Hypermagnetic field as a seed of Maxwellian field in early Universe: magnetic helicity transfer

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Abstract

We study how the helicity of hypermagnetic fields is related to the magnetic helicity of Maxwellian fields during the electroweak phase transition (EWPT). We show that during this phase transition on the surface separating the phases a separation of magnetic helicity exists. The magnetic helicity being collected in Maxwellian phase in the course of this separation is conserved then in further expansion of the Universe and the subsequent formation of galaxies.

The magnetic helicity is a relatively new and very attractive point of interest in cosmic Magneto-HydroDynamics (MHD) and dynamo theory. The point is that the magnetic helicity $\int \mathbf{AH}\mathrm{d}^3x$ where $\mathbf{H}$ is the magnetic field and $\mathbf{A}$ is the vector potential is an inviscid integral of motion in MHD. Relatively recently experts in cosmic MHD recognized that the magnetic helicity conservation is much more restrictive in astrophysical objects than the energy conservation [1].

The cosmological magnetic field as a seed of galactic magnetic fields and its helicity can be formed as a result of phase transitions in the early Universe and, in particular, in the electroweak phase transition (EWPT). In this phase transition the hypermagnetic field converts into the Maxwellian electromagnetic field.

For dynamo mechanism that provides the amplification of galactic magnetic fields [2] the helicity coefficent α is the most important parameter. In the absence of rotation and vortices in early Universe such hydrodynamical *pseudoscalar* parameter $\alpha_{MHD} = - < \mathbf{V}\cdot(\nabla\times\mathbf{V}) > /3$ vanishes while a new helicity *scalar* coefficent α originated by the parity violation in SM of particle physics arises [3, 4, 5].

The helicity coefficent α_Y for a long-ranged massless hypercharge field Y_μ has different forms in dependence on whether we take into account the anomalous Chern-Simons term in SM Lagrangian [3, 6], or not as we did in paper [5]. For the problem of the conversion of hypermagnetic helicity into the magnetic one such a concrete choice of α_Y does not matter since the final result for transferred helicity does not depend on α_Y as we see below.

This *scalar* coefficent enters the parity violating (last) term in generalized Maxwell equation [3, 5]:

$$-\frac{\partial \mathbf{E}_Y}{\partial t} + \nabla\times\mathbf{B}_Y = \sigma_{cond}\left[\mathbf{E}_Y + \mathbf{V}\times\mathbf{B}_Y + \alpha_Y\mathbf{B}_Y\right] \ . \tag{1}$$

Neglecting in MHD the displacement current $\partial\mathbf{E}_Y/\partial t$ and using the Maxwell equation $\partial_t\mathbf{B}_Y = -\nabla\times\mathbf{E}_Y$ we easily derive in the rest frame of the medium as a whole ($\mathbf{V}=0$) both the Faraday equation for the the hypermagnetic field $\mathbf{B}_Y$ before EWPT,

$$\frac{\partial \mathbf{B}_Y}{\partial t} = \nabla\times\alpha_Y\mathbf{B}_Y + \eta_Y\nabla^2\mathbf{B}_Y, \tag{2}$$

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and the analogous one for the Maxwellian field $\mathbf{B}$ after EWPT but with different coefficent α [4]. Here $\eta_Y \approx \eta = (4\pi\sigma_{cond})^{-1}$ is the hypermagnetic (magnetic) diffusion coefficent.

Let us consider a bubble (an embryo of the Maxwell phase) of the radius R, inside of the hot plasma in the early Universe at the EWPT moment with the temperature $T_{EW} \sim 100$ GeV. Let us assume that this bubble is growing with the constant velocity, $R(t) = v(t - t_{EW})$, where the velocity v itself ($v = 0.1 - 1$ according to [7]) is unessential (to be cancelled) in the solution to our problem.

It is important for our calculation that the value $(t - t_{EW})/t_{EW} \ll 1$ is small, or that the temperature during the phase transition remains constant at the moment $t_{EW} = M_0/2T_{EW}^2 = 0.23 \times 10^{-10}$ c , where $M_0 = M_{Pl}/1.66\sqrt{g^*}$ is given by the Plank mass $M_{Pl} = 1.2 \times 10^{19}$ GeV and by the degree of freedom $g^* \sim 100$. This implies that the radius of the bubble is much less than the horizon size ($2t_{EW} = l_H = 1.44$ cm), $R \ll l_H$. More precisely, we shall assume that the radius of the bubble is much less than the scale of the mean hypermagnetic field, $R \ll \eta_Y/\alpha_Y \ll l_H$.

Multiplying Eq. (2) and its analogue for Maxwellian field by the corresponding vector potential and adding the analogous construction produced by evolution equation governing the vector potential (multiplied by hypermagnetic or magnetic field) after the integration over the space we get the evolution equation for the total helicity $H = \int(\mathbf{B}\cdot\mathbf{A})d^3x + \int(\mathbf{B}_Y\cdot\mathbf{Y})d^3x$, where the integration is carried over the domains with Maxwell phase and hypermagnetic field correspondingly. This equation takes the form:

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{dt}} = \quad & -2\int(\mathbf{E}\cdot\mathbf{B})\mathrm{d}^3x - \\ & - \oint_S((\mathbf{E}\times\mathbf{A} + A_0\mathbf{B})\cdot\mathbf{n})\mathrm{d}^2S + \ldots, \end{aligned} \tag{3}$$

where the dots mean analogous terms for hypermagnetic field. We take into account the surface integrals which are omitted in problems for a monophase medium [8] as integrals over an infinite boundary of the domain. In our problem namely these integrals determine the flow of the helicity through the boundary of a bubble of the radius R, on which a separation of the helicity takes place. Accounting for the boundary condition $A_\mu = \cos\theta_W Y_\mu$, where $\sin^2\theta_W = 0.23$ is the parameter of the standard Weinberg-Salam model, the integrals above are calculated over the surface as the following sum of them [9]:

$$\frac{\mathrm{d}H_Y}{\mathrm{dt}} = -\sin^2\theta_W\oint(\mathbf{E}_Y\times\mathbf{Y} + Y_0\mathbf{B}_Y)\mathbf{n}_Y\mathrm{d}^2S, \tag{4}$$

where the unit normal vector $\mathbf{n}_Y = -\hat{\mathbf{e}}_r = (-1,0,0)$ is directed inwards the bubble with Maxwellian phase (the phase with broken symmetry).

The flow of hypermagnetic helicity density, penetrated inside the bubble through the surface at the moment of electroweak phase transition, is the pseudovector given by the formula

$$\mathbf{S} = \mathbf{n}_Y h_Y(t) = \mathbf{n}_Y\left(\frac{1}{4\pi R^2(t)d}\right)\int_{t_{EW}}^{t}\mathrm{dt}\frac{\mathrm{d}H_Y(t)}{\mathrm{dt}}. \tag{5}$$

This flow is analogous to the vector flow of the energy of a flat electromagnetic wave $\mathbf{S} = W\mathbf{n}$, where $W = (E^2 + B^2)/8\pi$ is the energy density of the field. Here $4\pi dR^2(t)$ is the volume of a thin spherical layer with the thickness d of the domain wall separating the two phases.

It is not difficult to prove that the surface integrals are equal to zero, i.e. there is no separation of the helicity if we substitute the Chern-Simons wave for hypermagnetic field, $Y_0 = Y_z = 0$, $Y_x = Y(t)\sin k_0 z$, $Y_y = Y(t)\cos k_0 z$. But in the case of 3D- field with nonzero helicity the considered integrals are nontrivial. Let us consider the following potential of the hypermagnetic field with the number of the linked loops equal to n :

$$\begin{aligned}
Y_r(t,\rho,\theta) = &\quad \frac{-Y(t)\cos\theta}{(\rho^2+1)^2},\\
Y_\theta(t,\rho,\theta) = &\quad \frac{Y(t)\sin\theta}{(\rho^2+1)^2}\Big[1+B(\rho-1)^2+b(\rho-1)^3\Big],\\
Y_\phi(t,\rho,\theta) = &\quad \frac{-Y(t)n\sin\theta}{(\rho^2+1)^2}\Big[\rho+C(\rho-1)^2+\\
&\quad +(C+c)(\rho-1)^3\Big],
\end{aligned} \tag{6}$$

where $Y(t)$ is the hypermagnetic field amplitude and $\rho = r/R$.

From the Faraday equation Eq. (2) governing the hypermagnetic field $\mathbf{B}_Y = \nabla\times\mathbf{Y}$ we can find the constant coefficents $B=-1$, $b=-2$, $c=-5/3$ as well as $C(t)$ changing over time due to the bubble expansion $R(t)$ [9]:

$$C(t) = \frac{2(n-n^{-1})\alpha_Y R^{-1}+4\eta_Y R^{-2}}{n\alpha_Y R^{-1}+2\eta_Y R^{-2}}. \tag{7}$$

A straightforward calculation of the surface term (4) we are looking for gives the following equation:

$$\frac{\mathrm{d}H_Y(t)}{\mathrm{dt}} = \frac{2\pi\sin^2\theta_W n}{3}R(t)Y(t)\int_{t_{EW}}^{t}\frac{Y(t')}{R(t')}\mathrm{dt}', \tag{8}$$

where we substituted in the expression $\mathbf{E}_Y = -\partial\mathbf{Y}/\partial t - \nabla Y_0$ the gradient ∇Y_0,

$$\begin{aligned}
\nabla Y_0 = &\quad \frac{1}{R(t)}\int_{t_{EW}}^{t}\frac{Y(t')\mathrm{d}t'}{R(t')}\Big[\frac{4\sin\theta\hat{\mathbf{e}}_\theta}{(\rho^2+1)^3} -\\
&\quad -\frac{4\cos\theta(1-5\rho^2)\hat{\mathbf{e}}_r}{(\rho^2+1)^4}\Big],
\end{aligned} \tag{9}$$

and took into account that in the case of the axial-symmetric configuration (6) the vector B_r^Y is independent of the coordinate ϕ. The values in Eq. (8) including $\nabla Y_0\times\mathbf{Y} = \hat{\mathbf{e}}_r(\nabla Y_0)_\theta Y_\phi$ are calculated at the surface of bubble $\rho = 1$.

Hence the problem is reduced to the calculation $Y(t)$ from the Faraday equation (2) which for the considered potential (6) takes the form (coming from the azimuthal component $\partial_t B_\phi = ...$) :

$$\frac{\dot{Y}}{Y} - \frac{\dot{R}}{R} = \frac{\alpha_Y n(2-C)}{R}\ . \tag{10}$$

Substituting the parameter (7) into Eq. (10) one obtains the ordinary differential equation for the amplitude $Y(t)$,

$$\frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{R}(t)}{R(t)} = \frac{2\alpha_Y^2}{n\alpha_Y R(t)+2\eta_Y}. \tag{11}$$

In the realistic situation of finite conductivity a scale of the mean hypermagnetic field $\Lambda = \kappa\eta_Y/\alpha_Y$, where $\kappa\geq 1$, should be much bigger than the diameter of the bubble in the new phase, i.e. the following inequality has to be satisfied: $\alpha_Y R(t)\ll\kappa\eta_Y$. If a more stronger condition $\alpha_Y R(t)\ll 2\eta_Y/n\leq\kappa\eta_Y$ is fulfilled, then from (11) for the function $B_Y(t) = Y(t)/R(t)$ we get

$$B_Y(t) = B_Y(t_{EW})\exp\left[\left(\frac{\alpha_Y^2}{\eta_Y}\right)(t-t_{EW})\right], \tag{12}$$

where $B_Y(t_{EW})$ is the hypermagnetic field amplitude on the scale of the bubble, $\alpha_Y = \alpha_Y(T_{EW})$, $\eta_Y = \eta_Y(T_{EW})$ are the constant coefficients at the moment of the phase transition,

$(t - t_{EW})/t_{RW} \ll 1$ is a small parameter for self-consistency of our problem (see above). This inevitably leads to a constant value of hypermagnetic field $B_Y(t) \approx B_Y(t_{EW})$ during EWPT, *or the helicity density transferred from the symmetric phase actually does not depend on a concrete choice of the helicity parameter* α_Y[1].

Substituting the amplitude of the hypercharge field $Y(t) = B_Y(t)R(t)$ on the surface of the phase separation (12) into the expression of the surface integral (8), after the integration over time and division by the volume of the spherical layer with the thickness d we get from (5) the value of the flow of hypermagnetic helicity density through the surface of the bubble,

$$\frac{h_Y(t)}{G^2 cm} = \frac{5 \times 10^{-3} n}{d(cm)} \left(\frac{B_Y(t_{EW})}{1\ G}\right)^2 \left(\frac{t - t_{EW}}{t_{EW}}\right)^2 . \tag{13}$$

Let us note that in order to avoid the screening of the hyperelectric field $\mathbf{E}_Y$ and the temporal component Y_0 over the surface of the bubble, the thickness d of the domain wall should be less than the Debye radius, $d < r_D = \sqrt{3T_{EW}/4\pi e^2 n_e} \sim 10/T_{EW}$, that allows to estimate the factor d^{-1} in the formula (13) as $d^{-1}(cm) > 10^{15}/2$. This means that a moderate hypermagnetic field $B_Y(t_{EW})$ provides a huge flow of the helicity density (13).

Indeed, substituting into (13) the value of hypermagnetic field at the moment of phase transition $B_Y(t_{EW})$ estimated in [10] as $B_Y(t_{EW}) \sim 5 \times 10^{17}\ G$, one gets $h/G^2 cm > 6.25 \times 10^{47}[(t - t_{EW})/t_{EW}]^2$. Such huge value estimated at the moment of the growth of a bubble of the new phase, e.g, for $R(t)/l_H < [(t - t_{EW})/t_{EW}] \sim 10^{-6}$, accounting for the following conservation of the net global helicity summed over different protogalactic scales, occurs much bigger than the helicity density of galactic magnetic field $h_{gal} \sim 10^{11}\ G^2 cm$, (see also estimates of the primordial magnetic helicity in paper [11]).

The single bubble of the Maxwellian phase inside of ambient symmetric phase with the potential given by Eq. (6) near the boundary, is a reasonable approximation during the beginning of the phase transition before percolation (junction of bubbles). One can consider also another final step of the phase transition, when a new phase with broken symmetry prevails and a single bubble of the symmetric phase with hypermagnetic field inside exists. It is not hard to check that in this case the change of sign $\rho - 1 > 0$ to $\rho - 1 < 0$ in the potential (6) gives the same components of hypermagnetic field inside the bubble $\rho < 1$. Let us note that in the considered approximation (6) magnetic charges near the surface of the phase transition and over this surface itself are absent, $\nabla \cdot \mathbf{B}_Y = 0$.

For a single bubble of the symmetric phase the flow of the helicity density through the surface (5) preserves the value (13). Moreover, this flow does not change the sign after the direction of the flow is changed, $\mathbf{n}_Y \to -\mathbf{n}_Y = \hat{\mathbf{e}}_r = (1, 0, 0)$. This well corresponds to the meaning of the problem: magnetic helicity of the Maxwellian field rises, unless helicity of the hypermagnetic field inside the bubble goes down.

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