

Some two-loop threshold corrections and three-loop renormalization group analysis of the MSSM

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Abstract

Two-loop threshold corrections for both the strong coupling constant and Yukawa couplings of heavy SM fermions are considered in the context of Minimal Supersymmetric Standard Model (MSSM). With the help of the well-known SOFTSUSY code the dependence of the corrections on universal MSSM parameters is analyzed. For a consistent study of the influence of the corrections on SUSY mass spectrum three-loop renormalization group equations are implemented. A particular scenario (SPS4) is considered and the shifts of the particle masses due to contributions of different threshold corrections are presented. Additionally, the impact on certain forbidden regions of the parameter space is studied.

1 Introduction

Minimal Supersymmetric Standard Model possesses many remarkable features that allows one to think of it as of the most viable candidate for a theory that describes physics beyond the Standard Model. Unfortunately, it has a lot of parameters which are related to unknown masses of SUSY particles. One way to deal with the problem is to use universal parameters at some high-energy scale and extrapolate them to low energies with the help of renormalization group equations (RGEs). Most of the computer codes [1] used to obtain the SUSY spectrum incorporate two-loop RGEs together with one-loop threshold corrections. The latter allow one to calculate boundary values for gauge and Yukawa couplings from the given low-energy input in a consistent way.

The necessity of threshold corrections is tightly related to the fact that the theories defined in a minimal subtraction scheme cease to satisfy the Appelquist-Carazzone theorem [2] in a naive form. Heavy degrees of freedom contribute to low energy observables even in the limit when the corresponding masses tend to infinity introducing potentially large logarithmic corrections. However, the latter are local and can be absorbed in the definition of the couplings of the effective theory and summed up with the help of a renormalization group. There exists a so-called matching (or decoupling) procedure that allows one to routinely calculate threshold corrections.

One-loop decoupling corrections can be found in [3]. It turns out that sometimes they can significantly change (e.g. by 40 % for the b -quark mass) the value of the parameters defined in the MSSM with respect to that in the SM. In such cases it is reasonable to calculate contributions from the next order of perturbation theory (PT).

It took some time to find leading two-loop corrections to matching relations between the strongest SM and MSSM couplings. Strong coupling was considered in [4]. The corresponding results for heavy SM fermion masses were found in [5, 6]

It is worth mentioning that the low-energy input to the MSSM can be given in terms of running SM parameters at the electroweak scale. However, there exists a distinction in

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minimal renormalization schemes usually used in the SM and MSSM. It comes from the fact that supersymmetry requires vector bosons to be accompanied by so-called ε -scalars. The latter maintain the balance between fermionic and bosonic degrees of freedom in a dimensionally regularized theory. As a consequence, most calculations in the context of the MSSM made use of the so-called $\overline{\text{DR}}$ -scheme [7] with ε -scalars implicitly (or explicitly) taken into account. This fact leads to the problem of $\overline{\text{DR}} \rightarrow \overline{\text{MS}}$ conversion since for the Standard Model the $\overline{\text{MS}}$ scheme is more suitable. The corresponding parameter redefinition can be done in a conventional way by comparison of certain (invariant) quantities calculated in both schemes within the same model (SM or MSSM). However, this route leads to appearance of unphysical “evanescent” couplings if such relations are considered in the SM [8] or to supersymmetry breaking at rigid level in the MSSM. Recently, it was proposed that $\overline{\text{DR}}$ parameters of the MSSM and $\overline{\text{MS}}$ parameters of the SM can be directly related since unphysical ε -scalars can be treated along the same lines as heavy degrees of freedom during calculation of decoupling corrections [9].

2 Two-loop matching

Let me introduce necessary notation and briefly discuss practical prescription for the matching procedure [10]. The task is to find relations of the type

$$\underline{A}(Q) = \zeta_A(\bar{\mu}) \cdot A(Q) \quad (1)$$

where $\underline{A}(Q)$ and $A(Q)$ are some running parameters (e.g., α_s) defined at the renormalization scale Q in the effective (SM) and the fundamental (MSSM) theories, respectively. The quantity

$$\zeta_A(Q) = 1 + \delta\zeta_A^{(1)}(Q) + \delta\zeta_A^{(2)}(Q) \dots \quad (2)$$

is called “decoupling constant” for the parameter A and can be calculated order-by-order in PT. The relation (1) allows one to express A in terms of \underline{A} and other parameters of the fundamental theory at any scale Q . In practice, however, the scale is chosen in a way to minimize uncertainties due high orders of PT.

A convenient way to find the expression for $\zeta_A(Q)$ is to consider bare quantities and find “bare” decoupling constant $\zeta_{A,0}$

$$\underline{A}_0 = \zeta_{A,0} \cdot A_0 \quad (3)$$

by demanding that bare Green functions calculated in both theories coincide¹ in the limit of vanishing external momenta and masses of the considered effective field theory. In this case all the diagrams without heavy degrees of freedom also vanish and the problem is reduced to calculation of bubble integrals with at least one heavy line with mass denoted here by M . Given (3) one expresses bare quantities in terms of renormalized ones

$$\begin{aligned} \underline{A}_0 &= \underline{Z}_A(\underline{A}) \underline{A}, & A_0 &= Z_A(A, B) A, \\ \zeta_A &= \left[Z_A(A, B) \right] \left[\underline{Z}_A(\underline{A}) \right]^{-1} \zeta_{A,0}(Z_A A, Z_B B, Z_M M). \end{aligned} \quad (4)$$

and after a proper re-expansion obtains (1). In (4) B stands for couplings presented in the fundamental theory that are absent in the effective one.

Let us consider one- and two-loop contributions to the decoupling constants for the strong coupling, ζ_{α_s} , the mass of bottom-quark, ζ_{m_b} , and the tau-lepton mass, ζ_{m_τ} . Due to the fact that full two-loop calculations require evaluation of many thousands of diagrams, it is reasonable to consider a simplified setup and neglect electroweak gauge couplings. In this case, five-flavor QCD with free tau-lepton plays the role of effective theory instead of the SM. The corresponding limit of the MSSM (“gauge-less” limit) is treated as a more fundamental theory and is used to calculate threshold corrections.

¹Field redefinition is also required in this case.

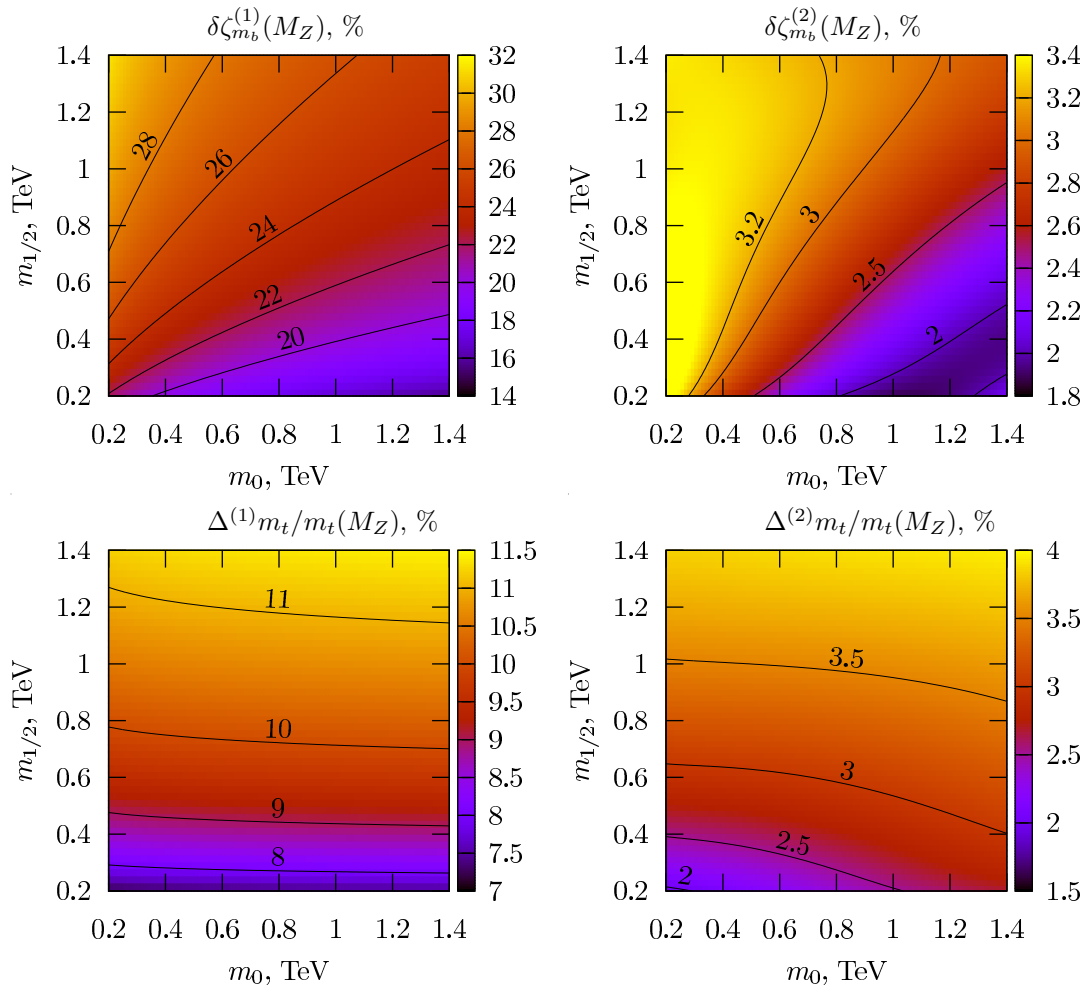


Figure 1: One- and two-loop corrections to the b -quark decoupling constant ζ_{m_b} and the pole mass M_t of t -quark as functions of universal parameters m_0 and $m_{1/2}$ for $\tan\beta = 50$ and $A_0 = 0$.

It is worth mentioning that since the t -quark mass is of the order of electroweak scale, it is reasonable to use the pole mass M_t to find the boundary condition for corresponding running parameter $m_t^{\overline{\text{DR}}}(\bar{\mu})$ in the MSSM

$$M_t = m_t^{\overline{\text{DR}}}(\bar{\mu}) \left(1 + \frac{\Delta^{(1)}m_t}{m_t} + \frac{\Delta^{(2)}m_t}{m_t} + \dots \right)$$

Here $\Delta^{(l)}m_t/m_t$ stands for l -loop contribution to the relation between M_t and $m_t^{\overline{\text{DR}}}$.

Since two-loop expressions are very lengthy, it is convenient to analyze them numerically. Figure 1 shows the dependence of quark mass corrections on the MSSM universal parameters m_0 and $m_{1/2}$. Large value of $\tan\beta = 50$ has been chosen since in this case loop corrections to the bottom-quark (and tau-lepton) mass are significantly enhanced. For the t -quark, contribution due to Yukawa couplings is suppressed by inverse $\tan\beta$ and is not taken into account.

I would like to stress that small values of $\delta_{m_b}^{(2)}$ result from large cancellations of contributions proportional to strong and Yukawa couplings. For example, in the region of small m_0 and large $m_{1/2}$ the two-loop $\mathcal{O}(\alpha_s^2)$ contribution can reach 10 %. However, negative corrections due to Yukawa couplings lower this value down to a few per cent only.

In the case of τ -lepton (see upper row in Fig. 2) there are no corrections due to strong interactions and the net effect is negative. The two-loop corrections are naturally small but

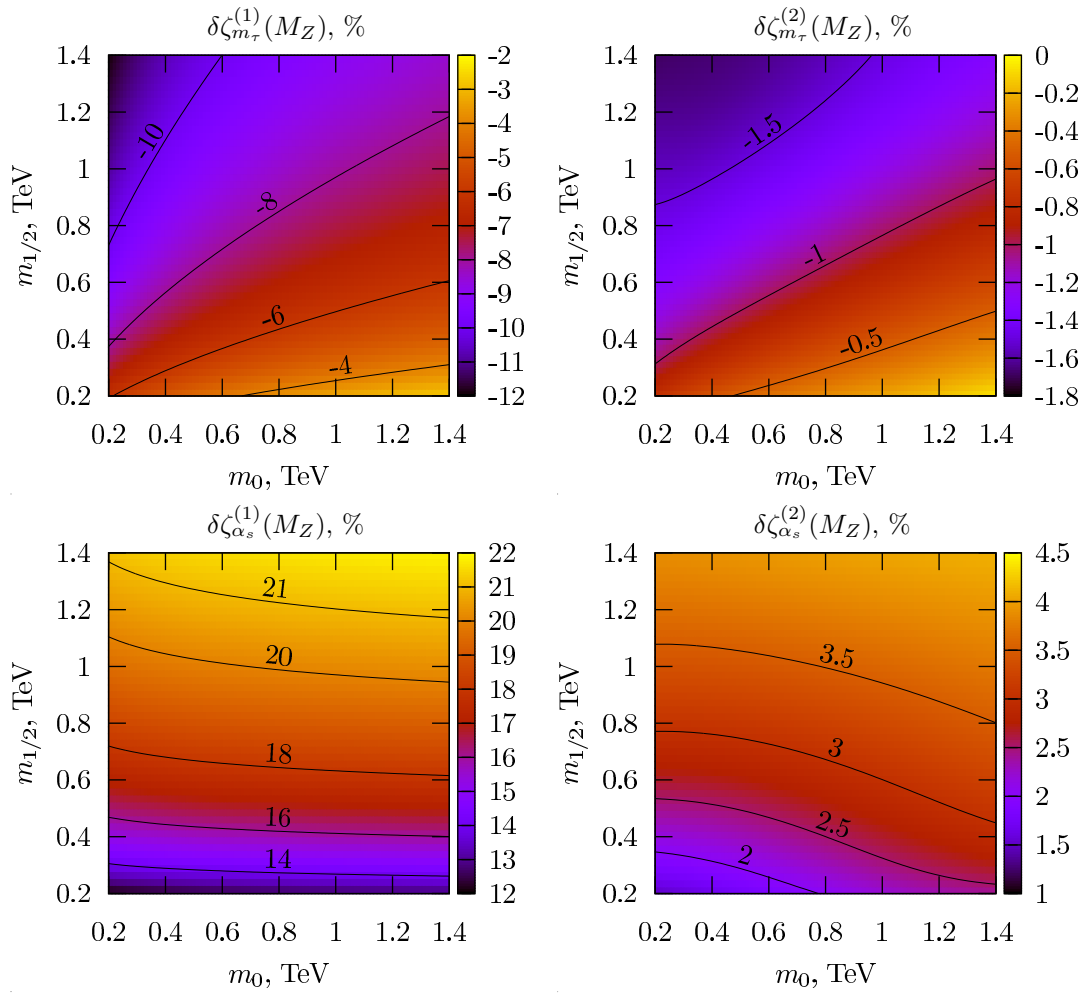


Figure 2: The dependence of threshold corrections to τ -lepton mass (upper row) and strong coupling α_s (lower row) on m_0 and $m_{1/2}$. Large value of $\tan\beta = 50$ is assumed, and $A_0 = 0$.

exceed the uncertainty in the experimental value $M_\tau = 1776.84(17)$ MeV.

Finally, two-loop decoupling corrections for the strong coupling α_s were obtained in [4]. Numerical values can be found in Fig. 2 (lower row). The corrections are small in comparison with one-loop result. Nevertheless, they can be important in studying gauge constant unification.

It is interesting to note that the two-loop corrections for α_s , m_b and m_t can be approximated by the same value of 2-3 % for the whole considered region² in the m_0 - $m_{1/2}$ plane.

All the above-mentioned corrections were taken into account in the three-loop renormalization group analysis of the MSSM that will be presented in the next section.

3 Three-loop analysis of the MSSM

It is known that for a self-consistent L -loop study of the MSSM one needs to know threshold corrections up to $(L - 1)$ order. Three-loop beta-functions for a rigid (supersymmetric) part of the MSSM Lagrangian were calculated by means of superfield Feynman diagram technique [11]. With the help of the spurion formalism developed in [12] beta-functions for soft supersymmetry-breaking terms can be easily found from the results of [11].

²The situation changes only slightly with variation of $\tan\beta$.

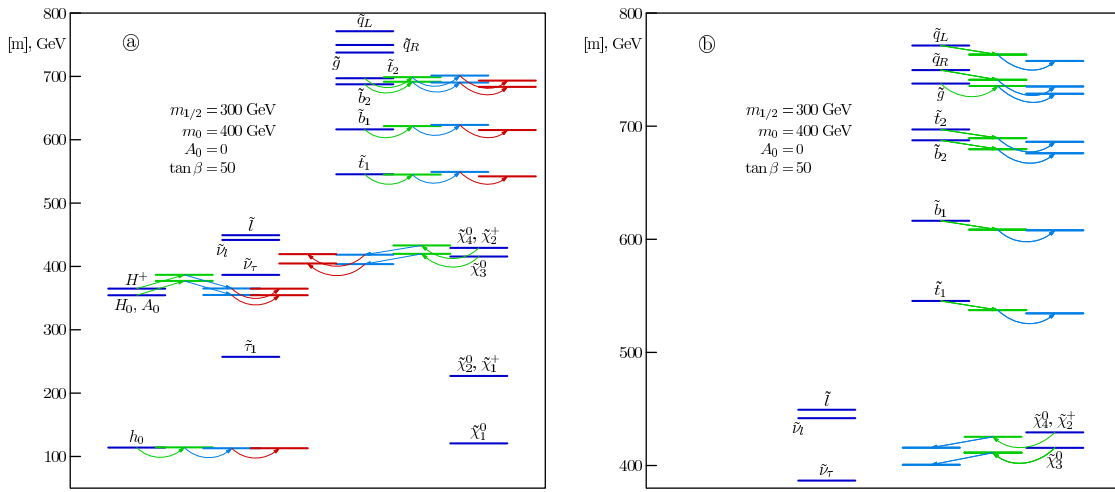


Figure 3: Initial spectrum for SPS4 scenario obtained with original SOFTSUSY code is shown together with shifts due to calculated two-loop decoupling corrections (a) and three-loop terms in RGEs (b). Left plot (a) corresponds to subsequent addition of $\delta\zeta_{m_b}$ (green), $\Delta m_t/m_t$ (blue), and $\delta\zeta_{\alpha_s}$ (red) into the code without modification of RGEs. Right plot (b) demonstrates the impact of three-loop RGEs alone (green arrows) and the subsequent inclusion of all the threshold corrections (blue). Corrections that are less than 1% at all the steps are not shown.

The first three-loop analysis of the MSSM was performed in [13] and it was found that the effect of three-loop running on the SUSY spectrum is small for weakly interacting particles but is larger for squark masses (1-5 %). However, two-loop threshold corrections were not available at those times. In this talk I will present the results of a more consistent study based on modified version of the SOFTSUSY [14] code³ which takes into account both three-loop RGEs and two-loop decoupling corrections discussed earlier.

As a benchmark scenario the so-called SPS4 point [15] was chosen. For this point one has $m_0 = 400$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$ at the GUT scale, and $\tan\beta = 50$ at the electroweak scale. In order to visualize the impact of additional two-loop terms in decoupling corrections and three-loop contributions to beta-functions, the spectrum produced by the SOFTSUSY code is presented for four cases.

In Fig. 3a one sees how two-loop corrections to b - (green arrows) and t -quark (blue arrows) masses modify the initial spectrum of SPS4 obtained with the original SOFTSUSY 3.1 code. Quark Yukawa couplings influence significantly the running of soft masses for the corresponding Higgs bosons. This, in turn, leads to the relatively large shifts in the masses of heavy higgses (4-5 %) and higgsinos (2-3 %). However, the overall result for heavy higgs masses is small (less than 1 %) due to cancellations. Inclusion of $\delta^{(2)}\zeta_{\alpha_s}$ (red arrows) slightly lowers the masses of third generation squarks below the initial value.

This picture for quark masses and strong coupling was obtained without the inclusion of three-loop terms in RGEs. Figure 3b shows the impact of these additional terms. First of all, for comparison with the results of Ref. [13] the spectrum was calculated without two-loop threshold corrections (green arrows). As it was noticed by the previous authors the corrections due to three-loop terms are small and mostly influence strongly interacting particles (by 1-2 %). After the inclusion of calculated two-loop decoupling corrections one can see additional shifts for squarks, gluino, and neutralino/stop squark with large higgsino component. In the end, one has 1-2 % overall correction to the masses of strongly interacting particles and 3 % correction to the higgsino masses. It is also interesting to note that the inclusion of three-loop RGEs

³Available from the author.

lowers the value of the lightest higgs boson mass from 114 GeV down to 113 GeV (not shown in Fig. 3b).

4 Summary and Conclusion

Some two-loop threshold corrections to the SM parameters were calculated and numerically studied in a wide region of the parameters space of the MSSM. It turns out that for the considered region two-loop contributions to the decoupling constants of the strong coupling and heavy quark masses are of the order of 2-4 % and do not depend significantly on $\tan\beta$. Since tau-lepton does not participate in strong interactions, the corresponding correction to its mass is smaller than that of quarks. Nevertheless, for large $\tan\beta$ it can reach the value of 1 % which exceeds current experimental uncertainty of the tau-lepton pole mass.

A proper way to use the above-mentioned two-loop quantities is to incorporate them together with three-loop RGEs in a code used to calculate the SUSY mass spectrum. For this purpose the SOFTSUSY package has been modified and it was found how the corrections influence the spectrum. A numerical study was performed for a particular scenario SPS4 with large $\tan\beta$ for which certain decoupling corrections are expected to be large. The overall effect of the three-loop running on the mass spectrum turns out to be small and does not exceed a few per cent. In comparison with previous studies it was found that the inclusion of the decoupling corrections besides lowering squark masses leads to a decrease in gluino and higgsino masses.

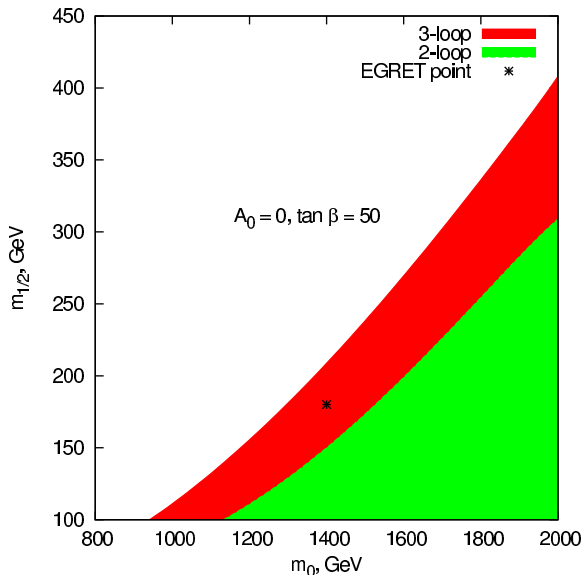


Figure 4: Forbidden regions in the $m_0 - m_{1/2}$ plane where no EWSB occurs. The green region was obtained with the help of two-loop RGEs and the red one — with three-loop terms. It is clear that the so-called EGRET [16] point is not allowed by three-loop analysis.

It is also interesting to study the influence on allowed regions in the parameter space. For example, the boundary that separates the regions with and without electroweak symmetry breaking (EWSB) can be significantly shifted. Figure 4 shows how the forbidden region increases after taking into account three-loop RGEs and the corresponding threshold effects. The so-called EGRET point proposed in [16] is also showed and it is clear that three-loop evolution excludes it. This result should not be taken seriously. However, one should be careful when choosing particular values of parameters near boundaries of the allowed region.

Moreover, although being small the calculated corrections give us an opportunity to estimate

theoretical uncertainties of the SUSY parameters fitted with the help of two-loop RGEs. This seems to be more reliable than (or at least complementary to) the comparison between the results of different computer codes.

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