Quasiattractor in models of new and chaotic inflation

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Abstract

We use a new method parametric attractor with drifting critical points to describe inflation with $\lambda(\phi^2 - v^2)^2$ potential of the scalar field. This method allows us to easily consider the inflaton fluctuations. We find the values of potential parameters and the mass of scalar field.

1 Introduction

Inflation is the exponential expansion of the Universe, which has become the standard model for the early stage of the Universe evolution before the Big Bang [1]-[5]. The inflation hypothesis was devised to explain the classic problem of the big bang cosmology. These are the flatness of the Universe, homogeneous and isotropic Universe in accordance with the cosmological principle. The scalar field which is responsible for inflation is called the inflaton. This hypothesis also explains the origin of the large-scale structure of the Universe. Quantum fluctuations of the inflaton in the early epoch became the galaxies and its clusters.

Presently, in cosmology there is a problem in determining the parameters of the inflaton. In this respect it would be useful to have a complete arsenal of effective methods in order to describe various characteristics at the inflationary stage. We use the new method quasiattractor. The notion of "quasiattractor" refers to the stable critical point of an autonomous system with external parameters slowly drifting with the evolution. This method was offered for the case of a quadratic potential [6], in order to generalize and develop investigations considering the dependence of cosmological evolution on initial data [7]-[11]. We apply this approach to $\lambda(\phi^2 - v^2)^2$ potential. Such a potential allows us to essentially expand the region of admissible values of the potential parameters consistent with the data. This fact significantly increases the viability of the model. In section 2 we give mathematical aspects of the model. In section 3 we compare data with experiments and derive the parameters of the inflaton. In section 4 we discuss the results. This work is based on the paper [12].

2 Mathematical aspects

Let us consider the action of the inflaton in this form

$$S = \int dx^4 \sqrt{-g} \left\{ \frac{1}{2} \, \partial_\mu \phi \, \partial^\mu \phi - V(\phi) \right\}, \tag{1}$$

with the potential

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$$V = \frac{\lambda}{4}(\phi^2 - v^2)^2. \tag{2}$$

The evolution of the isotropic Universe is described by a Friedmann- Lemaitre-Robertson-Walker metric

$$g_{\mu\nu} = \mathrm{diag}(1, -a^2(t), -a^2(t), -a^2(t)).$$

Where $a(t)$ is the scale factor.

The equations of motion read off as

$$\ddot{\phi} = -3H\dot{\phi} - \lambda\phi(\phi^2 - v^2), \tag{3}$$

$$\dot{H} = -4\pi G\dot{\phi}^2. \tag{4}$$

The Friedmann relation which can be derived from equations of motion is

$$H^2 = \frac{4\pi G}{3}\left\{\dot{\phi}^2 + \frac{1}{2}\lambda(\phi^2 - v^2)^2\right\}. \tag{5}$$

Where the Hubble constant $H = \dot{a}/a$. We can say that the system "rolls" in the potential well. If the evolution occurs from large field to the global minimum it is called chaotic inflation, if the field "falls" from the vicinity of a local maximum at small field then it is new inflation.

At present, the basic tool of such studies is the slow-roll approximation in the field equations of the inflation [5, 13, 14]. In this method the acceleration term $\ddot{\phi}$ can be neglected in comparison with the friction term $-3H\dot{\phi}$ in the equation of motion (3). But we follow another way which was suggested in [6] for the quadratic potential and applied to the quartic potential [15] (see also [13]-[20]).

Let us introduce new dimensionless variables with presumed properties of scaling

$$x = \frac{\kappa}{\sqrt{6}}\frac{\dot{\phi}}{H}, \tag{6}$$

$$y = \sqrt[4]{\frac{\lambda}{12}}\sqrt{\frac{\kappa}{H}}\sqrt{|\phi^2 - v^2|}, \tag{7}$$

$$z = \frac{\sqrt[4]{3\lambda}}{\sqrt{\kappa H}}, \tag{8}$$

$$u = \frac{\kappa v}{\sqrt{6}}, \tag{9}$$

where $\kappa^2 = 8\pi G$.

The differential equations of motion take the form

$$x' = 3x^3 - 3x - 2y^2 z\sqrt{y^2 + u^2 z^2}, \tag{10}$$

$$yy' = \frac{3}{2}x^2 y^2 + xz\sqrt{y^2 + u^2 z^2}, \tag{11}$$

$$z' = \frac{3}{2}x^2 z. \tag{12}$$

The Friedmann equation is

$$x^2 + y^4 = 1. \tag{13}$$

We can say that the terms x^2 and y^4 are kinetic and potential terms respectively. Here the prime denotes the derivative with respect to $N = \ln(a/a_{\mathrm{initial}})$. We marked the initial state by index "initial" and the end of inflation by index "end". The equations are simplified, since they are already differential equations of the first order, though they are nonhomogeneous, but they are easier for analysis than the initial ones. The equations (10) and (11) can be considered as an autonomous system. The system could reach stable critical points on a phase plane $\{x, y\}$. The paths converge to these points, being the attractors. The position of the critical point is not fixed, since it is determined by the control parameter, which evolves and displaces the critical point. But the evolution velocity of the control parameters is slow enough in order to consider the displacement of the point in the phase space as driftage. Thus, the system motion is the following: the system very quickly "falls" to the quasiattractor in the phase space, and then the critical point slowly drifts during the evolution. The numerical analysis shows that the system is stable under some definite conditions. The control parameter of an autonomous system is the slowly varying quantity z. The system motion is appropriated by the evolution of control parameters, and the system seems to lose some degrees of freedom.

Our first task is to search for such critical points. The equations for the critical point in the physical case are $x' = 0,\ y' = 0$ (notice that the solution $x = 0,\ y = 0$ is not physical). The equations for the critical point are reduced to the single equation in x with parameter z, namely

$$\frac{3}{2}x_c\sqrt{1-x_c^2} + z\sqrt{\sqrt{1-x_c^2}+u^2z^2} = 0. \tag{14}$$

The Friedmann relation gives $x_c\delta x + 2y_c^3\delta y = 0$ which means that there is the eigenvector $(2y_c^3 \quad -x_c)^T$. The evolution goes according to the law

$$\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = C \begin{pmatrix} 2y_c^3 \\ -x_c \end{pmatrix} e^{\mathcal{B}N}, \tag{15}$$

where δx and δy are deviations from the critical point $x_c,\ y_c$, i.e. $x = x_c + \delta x$ and $y = y_c + \delta y$. And

$$\mathcal{B} = -3 + 6x_c^2 - \frac{2}{3}\frac{z^2}{y_c^2}. \tag{16}$$

Then, there is a question of the stability of given system. We require $\mathcal{B} < 0$ for the stability of attractor. This condition is valid at small values of x and z and $\mathcal{B}$ is certainly less than zero.

Let us consider Universe inflation due to the inflaton with the chosen potential. The condition of accelerated expansion is the following $\ddot{a} > 0$ which means that $3x^2 < 1$. Accordingly, such the expansion regime ends up with

$$x_{\text{end}}^2 = \frac{1}{3}, \tag{17}$$

$$y_{\text{end}}^4 = \frac{2}{3}, \tag{18}$$

$$z_{\text{end}}^2 = \frac{\sqrt{3u^2+1}-1}{u^2\sqrt{6}}. \tag{19}$$

3 Comparing data with the experiments

In order to find numerical values of the theory parameters one should compare it with the observational data. Experiment measures the inhomogeneity of the cosmic microwave background radiation, related to the inhomogeneity of matter, hence we need to find the distribution of the inflation inhomogeneity, which leads to the matter inhomogeneity at the stage of reheating. Such inhomogeneity is given by the quantum fluctuations of the inflaton. Then, the spectral densities of scalar and tensor perturbations are

$$P_S(k) = \left(\frac{H}{2\pi}\right)^2\left(\frac{H}{\dot{\phi}}\right)^2 = \frac{\lambda}{8\pi^2}\frac{1}{x_c^2z^4}, \tag{20}$$

$$P_T(k) = 8\kappa^2\left(\frac{H}{2\pi}\right)^2 = \frac{6\lambda}{\pi^2}\frac{1}{z^4}, \tag{21}$$

where the wave vector k is given by the Hubble rate at the exit of the fluctuations from the horizon.

Consider the ratio r determining the relative contribution of tensor spectrum with respect to scalar spectrum and introduce the spectral index n_S as

$$r = \frac{P_T(k)}{P_S(k)} = 48\, x_c^2, \tag{22}$$

$$n_S - 1 = \frac{d \ln P_S}{d \ln k} = \frac{4(9x_c^2 - z^2)}{3(3x_c^2 - 1)}, \tag{23}$$

one can see that $\ln(k/k_{\text{end}}) = N - 2\ln(z/z_{\text{end}})$, so the differentiation with respect to k is reduced to the differentiation with respect to parameter z.

The expression for $N_{\text{total}} = \ln(a_{\text{end}}/a_{\text{initial}})$ is simplified to

$$N_{\text{total}} = \frac{2}{3}\int_{z_{\text{initial}}}^{z_{\text{end}}} \frac{dz}{x_c^2 z} \approx \frac{3}{4}\left(\frac{1}{z_{\text{initial}}^2} - u^2 \ln \frac{1 + u^2 z_{\text{initial}}^2}{u^2 z_{\text{initial}}^2}\right). \tag{24}$$

We considered the chaotic inflation, and at $x_c^2 \ll 1$ one gets $x_c^2 \approx \frac{4}{9} z^2(1 + u^2 z^2)$, but for the new inflation we should put $x_c^2 \approx \frac{4}{9} z^2(-1 + u^2 z^2)$ because in this case the sign of y_c^2 is changed.

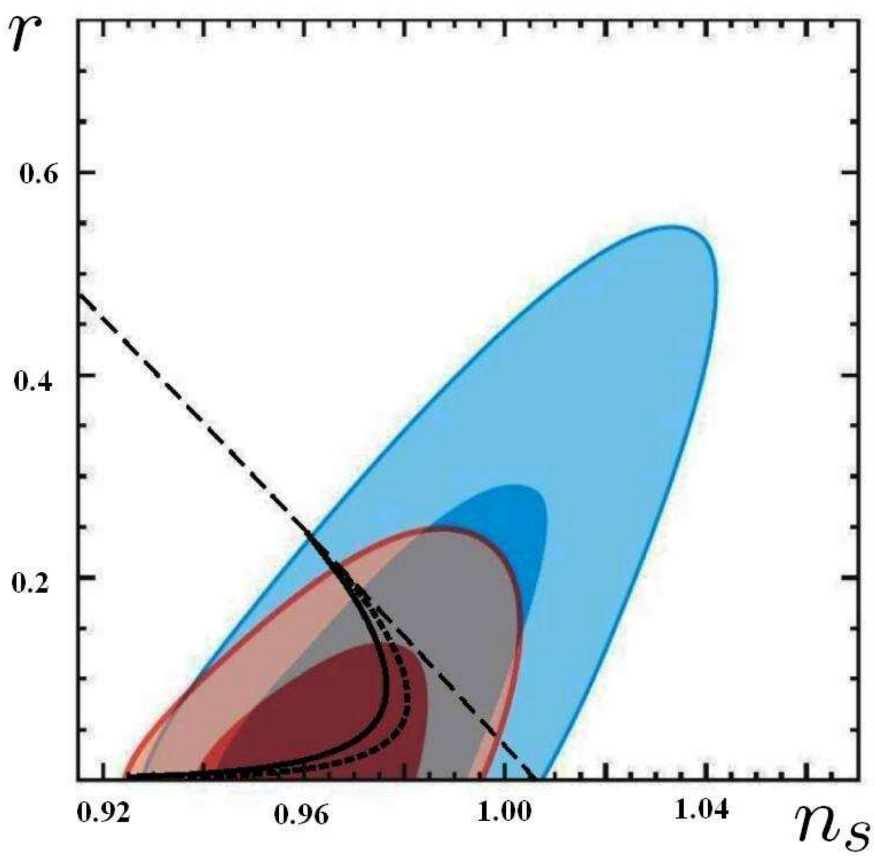

Figure 1: Experimental data

Limiting cases are in complete consistency with the consideration of these cases in other approaches

$$r = \frac{16}{N}, \quad n_S - 1 = -\frac{3}{N} \qquad \text{at} \quad u^2 \to 0, \tag{25}$$

$$r = \frac{8}{N}, \quad n_S - 1 = -\frac{2}{N} \qquad \text{at} \quad u^2 \to \infty. \tag{26}$$

Data of the WMAP collaboration [21]-[24] in the plane of the spectral parameter and the fraction of the tensor term $\{n_S, r\}$ in fluctuations of density in comparison with theoretical predictions at different values of e-folding, namely $N = 60$ (thick solid line) and $N = 70$ (dotted line) in Fig. 1. The panel shows the WMAP data after 5 years of data taking the confidence levels equal to $1-\sigma$ and $2-\sigma$ in comparison with further constraints following from BAO [25] and SN [26]-[29] experiments. Dashed line is the border of the applicability region for this potential. The equation of this line is $n_S - 1 = -3r/(16-r)$ and it corresponds to $u^2 = 0$. The region above correspond to $u^2 < 0$, it is irrelevant to the present work.

From the analysis of data we can obtain quite wide limits of possible values for parameters of the model potential. Namely, using the WMAP data we obtain at $1-\sigma$ level

$$N = 60^{+40}_{-20} \qquad 25 \leqslant u^2 \leqslant \infty, \tag{27}$$

so $v > 12 m_{Pl}$, where $m_{Pl} = 1/\sqrt{8\pi G}$ is Planck mass. However, the amount of e-folding is, in fact, limited by the actual history of the Universe evolution after inflation, so that the analysis leads to the typical value of $N \approx 60$ [30].

Then, for $N = 60$

$$0 \leqslant \lambda \leqslant 9.0 \cdot 10^{-14}, \tag{28}$$

and we can calculate the inflaton mass $m^2 = 2\lambda v^2$

$$1 \cdot 10^{13}\ \text{GeV} \leqslant m \leqslant 1.7 \cdot 10^{13}\ \text{GeV}. \tag{29}$$

You can see, that one could extract the mass of the inflation corresponding to maximal definiteness for all of the potential parameters. Our results are in agreement with the precise analysis of a complete data set previously performed in [31, 32].

4 Conclusion

Thus, the model has allowed us to consider the scenarios of chaotic and new inflation in the framework of the quasiattractor method, which has enabled us to quite elegantly calculate the recently observed inhomogeneity of the cosmic microwave background and distribution of matter in the Universe. We have shown that such model is consistent with the observational data. One can see, of course, that this model of the potential parametrically cannot satisfy all of the experimentally admissible values of n_S and r within the empirical uncertainties, but these restrictions are not critical within the accuracy of measurements, and the given potential seems to be consistent with the current data. We have obtained also, that observational data on the inhomogeneity of the Universe corresponds to the time of forming the inflation fluctuations, when the Universe expands approximately e^{60} times to the end of inflation, which is in agreement with other estimations. We have also precisely enough determined the inflaton mass.

References

[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[2] A. D. Linde, Phys. Lett. B **108**, 389 (1982).

[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[4] A. D. Linde, Phys. Lett. B **129**, 177 (1983).

[5] A. Linde, Lect. Notes Phys. **738**, 1 (2008) [arXiv:0705.0164 [hep-th]].

[6] L. A. Urena-Lopez and M. J. Reyes-Ibarra, arXiv:0709.3996 [astro-ph].

[7] V. A. Belinsky, I. M. Khalatnikov, L. P. Grishchuk and Y. B. Zeldovich, Phys. Lett. B **155**, 232 (1985).

[8] T. Piran, R. M. Williams, Phys. Lett. B **163**, 331 (1985).

[9] L. A. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Lett. B **157**, 361 (1985).

[10] A. de la Macorra and G. Piccinelli, Phys. Rev. D **61**, 123503 (2000) [arXiv:hep-ph/9909459].

[11] R. H. Brandenberger and J. H. Kung, Phys. Rev. D **42**, 1008 (1990).

[12] V. V. Kiselev and S. A. Timofeev, Gen. Rel. Grav. **42**, 183 (2010) [arXiv:0905.4353 [gr-qc]].

[13] D. Boyanovsky, H. J. de Vega and N. G. Sanchez, Phys. Rev. D **73**, 023008 (2006) [arXiv:astro-ph/0507595].

[14] D. Boyanovsky, H. J. de Vega and D. J. Schwarz, Ann. Rev. Nucl. Part. Sci. **56**, 441 (2006) [arXiv:hep-ph/0602002].

[15] V. V. Kiselev and S. A. Timofeev, arXiv:0801.2453 [gr-qc].

[16] C. Wetterich, Nucl. Phys. B **302**, 668 (1988).

[17] E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. D **57**, 4686 (1998) [arXiv:gr-qc/9711068].

[18] P. G. Ferreira and M. Joyce, Phys. Rev. D **58**, 023503 (1998) [arXiv:astro-ph/9711102].

[19] A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, 2076 (2000) [arXiv:astro-ph/9908085].

[20] V. V. Kiselev, JCAP **0801**, 019 (2008) [arXiv:gr-qc/0611064].

[21] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **148**, 175 (2003) [arXiv:astro-ph/0302209]

[22] D. N. Spergel *et al.*, arXiv:astro-ph/0603449.

[23] J. Dunkley *et al.* [WMAP Collaboration], arXiv:0803.0586 [astro-ph].

[24] E. Komatsu *et al.* [WMAP Collaboration], arXiv:0803.0547 [astro-ph].

[25] W. J. Percival, S. Cole, D. J. Eisenstein, R. C. Nichol, J. A. Peacock, A. C. Pope and A. S. Szalay, Mon. Not. Roy. Astron. Soc. **381**, 1053 (2007) [arXiv:0705.3323 [astro-ph]].

[26] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astrophys. J. **607**, 665 (2004) [arXiv:astro-ph/0402512].

[27] A. G. Riess *et al.*, Astrophys. J. **659**, 98 (2007) [arXiv:astro-ph/0611572].

[28] P. Astier *et al.* [The SNLS Collaboration], Astron. Astrophys. **447**, 31 (2006) [arXiv:astro-ph/0510447].

[29] W. M. Wood-Vasey *et al.* [ESSENCE Collaboration], Astrophys. J. **666**, 694 (2007) [arXiv:astro-ph/0701041].

[30] A. R. Liddle and S. M. Leach, Phys. Rev. D **68**, 103503 (2003) [arXiv:astro-ph/0305263].

[31] C. Destri, H. J. de Vega and N. G. Sanchez, Phys. Rev. D **77**, 043509 (2008) [arXiv:astro-ph/0703417].

[32] C. Destri, H. J. de Vega and N. G. Sanchez, Phys. Rev. D **78**, 023013 (2008) [arXiv:0804.2387 [astro-ph]].