge and RG invariant. Thus, classification of diagrams in terms of their factorizability is consistent and gives a powerful tool of the quantitative analysis.

We turn now to the non-factorizable contributions. The first one comes from the pQCD diagram shown in Fig. 5.

The NLO analysis of non-factorizable contributions within perturbation theory amounts to the calculation of a set of three-loop diagrams that is a nontrivial task.

There are also non-factorizable condensate contributions that are simpler to compute as they are given by one-loop or even tree-level diagrams.

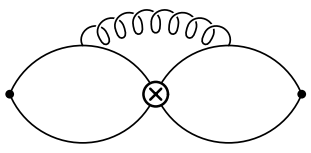


Figure 5: A non-factorizable diagram at NLO

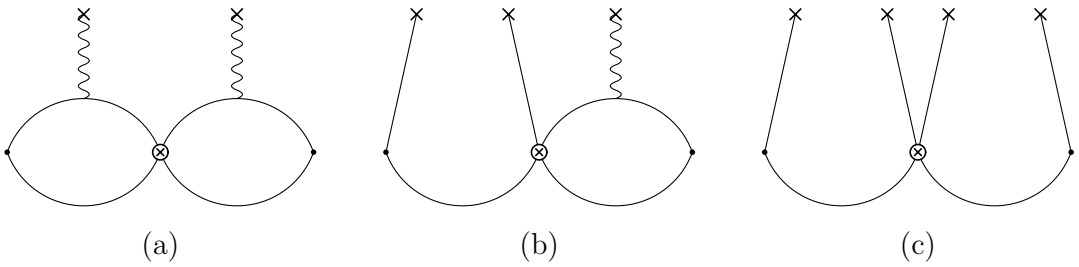


Figure 6: Non-factorizable condensate contributions

I start my analysis of the sum rules for the three-point correlator with nonPT terms, or vacuum condensates.

5.2 Condensates corrections to factorization

The general OPE result for the spectral density has the form

$$\Delta\rho_i(s_1,s_2) = \Delta\rho_i^{\rm GG}(s_1,s_2)\langle GG\rangle + \Delta\rho_i^{\rm sGs}(s_1,s_2)\langle\bar{s}Gs\rangle + \dots$$

for each of the eight cases (AV or PS current for Q, Q_S, R_2, R_3 operators. The result for the gluon condensate contribution for the leading order operator Q has been known since long ago [2].

As an example I give here the explicit expression for the Q_S operator with PS current because it is reasonably short

$$\Delta\rho_{\rm PS}(s_1,s_2) = \frac{1}{48\pi^2}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{s_1s_2}\left(\frac{s_1s_2}{2}(6-3z_1-3z_2+z_1z_2)+(p_1p_2)^2z_1z_2\right)$$

$$+\frac{m_b}{16\pi^2}\langle\bar{s}Gs\rangle\left((-2+z_1)\delta(s_2-m_b^2)+(-2+z_2)\delta(s_1-m_b^2)\right), \quad z_i = m_b^2/s_i.$$

Fig. 7 shows the results of the Borel sum rules in HQET approximation [1]. The formal procedure of taking the HQET limit is as follows: we write $M_B = m_b + \bar{\Lambda}$, $s = (m_b+E)^2$, $s_0 = (m_b+E_0)^2$ and expand QCD sum rules in $1/m_b$.

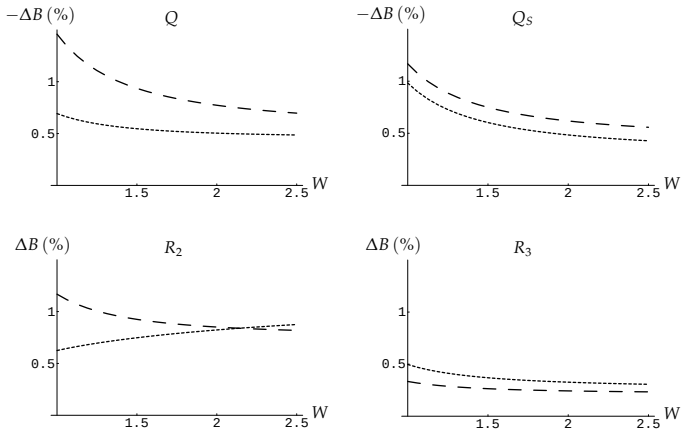


Figure 7: Plots of $(-)\Delta B$ vs. W obtained with the Borel sum rules in HQET.

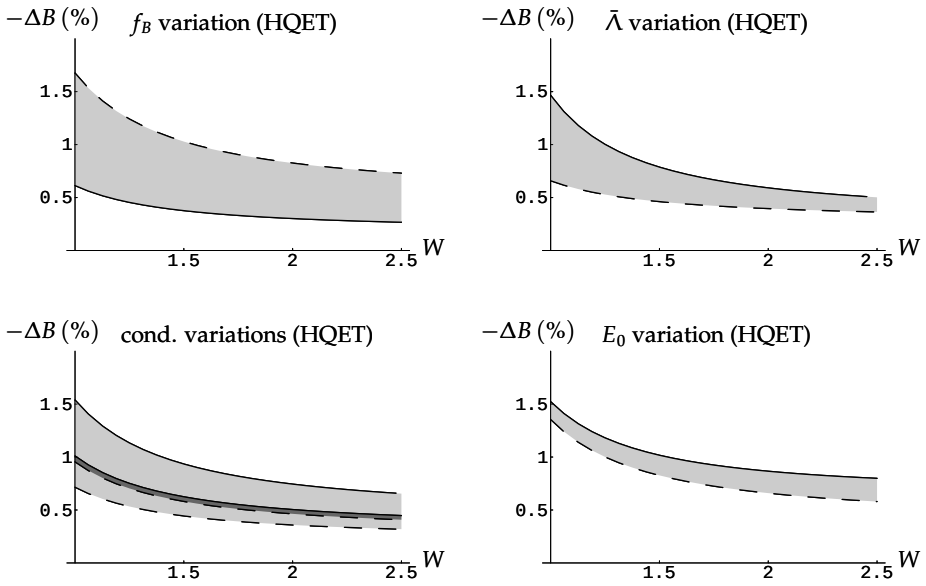


Figure 8: $-\Delta B$ for Q_S operator, AV current, in the HQET sum rule. For the condensate variations, the dark-gray band corresponds to the gluon condensate and the larger light-gray band to the quark-gluon condensate variation.

Numerical results and uncertainties of the analysis are presented in Fig. 8. The parameters are 210 MeV $< f_{B_s} <$ 270 MeV, $m_b = 4.2 \pm 0.2$ GeV, $32 < s_0 < 40$ GeV^2 in QCD, and $m_b = 4.8$ GeV, 1 GeV $< E_0 <$ 1.5 GeV in HQET. Condensates are varied by $\pm 30\%$.

Uncertainty due to each parameter variation are given on the example of the Q_S operator in HQET. The largest errors are associated with the value of the decay constant f_B. The dependence on $\bar{\Lambda}$ and E_0 is moderate. The results depend linearly on the condensates and the uncertainty due to the condensates is comparable with that due to f_B.

In all cases (except for R_2), our central values for ΔB in QCD and HQET turned out to be (nearly) equal. The reason is a slow convergence in $1/m_b$ for R_2.

The final results are given in the table.

Operator	$\Delta B(\%)$ QCD	$\Delta B(\%)$ HQET
Q	-0.6 ± 0.5	-0.6 ± 0.5
Q_S	-0.5 ± 0.4	-0.6 ± 0.4
R_2	0.3 ± 0.3	0.8 ± 0.7
R_3	0.3 ± 0.2	0.3 ± 0.2

These have been condensates contributions only.

Non factorizable PT terms involve three-loop diagrams and their computation is a non-trivial task. Results are nonetheless available for the operator Q.

5.3 Non factorizable PT corrections: pQCD analysis of three-point correlator at three loops

Due to technical difficulties of computing the spectral density of three-loop diagrams we instead compute moments of the correlation function at $p_1^2 = p_2^2 = 0$ at the point $q^2 = 0$ as has been done in ref. [3]

$$M(i,j) \equiv \frac{\partial^{i+j}\Pi(p_1^2, p_2^2, 0)}{i!j!\partial p_1^{2i}\partial p_2^{2j}} = \int \frac{\rho(s_1, s_2, 0) ds_1 ds_2}{s_1^{i+1} s_2^{j+1}} \,.$$

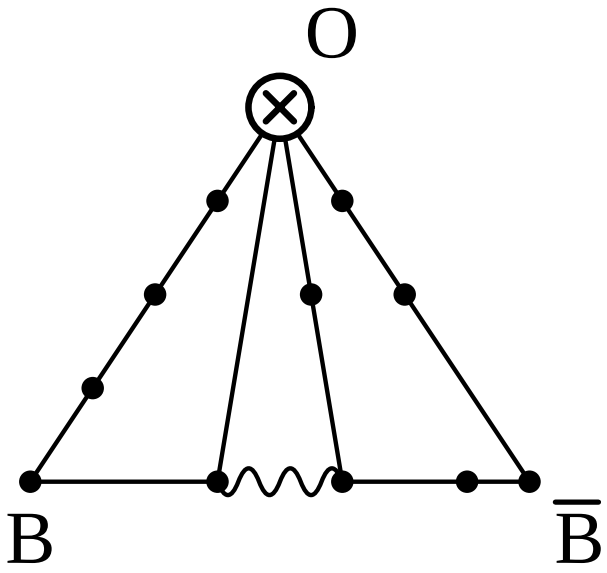


Figure 9: $B^0 - \bar{B}^0$ moments topology

Computation of moments in QCD reduces to an evaluation of single scale vacuum diagrams. This calculation is done analytically at the three-loop level with available tools for the automatic computation of multi-loop diagrams. These diagrams have been computed using the package MATAD for automatic calculation of Feynman diagrams. The combinatorics of disentangling the tensorial structures has been solved and all diagrams have been reduced to scalar integrals.

Theoretical moments are given in the form

$$M_{th}(i,j) = \frac{m^6 a_{ij}}{m^{2(i+j)}} \left(1 + \frac{\alpha_s}{4\pi}\left(b_{ij}^f + b_{ij}^{nf}\right)\right) \tag{1}$$

with the unique decomposition that is Renormalization Group and gauge invariance consistent. The quantities in eq. 1 are

a_{ij} is Leading Order contribution,

b_{ij}^f is NLO factorizable piece,

b_{ij}^{nf} is NLO nonfactorizable contribution .

We have found NLO nonfactorizable contributions b_{ij}^{nf} with $i + j \leq 7$ analytically. Calculation required about 24 hours of computing time on a dual-CPU 2 GHz Intel Xeon machine. The calculation of higher moments is feasible but requires considerable optimization of the code. This work is in progress. The analytical results for the lowest finite moment $M_{th}(2,2)$:

$$a_{22} = \frac{1}{(16\pi^2)^2}\left(\frac{8}{3}\right), \quad b_{22}^f = \frac{40}{3} + \frac{16\pi^2}{9},$$

$$b_{22}^{nf} = S_2 \frac{8366187}{17500} - \zeta_3 \frac{84608}{875} - \pi^2 \frac{33197}{52500} - \frac{426319}{315000}.$$

Here

$$S_2 = \frac{4}{9\sqrt{3}} \mathrm{Cl}_2\left(\frac{\pi}{3}\right) = 0.2604\ldots, \quad \zeta_3 = \zeta(3)$$

Numerical values for higher moments are

$$b_{2(2345)}^{nf} = \{0.68, 1.22, 1.44, 1.56\}$$

and

$$b^{nf}_{3(34)} = \{1.96, 2.25\}$$

To find the nonfactorizable addition to B_B we form ratios of the total and factorizable contributions

$$\frac{M_{th}(i,j)}{M^f_{th}(i,j)} = 1 + \frac{\alpha_s}{4\pi}\frac{b^{nf}_{ij}}{1+\frac{\alpha_s}{4\pi}b^f_{ij}}$$

Writing $B_B = 1 + \Delta B$ we extract ΔB by a combined fit of several "theoretical" and "phenomenological" moments. The final formula for the determination of ΔB reads

$$\frac{\alpha_s}{4\pi}b^{nf}_{ij} = \Delta B + \Delta R(z^{j-2} + z^{i-2}) + \Delta C z^{i+j-4}$$

where ΔR and ΔC are parameters of the fit related to continuum model, $z = m_B^2/(m_B^2 + \Delta)$. Using least-square fit for all available theoretical moments and estimating all uncertainties we finally find the NLO non-factorizable pQCD correction to ΔB

$$\Delta B = (6 \pm 1)\frac{\alpha_s(m)}{4\pi}.$$

For $m = 4.8$ GeV, $\alpha_s(m) = 0.2$ it leads to $\Delta B = 0.095 \approx 0.1$.

6 Conclusion

To summarize,

- SR is a powerful tool for analysing ME of local operators relevant to flavor physics. Factorization results are reproduced at diagram level (not only LO).
- Non-factorizable contributions due to nonPT condensates to bag parameters are small $\Delta B_i = 0.5 - 1\%$ for all operators $\{Q, Q_S, R_2, R_3\}$.
- Non-factorizable PT diagrams are technically difficult – three-loop massive Feynman diagrams. Moments of the spectral density for operator Q have been computed analytically at three-loop level. Phenomenological analysis within QCD sum rules gives

$$\Delta B(m) = 0.10 \pm 0.02$$

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