

# Weak interaction contribution to the inclusive hadron-hadron scattering cross sections at high $p_T$

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## Abstract

It is demonstrated that the strong power-like scaling violation in the transverse momentum distribution of inclusive hadron production, observed by CDF Collaboration in  $\bar{p}p$  collisions at Tevatron is caused by contribution of weak interaction. The contribution of weak interaction is increasing with energy at high energies.

The CDF Collaboration have measured the inclusive cross sections of charged hadron production at high transverse momentum  $p_T$  at  $\bar{p}p$  collisions at c.m. energy 1.96 TeV [1]. Surprisingly the strong power-like scaling violation was observed at  $p_T > 30$  GeV: at  $p_T \approx 100$  GeV the data indicate that the scaling law  $Ed\sigma/d^3p \sim 1/p_T^4$  is violated more than by one order of magnitude. The scaling law  $Ed\sigma/d^3p \sim 1/p_T^4$  for inclusive hadron production in hadron-hadron scattering was proved basing on very general grounds – the light-cone dominance of hard processes in strong interaction [2]. Therefore, the observation of the violation of the scaling law resulted to strong confusion. Theoretically the observed phenomenon was discussed in the paper by Albino et al [3]. The authors of Ref.[3] addressed the scaling violation to factorization breaking at high transverse momentum charged hadron production. Such explanation is not satisfactory: QCD has no scale parameters besides  $\Lambda_{QCD}$  and inclusive cross sections are infra-red stable in QCD. In principal there is the dimensional parameter in the problem in view – the energy of the collision. But, as it is well known [2],[4], the energy is related to the longitudinal size of the collision region, but not to the transverse size, which determines the cross section. In recent paper [5] an attempts were done to construct the models, describing the data, but as well as in [3] no success was achieved. At the same time the measurements of inclusive jet production [6],[7] demonstrate good agreement with scaling law and theoretical expectations.

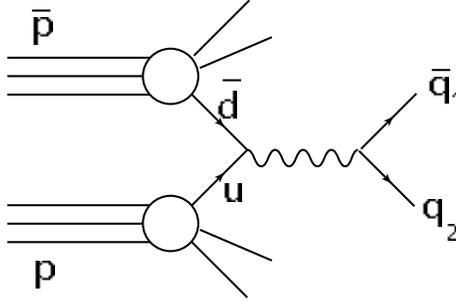
In this paper it is shown, that the scaling law violation in inclusive cross sections of charged hadron production at high  $p_T$ , observed by CDF Collaboration, is described by contribution of weak interaction. The idea is that weak interaction has the scale parameters – the masses of  $W$  and  $Z$  bosons. At high  $p_T$  the contribution of weak interaction to the inclusive cross section is strongly enhanced by the presence of  $W$  and  $Z$  resonances in comparison with strong interaction contribution which falls steeply with  $p_T$ . Weak interaction contribution has a peak at  $p_T = m_W/2$ . Due to these circumstances the weak interaction contribution becomes compatible with strong ones at  $p_T \gtrsim 30$  GeV. The weak interaction Lagrangian is the ones of the Standard Model:

$$\begin{aligned}
 L = \frac{g}{\sqrt{2}} \left\{ \left[ W_\mu^+ \bar{u} \gamma_\mu \frac{1}{2} (1 + \gamma_5) d + W_\mu^- \bar{d} \gamma_\mu \frac{1}{2} (1 + \gamma_5) u \right] + \right. \\
 + Z_\mu \frac{1}{\cos\theta_W} \left[ \bar{u} \gamma_\mu \frac{1}{2} (1 + \gamma_5) \left( \frac{1}{2} - \frac{2}{3} \sin^2\theta_W \right) u + \bar{u} \gamma_\mu \frac{1}{2} (1 - \gamma_5) \left( -\frac{2}{3} \sin^2\theta_W \right) u + \right. \\
 \left. \left. + \bar{d} \gamma_\mu \frac{1}{2} (1 + \gamma_5) \left( -\frac{1}{2} + \frac{1}{3} \sin^2\theta_W \right) d + \bar{d} \gamma_\mu \frac{1}{2} (1 + \gamma_5) \left( \frac{1}{3} \sin^2\theta_W \right) d \right] + (u \rightarrow c, d \rightarrow s) \right\}. \quad (1)
 \end{aligned}$$

Here  $u$  and  $d$  are fields of  $u$  and  $d$  quarks,  $\theta_W$  is the Weinberg angle,  $\sin^2\theta_W \approx 0.230$ . The coupling constant  $g$  is equal

$$g^2 = \frac{e^2}{\sin^2\theta_W}, \quad e^2 = \frac{1}{137} \quad (2)$$

The matrix element of weak interaction contribution to the inclusive cross section in  $\bar{p}p$  collision is represented by the diagram of Fig.1



**Fig.1.** The diagram, describing the quark pair production at high  $p_T$  in case of  $W^+$  exchange in annihilation channel.

There are also the diagrams, where  $\bar{p}$  fragments into  $u, \bar{s}, c$  and  $p$  – into  $\bar{d}, c, \bar{s}$ , correspondingly, as well the diagrams with  $W^-$  and  $Z$  in annihilation channel. (The contribution of  $W$  and  $Z$  exchange in  $t$ -channel is negligible.) In order to compare the results with CDF data let us calculate the inclusive cross section integrated over pseudorapidity

$$\eta = \frac{1}{2} \ln \frac{E' + p'_{\parallel}}{E' - p'_{\parallel}}, \quad (3)$$

where  $E' = \sqrt{p'^2_{\parallel} + p'^2_T}$  is the energy of detected charged particle,  $p' = (p'_{\parallel}, \mathbf{p}'_T)$ ,  $p'_{\parallel}$  and  $\mathbf{p}'_T$  are projections of its momenta parallel and perpendicular to beam direction. The contribution to the inclusive cross section of the diagram of Fig.1 is equal

$$E' \int \frac{d\sigma_{weak}}{d^3p'} d\eta' \Big|_{|\eta'| < \eta} = \frac{9}{8} \frac{g^4}{(2\pi)^2} \frac{p_T}{E} \int \frac{dx_1 dx_2 dx_3}{x_1 x_2} \frac{(x_1^2 + x_2^2)(1 + th^2\eta)}{(4E^2 x_1 x_2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \times$$

$$\frac{1}{(x_1 + x_2)x_3 \operatorname{sech} \eta - \frac{p_T}{E}} F_u(x_1) F_d(x_2) \sum_i \left[ D_u^i(x_3) + D_d^i(x_3) + D_s^i(x_3) + D_c^i(x_3) \right]. \quad (4)$$

Here  $E$  is the proton or antiproton energy in c.m.s.,  $m_W$  and  $\Gamma_W$  are  $W$  mass and width,  $F_u(x_1), F_d(x_2)$  – are  $u$  and  $d$ -quark distributions in proton,  $D_u^i, D_d^i, D_s^i, D_c^i$  are the fragmentation functions of  $u, d, s, c$  quarks into  $i$ -th charged particle, the sum is performed over all charged particles. The integration domain in variables  $x_1, x_2, x_3$  is restricted by

$$x_1 x_2 > \left( \frac{p_T}{E} \right)^2$$

$$(x_1 + x_2)x_3 \operatorname{sech} \eta > \frac{p_T}{E} (1 + x_3 \operatorname{sech} \eta) \quad (5)$$

As well known, (see e.g. [8] and references herein) in case of production of narrow vector resonances (like  $\omega, \varphi, J/\psi, \Upsilon$ ) in  $e^+e^-$ -annihilation  $e^+e^- \rightarrow V$  the radiative effects due to emission of real or virtual photons by initial  $e^+$  and  $e^-$  are very important. The resonance curves

are widened and resonance maxima are suppressed. The cross section of the process  $e^+e^- \rightarrow V$  without radiative effects is described by Breit-Wigner formula

$$\sigma(e^+e^- \rightarrow V) = \frac{12\pi}{s} \frac{\Gamma_{e^+e^-}^V}{M} \text{Im}f_0(\sqrt{s}), \quad (6)$$

where  $\Gamma_{e^+e^-}^V$  is the electron width of  $V$ -resonance,  $M - s$  its mass,

$$f_0(\sqrt{s}) = \frac{(1/2)M}{-\sqrt{s} + M - i\Gamma/2}, \quad (7)$$

$\sqrt{s}$  is the total energy of  $e^+e^-$  pair in their c.m.s. and  $\Gamma$  is  $V$  total width. In [8] it was shown, that the account of radiative effects results to substitution

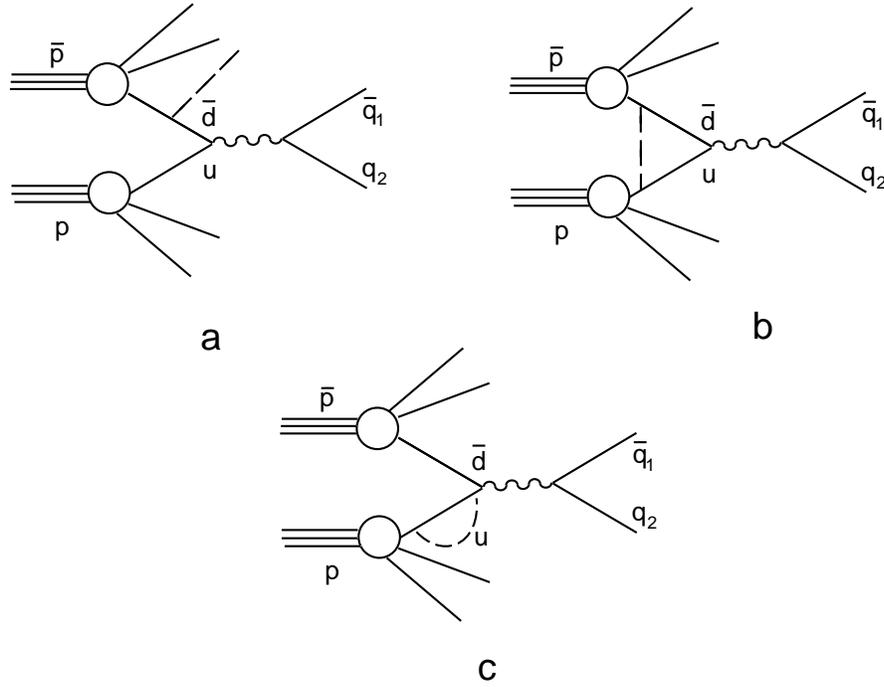
$$f_0(\sqrt{s}) \rightarrow f(\sqrt{s}) = [f_0(\sqrt{s})]^{-\beta_{QED}} \quad (8)$$

where

$$\beta_{QED} = \frac{4\alpha}{\pi} \left[ \ln \frac{\sqrt{s}}{m_e} - \frac{1}{2} \right], \quad (9)$$

$\alpha = e^2 = 1/137$ ,  $m_e$  - is the electron mass.

The similar situation takes place in production of  $W$  or  $Z$  bosons by annihilation of quark pair. The diagrams corresponding to gluon corrections of the first order are shown on Fig.2.



**Fig.2.** Gluon corrections to the process  $\bar{q}q \rightarrow W$ . Dashed lines correspond to gluons.

The diagrams Fig.2,b,c contribute through interference with the diagram of Fig.1. The gluon emission by final quarks  $\bar{q}_1, q_2$  can be neglected, since these quarks are carrying large  $p_T$ . In analogy with (8),(9) the account of gluon corrections results to substitution in Eq.4:

$$\frac{1}{(4x_1x_2E^2 - m_W^2)^2 + m_W^2\Gamma^2} \rightarrow \frac{1}{m_W^3\Gamma_W} \left[ \frac{m_W^3\Gamma_W}{(4x_1x_2E^2 - m_W^2)^2 + m_W^2\Gamma_W^2} \right]^{1-\beta_{QCD}} \quad (10)$$

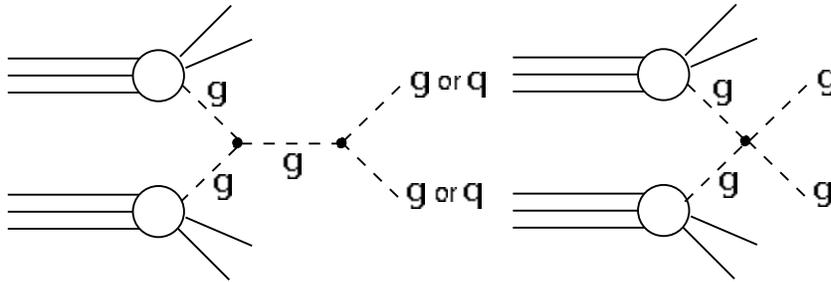
where

$$\beta_{QCD} = \frac{8}{3} \frac{\alpha_s(s)}{\pi} \left[ \ln \frac{s}{M_{char}^2} - 1 \right], \quad s = 4x_1x_2E^2, \quad (11)$$

and  $M_{char}$  is the characteristic mass of strong interaction,  $M_{char} \sim 1$  GeV. At the derivation of (10) it was assumed that  $\alpha_s(2p_T)$  is small,  $\alpha_s(s) \ll 1$ , the  $\ln[s/m_{char}^2]$  is large, the product  $\alpha_s(s) \ln[s/M_{char}^2]$  is of order of 1 and the terms  $\sim (\alpha_s \ln[s/M_{char}^2])^n$  are summed. With account of gluon corrections we have instead of (4):

$$\begin{aligned}
E' \int \frac{d\sigma_{weak}}{d^3p'} d\eta' \Big|_{|\eta'| < \eta} &= \frac{9}{8} \frac{g^2}{(2\pi)^2} \frac{p_T}{E} \int \frac{dx, dx_2, dx_3}{x_1 x_2} \frac{(x_1^2 + x_2^2)(1 + th^2\eta)}{m_W^3 \Gamma_W} \times \\
&\times \left[ \frac{m_W^3 \Gamma_W}{(4x_1 x_2 E^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \right]^{1-\beta_{QCD}} \frac{1}{(x_1 + x_2)x_3 \operatorname{sech} \eta - p_T/E} F_u(x_1) F_d(x_2) \times \\
&\times \sum_1 [D_u^i(x_3) + D_d^i(x_3) + D_s^i(x_3) + D_c^i(x_3)] \tag{12}
\end{aligned}$$

At  $p_T \approx 40 - 100$  GeV  $\alpha_s(2p_T) = 0.12 - 0.10$  and  $\beta_{QCD} \approx 0.6 - 0.8$ . So, the account of gluon corrections drastically changes the results. At small  $x$  the quark distributions and fragmentation functions behave as  $(1/x)^\gamma$ , where  $\gamma$  is equal or larger than 1 and small  $x_1, x_3$  or  $x_2, x_3$  are dominating in (12). The consequence of this fact is that the cross section (12) increases with beam energy  $E$  and decreases with  $p_T$  more slowly, than  $1/p_T^4$ . Therefore the measurements of inclusive cross sections at high  $p_T$  at LHC are very promisable. The detailed calculation of weak contribution to the inclusive cross section at high  $p_T$ , using quark distributions found by MSTW2008 [9] and CTEQ 6.6.M [10] and available information on distribution functions will be presented in separate publication.



**Fig.3.** The diagrams representing the contributions to inclusive jets production due to gluon exchange.

Finally, let us now explain, why experimentally scaling violation is not observed in inclusive jet production [6],[7]. In this case in strong interaction mechanism the main role are playing the diagrams with gluon exchange, like ones presented on Fig.3 and the contribution of weak interaction is small in comparison with them.

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