Cosmological density perturbations from conformal scalar field.

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Abstract

We consider a scenario in which primordial scalar perturbations are generated when complex conformal scalar field rolls down its negative quartic potential. Initially, these are the perturbations of the phase of this field; they are converted into the adiabatic perturbations at a later stage. A potentially dangerous feature of this scenario is the existence of perturbations in the radial field direction, which have red power spectrum. We show, however, that to the linear order in the small parameter — the quartic self-coupling — the infrared effects are completely harmless, as they can be absorbed into field redefinition. We then evaluate the statistical anisotropy inherent in the model due to the existence of the long-ranged radial perturbations. To the linear order in the quartic self-coupling the statistical anisotropy is free of the infrared effects. The latter show up at the quadratic order in the self-coupling and result in the mild (logarithmic) enhancement of the corresponding contribution to the statistical anisotropy. The resulting statistical anisotropy is a combination of a larger term which, however, decays as momentum increases, and a smaller term which is independent of momentum.

1 Introduction and summary

The two basic properties of primordial scalar perturbations in the Universe are approximate Gaussianity and approximate flatness of the power spectrum [1]. The first property strongly suggests that these perturbations originate from amplified vacuum fluctuations of nearly linear (i.e., weakly coupled) quantum field(s): free quantum field in its vacuum state obeys the Wick theorem, the defining property of Gaussian random field, while linear evolution in classical backgrounds does not induce the non-Gaussianity. The flatness of the power spectrum calls for some symmetry behind it. The best known candidate is the symmetry of the de Sitter space-time under spatial dilatations supplemented by time translations. This is the approximate symmetry of the inflating Universe [2], and, indeed, the inflationary mechanism of the generation of scalar perturbations [3] produces almost flat power spectrum. Another example is the symmetry (of the field equation) of the scalar theory with negative exponential scalar potential in flat space-time. This is the symmetry under space-time dilatations suppelemented by the shifts of the field. This symmetry remains approximately valid in slowly evolving, e.g., ekpyrotic [4] or "starting" [5] Universe, and the resulting perturbation spectrum is again almost flat[1] [6] (see also Ref. [7]).

Yet another symmetry that may be responsible for the approximate flatness of the scalar spectrum has been proposed in Ref. [11]. This is a combination of conformal invariance and

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[1] There are other mechanisms capable of producing flat or almost flat scalar spectrum [8, 9, 10]. In some cases, there is no obvious symmetry that guarantees the flatness, i.e., the scalar spectrum is flat accidentally.

a global symmetry. The simplest model involves complex scalar field ϕ, which is conformally coupled to gravity and for long enough time evolves in negative quartic potential

$$V(\phi) = -h^2|\phi|^4 . \tag{1}$$

A necessary condition for the absence of strong non-linearities at the classical level and strong coupling at the quantum level is

$$h < 1 .$$

As discussed in Section 2.2, there may or may not be stronger bounds on h. The global symmetry in this simplest case is $U(1)$ acting as $\phi \to \mathrm{e}^{i\alpha}\phi$. One assumes that the background space-time is homogeneous, isotropic and spatially flat, $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$. Then, due to conformal invariance, the dynamics of the field

$$\chi = a\phi$$

is independent of the evolution of the scale factor and in terms of the conformal coordinates $(\eta, \mathbf{x})$ proceeds in the same way as in Minkowski space-time. One begins with the homogeneous background field $\chi_c(\eta)$ that rolls down the negative quartic potential. Its late-time behavior is completely determined by conformal invariance. As we review in Section 2, this rolling field produces effective "horizon" for perturbations $\delta\chi$: at early times the linear perturbations oscillate in conformal time as modes of free massless (quantum) scalar field[2], while at late times the oscillatory behavior ceases to hold. The perturbations of the phase $\theta = \mathrm{Arg}\,\phi$ freeze out at the time of the "horizon" exit, and their power spectrum is flat after that,

$$\sqrt{\mathcal{P}_{\delta\theta}} = \frac{h}{2\pi} . \tag{2}$$

As discussed in Ref. [11], this property is a consequence of conformal and global symmetries.

The scenario proceeds with the assumption that the scalar potential $V(\phi)$ has, in fact, a minimum at some large value of $|\phi|$, and that the modulus of the field ϕ eventually gets relaxed to the minimum, see Fig. 1. The simplest option concerning further evolution of the perturbations $\delta\theta$ is that they are superhorizon in the conventional sense by the time the conformal rolling stage ends. Then they remain frozen out[3], and their power spectrum remains flat. At some much later cosmological epoch, the perturbations of the phase are converted into the adiabatic scalar perturbations; we discuss possible mechanisms responsible for that in Section 2.2. These mechansims do not distort the power spectrum, so the resulting adiabatic perturbations have flat primordial power spectrum. If conformal invariance is not exact at the rolling stage, the scalar power spectrum has small tilt, which depends on both the strength of the violation of conformal invariance and the evolution of the scale factor at the rolling stage [14].

A peculiar, and potentially dangerous property of the model is that the modulus of the rolling field also acquires perturbations. Super-"horizon" modes of the modulus (i.e., radial direction) have red power spectrum (see Section 2.3 for details),

$$\sqrt{\mathcal{P}_{|\phi|}(k)} \propto k^{-1} . \tag{3}$$

One consequence is that there are perturbations of the energy density with red spectrum right after the conformal rolling stage, but before the modulus freezes out at the minimum of $V(\phi)$. These are not dangerous, provided that the energy density of the field ϕ is small compared to

[2] An assumption here is that the rolling stage is long enough in conformal time, so that the modes of interest are indeed sub-"horizon" at early times. This assumption is non-trivial: the conformal rolling should occur at a cosmological epoch, preceeding the hot Big Bang stage, when the standard horizon problem is solved, at least formally, see Ref. [11] and Section 2.

[3] For contracting Universe, this property of superhorizon modes holds if the dominating matter has stiff equation of state, $w > 1$. This appears to be necessary for the viability of the bounce scenario anyway, see the discussion in Refs. [12, 13].

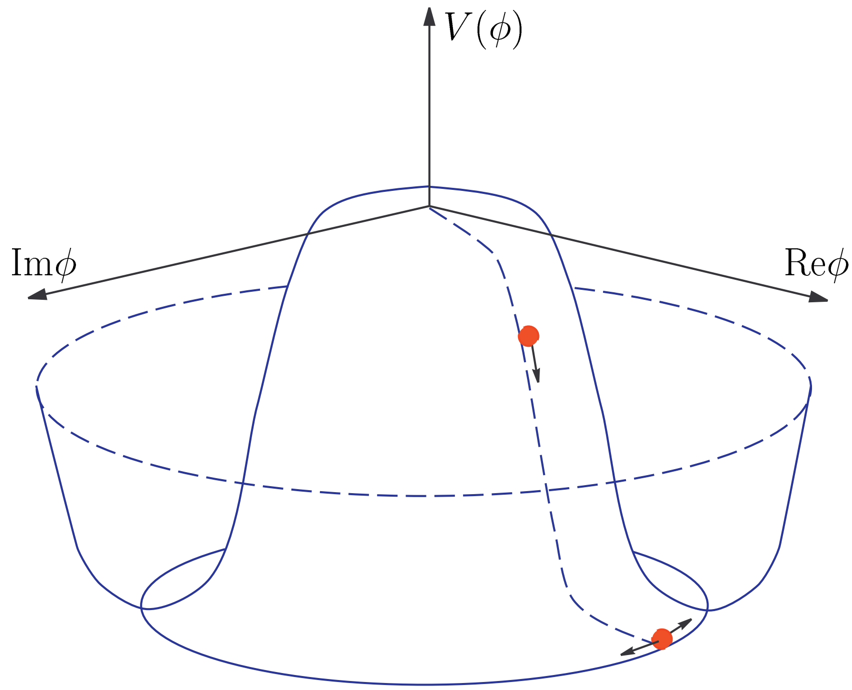

Figure 1: The scalar potential. Bullets show the evolution of the scalar field. Arrows at the end point at the bottom of the potential indicate that there are perturbations of the phase.

the total energy density at all times preceding the time when the modulus of this field settles down to the minimum of its potential, i.e., the cosmological evolution is governed by some other matter at that early epoch. In this paper we assume that this is indeed the case.

The second consequence is that the infrared radial modes interact with the perturbations of the phase, and in principle may have strong effect on the latter. This is precisely the issue we address in this paper. To this end, we study the perturbations of the phase in the presence of long-ranged perturbations of the modulus and make use of the spatial gradient expansion of the latter. We interpret the effects emerging at the zeroth and first orders in the gradient expansion as the local time shift and local Lorentz boost of the background field χ_c, the latter property meaning that the rolling background is approximately homogeneous in a local reference frame different from the cosmic frame where the metric has the standard Friedmann–Robertson–Walker form. Our interpretation makes it straightforward to obtain the expression for the perturbation of the phase, valid to these two orders of the gradient expansion. We then show that to the linear order in h, the infrared effects cancel out: the perturbations of the phase remain Gaussian random and have flat spectrum (2). Since the red power spectrum is characteristic of $\delta|\phi|$ and $\partial_i(\delta|\phi|)$, but not of higher spatial derivatives of $\delta|\phi|$, our analysis is sufficient to show that, in fact, there is no gross modification of the results of the linear analysis due to the effect of the infrared modes.

The large wavelength modes of $\delta|\phi|$ are not entirely negligible, however. The modes whose present wavelengths exceed the present Hubble size H_0^{-1} induce statistical anisotropy in the perturbations of the phase $\delta\theta$, and hence in the resulting adiabatic perturbations. To the linear order in h, the statistical anisotropy is generated at the second order in the gradient expansion, and hence it is free of the infrared effects. The infrared modes show up at the second order in h, and lead to mild enhancement of the statistical anisotropy at this order. Accordingly, the power spectrum of the adiabatic perturbation ζ has the following form,

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_0(k)\left(1 + c_1 \cdot h \cdot \frac{H_0}{k} \cdot \hat{k}_i \hat{k}_j w_{ij} - c_2 \cdot h^2 \cdot (\hat{\mathbf{k}}\mathbf{u})^2\right) . \quad (4)$$

In the first non-trivial term, w_{ij} is a traceless symmetric tensor of a general form with unit normalization, $w_{ij}w_{ij} = 1$, $\hat{\mathbf{k}}$ is a unit vector, $\hat{\mathbf{k}} = \mathbf{k}/k$, and c_1 is a constant of order 1 whose actual value is undetermined because of the cosmic variance. As we alluded to above, the deep infrared modes are irrelevant for this term. In the last term, $\mathbf{u}$ is some unit vector independent of w_{ij}, and the positive parameter c_2 is logarithmically enhanced due to the infrared effects. This is the first place where the deep infrared modes show up. Clearly, their effect is subdominant for small h.

The statistical anisotropy encoded in the second term in (4) is similar to that commonly discussed in inflationary context [15], and, indeed, generated in some concrete inflationary models [16]: it does not decay as momentum increases and has special tensorial form $(\hat{\mathbf{k}}\mathbf{u})^2$ with constant $\mathbf{u}$. On the other hand, the first non-trivial term in (4) has the general tensorial structure and decreases with momentum. The latter property is somewhat similar to the situation that occurs in cosmological models with the anisotropic expansion before inflation [17]. Overall, the statistical anisotropy (4) may be quite substantial, since there are no strong bounds on h at least for one mechanism of conversion of the phase perturbations into adiabatic ones, as discussed in Section 2.2.

Interestingly, the overall statistical anisotropy is a combination of a larger (order $\mathrm{O}(h)$) term which, however, decays as k increases, and a smaller (order $\mathrm{O}(h^2 \log \Lambda)$, where Λ is the infrared cutoff) term which is independent of k. We consider this feature as a potential smoking-gun property of our scenario.

It is worth noting that the non-linearity of the scalar potential gives rise to the non-Gaussianity of the perturbations of the phase $\delta\theta$, and hence the adiabatic perturbations in our scenario, over and beyond the non-Gaussianity that may be generated at the time when the phase perturbations get reprocessed into the adiabatic perturbations. In view of the result outlined above, this non-Gaussianity is not plagued by the infrared effects at the first non-trivial order in h, and must be fully calculable at this order. We do not consider the non-Gaussianity in this paper, since the derivative expansion approach we employ here is useless in this regard.

The paper is organized as follows. In Section 2 we review the linear analysis of the model. In Section 3 we study the effect of infrared modes of the modulus $\delta|\phi|$ on the perturbations of the phase $\delta\theta$ at the leading and subleading orders of the gradient expansion, and to the linear order in h. Statistical anisotropy, which is generated at the next order, is analysed in Section 4. We concude in Section 5.

2 Linear analysis

At the conformal rolling stage, the dynamics of the scalar field is governed by the action

$$S[\phi] = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + \frac{R}{6}\phi^*\phi - V(\phi)\right] .$$

where the scalar potential $V(\phi)$ is negative and has conformally invariant form (1). In terms of the field $\chi = a\phi$, the action in conformal coordinates is the same as in Minkowski space-time,

$$S[\chi] = \int d^3x \; d\eta \; \left[\eta^{\mu\nu}\partial_\mu\chi^*\partial_\nu\chi + h^2|\chi|^4\right] .$$

The field equation is

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|^2\chi = 0 . \tag{5}$$

Spatially homogeneous solutions approach the late-time attractor

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)} , \tag{6}$$

where η_* is an arbitrary real parameter, and we consider real solution, without loss of generailty. We take the solutuion (6) as the background. The meaning of the parameter η_* is that the field χ_c would run away to infinity as $\eta \to \eta_*$, if the scalar potential remained negative quartic at arbitrarily large fields.

2.1 Perturbations of phase

At the linearized level, the perturbations of the phase and modulus of the field ϕ decouple from each other. Let us begin with the perturbations of the phase, or, for real background (6), perturbations of the imaginary part

$$\chi_2 \equiv \text{Im}\ \chi\ .$$

They obey the linearized equation,

$$(\delta\chi_2)'' - \partial_i\partial_i\ \delta\chi_2 - 2h^2\chi_c^2\ \delta\chi_2 = 0\ , \tag{7}$$

where prime denotes the derivative with respect to η. Explicitly,

$$(\delta\chi_2)'' - \partial_i\partial_i\ \delta\chi_2 - \frac{2}{(\eta_* - \eta)^2}\ \delta\chi_2 = 0\ . \tag{8}$$

Let $\mathbf{k}$ be conformal momentum of perturbation. An important assumption of the entire scenario is that the rolling stage begins early enough, so that there is time at which the following inequality holds:

$$k(\eta_* - \eta) \gg 1\ . \tag{9}$$

Since the momenta k of cosmological significance are as small as the present Hubble parameter, this inequality means that the duration of the rolling stage in conformal time is longer than the conformal time elapsed from, say, the beginning of the hot Big Bang expansion to the present epoch. This is only possible if the hot Big Bang stage was preceded by some other epoch, at which the standard horizon problem is solved; the mechanism we discuss in this paper is meant to operate at that epoch. We note in passing that the latter property is inherent in most, if not all, mechanisms of the generation of cosmological perturbations.

Equation (8) is exactly the same as equation for minimally coupled massless scalar field in the de Sitter background. Nevertheless, let us briefly discuss its solutions. At early times, when the inequality (9) is satisfied, $\delta\chi_2$ is free massless quantum field,

$$\delta\chi_2(\mathbf{x}, \eta) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}} \left(\delta\chi_2^{(-)}(\mathbf{k}, \mathbf{x}, \eta)\hat{A}_{\mathbf{k}} + h.c.\right)\ ,$$

whose modes are (we keep the dependence on $\mathbf{x}$ for future convenience)

$$\delta\chi_2^{(-)}(\mathbf{k}, \mathbf{x}, \eta) = \mathrm{e}^{i\mathbf{k}\mathbf{x} - ik\eta}\ . \tag{10}$$

Here $\hat{A}_{\mathbf{k}}$ and $\hat{A}_{\mathbf{k}}^\dagger$ are aniihilation and creation operators obeying the standard commutational relation, $[\hat{A}_{\mathbf{k}}, \hat{A}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$. It is natural to assume that the field $\delta\chi_2$ is initially in its vacuum state.

The rolling background $\chi_c(\eta)$ produces an effective "horizon" for the perturbations $\delta\chi_2$. The oscillations (10) terminate when the mode exits the "horizon", i.e., at $k(\eta_* - \eta) \sim 1$. The solution to Eq. (8) with the initial condition (10) is

$$\delta\chi_2^{(-)}(\mathbf{k}, \mathbf{x}, \eta) = \mathrm{e}^{i\mathbf{k}\mathbf{x} - ik\eta_*} \cdot F(k, \eta_* - \eta)\ , \tag{11}$$

where

$$F(k, \xi) = -\sqrt{\frac{\pi}{2}k\xi}\ H_{3/2}^{(1)}(k\xi) \tag{12}$$

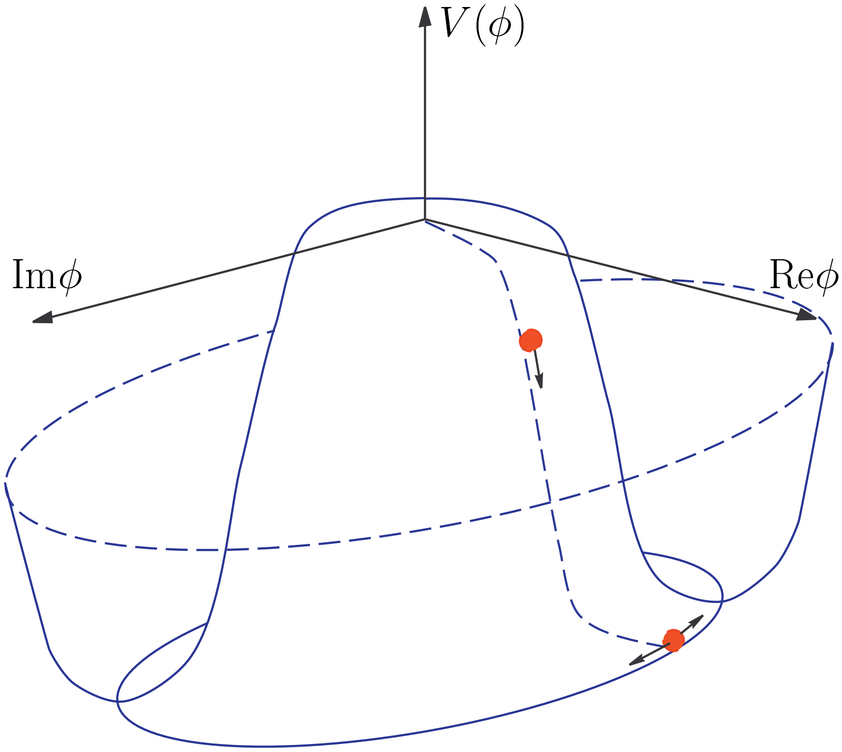

Figure 2: The scalar potential in the pseudo-Nambu–Goldstone scenario.

and $H^{(1)}_{3/2}$ is the Hankel function. In the late-time super-"horizon" regime, when $k(\eta_* - \eta) \ll 1$, one has

$$F(k, \eta_* - \eta) = \frac{i}{k(\eta_* - \eta)} \ . \tag{13}$$

Hence, the super-"horizon" perturbations of the phase $\delta\theta \equiv \delta\chi_2/\chi_c$ are time-independent,

$$\delta\theta(\mathbf{x}) = \frac{\delta\chi_2(\mathbf{x}, \eta)}{\chi_c(\eta)} = h \int \frac{d^3k}{4\pi^{3/2} k^{3/2}} \, i e^{i\mathbf{kx} - ik\eta_*} \hat{A}_{\mathbf{k}} + h.c. \tag{14}$$

This expression describes Gaussian random field (cf. Ref. [18]) whose power spectrum is given by (2).

2.2 Converting perturbations of phase into adiabatic perturbations

For the sake of completenetss, let us briefly discuss two possible ways to reprocess the perturbations of the phase $\delta\theta$ into the adiabatic perturbations.

One way is to make use of the curvaton mechanism [19] specified to pseudo-Nambu–Goldstone curvaton [20]. Namely, let us assume that θ is actually pseudo-Nambu–Goldstone field. Generically, conformal rolling ends up at a slope of its potential, see Fig. 2. The field θ together with its perturbations remains frozen out until the time at the hot Big Bang epoch when the Hubble parameter becomes of the order of the mass parameter of this field. Then θ starts to oscillate about the minimum of its potential. Provided this field interacts with conventional particles, its oscillations eventually decay, and their energy density is transfered to the cosmic plasma. At that epoch, the perturbations of the energy density of the field θ are converted into the adiabatic perturbations. The shape of the resulting adiabatic power spectrum is the same as that of the initial spectrum of $\delta\theta$, i.e., it is flat in the case of exact conformal invariance at the rolling stage. Generically, the amplitude of the adiabatic perturbations is of order

$$\sqrt{\mathcal{P}_\zeta} \sim r\frac{\sqrt{\mathcal{P}_{\delta\theta}}}{\theta_c} = r\frac{h}{2\pi\theta_c} ,$$

where r is the ratio of the energy density of the field θ to the total energy densiy at the time the θ-oscillations decay, and θ_c is the distance from the landing point of rolling to the minimum of the potential.

The pseudo-Nambu–Goldstone mechanism produces the non-Gaussianity of local form in the adiabatic perturbations. For generic values of the phase at landing, $\theta_c \sim \pi/2$, non-observation of the non-Gaussianity [1] implies

$$r \gtrsim 10^{-2} ,$$

so that the correct scalar amplitude is obtained for

$$h \lesssim 10^{-2} . \tag{15}$$

Thus, the scenario with the pseudo-Nambu–Goldstone mechanism of conversion of the phase perturbations into the adiabatic ones is viable for quite small scalar self-coupling only.

Another possibility is the modulated decay of heavy particles [21, 22, 23]. One assumes that the phase field θ interacts with some heavy particles in such a way that the masses and/or widths of the latter depend on θ,

$$M = M_0 + \epsilon_M\theta \quad \text{and/or} \quad \Gamma = \Gamma_0 + \epsilon_\Gamma\theta . \tag{16}$$

One assumes further that these particles survive at the hot Big Bang epoch until they are non-relativistic and dominate the cosmological expansion. When these particles decay, the perturbations in θ, and hence in M and/or Γ, induce adiabatic perturbations,

$$\zeta \sim \frac{\delta M}{M} = \frac{\epsilon_M\delta\theta}{M_0 + \epsilon_M\theta_c} \quad \text{and/or} \quad \zeta \sim \frac{\delta\Gamma}{\Gamma} = \frac{\epsilon_\Gamma\delta\theta}{\Gamma_0 + \epsilon_\Gamma\theta_c} .$$

The shape of the adiabatic power spectrum is again the same as that of the initial power spectrum of $\delta\theta$.

The modulated decay mechanism also induces non-Gaussianity in the adiabatic perturbations. However, once the dependence of the mass/width on θ is linear, as written in (16), the induced non-Gaussian part of the adiabatic perturbations is of order

$$(\delta M/M)^2 , \; (\delta\Gamma/\Gamma)^2 \sim \zeta^2 .$$

In other words, irrespectively of the value of the coupling h, the non-Gaussianity parameter is fairly small (see Refs. [22, 23] for details),

$$f_{NL} \sim 1 ,$$

in comfortable agreement with the existing limit [1]. Thus, the modulated decay mechanism by itself does not imply any bound on h.

2.3 Perturbations of modulus

Let us now come back to the conformal rolling stage and consider the radial perturbations — perturbations of the modulus of the field χ, or, with our convention of real backgronud χ_c, perturbations of the real part

$$\chi_1 \equiv \text{Re}\, \chi .$$

At the linearized level, they obey the following equation,

$$(\delta\chi_1)'' - \partial_i\partial_i\ \delta\chi_1 - 6\chi_c^2\delta\chi_1 \equiv (\delta\chi_1)'' - \partial_i\partial_i\ \delta\chi_1 - \frac{6}{(\eta_* - \eta)^2}\delta\chi_1 = 0\ .$$

Its solution that tends to properly normalized mode of free quantum field as $k(\eta_* - \eta) \to \infty$ is

$$\delta\chi_1 = \mathrm{e}^{i\mathbf{kx} - ik\eta_*} \cdot \frac{1}{4\pi}\sqrt{\frac{\eta_* - \eta}{2}} H_{5/2}^{(1)}\left[k(\eta_* - \eta)\right] \cdot \hat{B}_{\mathbf{k}} + h.c.\ ,$$

where $\hat{B}_{\mathbf{k}}$, $\hat{B}_{\mathbf{k}}^\dagger$ is another set of annihilation and creation operators. At late times, when $k(\eta_* - \eta) \ll 1$ (super-"horizon" regime), one has

$$\delta\chi_1 = \mathrm{e}^{i\mathbf{kx} - ik\eta_*} \cdot \frac{3}{4\pi^{3/2}} \frac{1}{k^{5/2}(\eta_* - \eta)^2} \cdot \hat{B}_{\mathbf{k}} + h.c.\ .$$

Hence, the super-"horizon" perturbations of the modulus have red power spectrum (3).

The dependnce $\delta\chi_1 \propto (\eta_* - \eta)^{-2}$ is naturally interpreted in terms of the local shift of the "end time" parameter η_*. Indeed, with the background field given by (6), the sum $\chi_c + \delta\chi_1$, i.e., the radial field including perturbations, is the linearized form of

$$\chi_c[\eta_*(\mathbf{x}) - \eta] = \frac{1}{h[\eta_*(\mathbf{x}) - \eta]}\ , \tag{17}$$

where

$$\eta_*(\mathbf{x}) = \eta_* + \delta\eta_*(\mathbf{x}) \tag{18}$$

and

$$\delta\eta_*(\mathbf{x}) = \frac{3h}{4\pi^{3/2}} \int \frac{d^3k}{k^{5/2}} \left(\mathrm{e}^{i\mathbf{kx} - ik\eta_*} \cdot \hat{B}_{\mathbf{k}} + h.c.\right)\ . \tag{19}$$

So, the infrared radial modes modify the effective background by transforming the "end time" parameter η_* into random field that slowly varies in space[4], as given in Eqs. (17), (18). Clearly, this observation is valid beyond the linear approximation: once the spatial scale of variation of $\chi_1(\mathbf{x}, \eta)$ exceeds the "horizon" size, spatial gradients in Eq. (5) are negligible, and the late-time solutions to the full non-linear field equation have locally one and the same form (6), modulo slow variation of η_* in space.

It is worth noting that the infrared modes contribute both to the field $\delta\eta_*(\mathbf{x})$ itself and to its spatial derivative. The contribution of the modes which are superhorizon today, i.e., have momenta $k \lesssim H_0$, to the variance of the latter is given by

$$\langle \partial_i\eta_*(\mathbf{x})\partial_j\eta_*(\mathbf{x})\rangle_{k\lesssim H_0} = \delta_{ij} \cdot \frac{3h^2}{4\pi} \int_{k\lesssim H_0} \frac{dk}{k} = \delta_{ij} \cdot \frac{3h^2}{4\pi} \log\frac{H_0}{\Lambda}\ , \tag{20}$$

where Λ is the infrared cutoff which parametrizes our ignorance of the dynamics at the beginning of the conformal rolling stage.

3 Effect of infrared radial modes on perturbations of phase: first order in h

The main purpose of this paper is to understand how the interaction with the infrared radial modes affects the properties of the perturbations of the phase $\delta\theta$. To this end, we consider perturbations of the imaginary part $\delta\chi_2$, whose wavelengths are much smaller than the scale

[4]There are corrections to Eq. (17) of order $\partial_i\partial_j\eta_*(\mathbf{x})$ and $[\partial_i\eta_*(\mathbf{x})]^2$, see Sections 4.1 and 4.2.

of the spatial variation of the modulus. Because of the separation of scales, perturbations $\delta\chi_2$ can still be treated in the linear approximation, but now in the background (17).

Since our concern is the infrared part of $\eta_*(\mathbf{x})$, we make use of the spatial gradient expansion, consider, for the time being, a region near the origin and write

$$\eta_*(\mathbf{x}) = \eta_*(0) - v_i x_i + \dots \ , \tag{21}$$

where

$$v_i = -\partial_i \eta_*(\mathbf{x})|_{\mathbf{x}=0} \ ,$$

and dots denote higher order terms in $\mathbf{x}$. Importantly, the field $\partial_i\partial_j\eta_*(\mathbf{x})$ has blue power spectrum, unlike $\eta_*(\mathbf{x})$ and $\partial_i\eta_*(\mathbf{x})$, so the major effect of the infrared modes is accounted for by considering the two terms of the gradient expansion written explicitly in (21). In this Section we work at this, first order of the gradient expansion. Furthermore, we assume in what follows that

$$|\mathbf{v}| \ll 1 \ , \tag{22}$$

and in this Section we neglect corrections of order $\mathbf{v}^2$. The expansion in $|\mathbf{v}|$ is legitimate, since the field $\mathbf{v}(\mathbf{x})$ has flat power spectrum, so the fluctuation of $\mathbf{v}$ is of order $h^2|\log\Lambda|$, where Λ is the infrared cutoff, and it is small for small h and not too large $|\log\Lambda|$. In other words, the expansion in $|\mathbf{v}|$ is the expansion in h, modulo infrared logarithms. We postpone to Section 4.2 the analysis of the leading effect that occurs at the order $\mathbf{v}^2$.

Keeping the two terms in (21) only, we have, instead of Eq. (8),

$$(\delta\chi_2)'' - \partial_i\partial_i\, \delta\chi_2 - \frac{2}{[\eta_*(0) - \mathbf{v}\mathbf{x} - \eta]^2}\delta\chi_2 = 0 \ . \tag{23}$$

We observe that the denominator in the expression for the background field

$$\chi_c = \frac{1}{h[\eta_*(0) - \eta - \mathbf{v}\mathbf{x}]} \tag{24}$$

contains the combination $\eta_*(0) - (\eta + \mathbf{v}\mathbf{x})$. We interpret this as the local time shift and Lorentz boost of the original background (6): the effective background is homogeneous in a reference frame (in conformal coordinates) other than the cosmic frame where the metric is spatially homogeneous. Note that the field (24) is a solution to the field equation (5) in our approximation. Our interpretation suggests that the solutions to Eq. (23) can be obtained by time translation and Lorentz boost of the original solution (11), (12). Indeed, it is straightforward to see that to the first order in $\mathbf{v}$, the solution to Eq. (23) obeying the initial condition (10) is

$$\delta\chi_2^{(-)}(\mathbf{k},\mathbf{x},\eta) = \mathrm{e}^{i\mathbf{q}(\mathbf{x}+\mathbf{v}\eta) - iq\eta_*(0)} \cdot F(q, \eta_*(0) - \eta - \mathbf{v}\mathbf{x}) \ , \tag{25}$$

where the function F is still defined by (12), the Lorentz-boosted momentum is

$$\mathbf{q} = \mathbf{k} + k\mathbf{v} \ , \quad q = |\mathbf{q}| = k + \mathbf{k}\mathbf{v} \ , \tag{26}$$

and it is understood that terms of order $\mathbf{v}^2$ must be neglected. The solution (25) can be cast into the following form,

$$\delta\chi_2^{(-)}(\mathbf{k},\mathbf{x},\eta) = \mathrm{e}^{i\mathbf{k}\mathbf{x} - ik\eta_*(\mathbf{x}) - i\mathbf{k}\mathbf{v}[\eta_*(\mathbf{x}) - \eta]} \cdot F(q, \eta_*(\mathbf{x}) - \eta) \ , \tag{27}$$

where

$$v_i = -\partial_i \eta_*(\mathbf{x}) \ .$$

This form, still valid to the first order in the gradient expansion and to the linear order in $\mathbf{v}$, does not make any reference to the origin of the coordinate frame and can be used for arbitrary

$\mathbf{x}$. We consider corrections of order $\partial_i \partial_j \eta_*(\mathbf{x})$ and $\mathbf{v^2}$ to this solution in Sections 4.1 and 4.2, respectively.

We find from Eqs. (24) and (25) (or, equivalently, (17) and (27)) that the perturbations of the phase again freeze out as $k[\eta_*(\mathbf{x}) - \eta] \to 0$, now at

$$\delta\theta(\mathbf{x}) = \frac{\delta\chi_2(\mathbf{x})}{\chi_c(\mathbf{x})} = i \int \frac{d^3k}{\sqrt{k}} \frac{h}{4\pi^{3/2} q} \, \mathrm{e}^{i\mathbf{k}\mathbf{x} - ik\eta_*(\mathbf{x})} \hat{A}_{\mathbf{k}} + h.c. \; , \tag{28}$$

where the relation between $\mathbf{k}$ and $\mathbf{q}$ is still given by Eq. (26). We recall that $\eta_*(\mathbf{x})$ here is a realization of infrared random field. The formula (28) implies that to the first order of the gradient expansion we limit ourselves in this Section, the properties of the random field $\delta\theta$ are the same as those of the linear field (14). Consider, e.g., the two-point correlation function

$$\langle \delta\theta(\mathbf{x}_1) \, \delta\theta(\mathbf{x}_2) \rangle = \int \frac{d^3k}{k} \frac{h^2}{16\pi^3 q^2} \mathrm{e}^{i\mathbf{k}(\mathbf{x}_1 - \mathbf{x}_2) - ik[\eta_*(\mathbf{x}_1) - \eta_*(\mathbf{x}_2)]} + c.c. \tag{29}$$

Infrared modes of $\eta_*(\mathbf{x})$ we consider have momenta much lower than k, i.e., the spatial scale of their variation much exceeds $|\mathbf{x_1} - \mathbf{x}_2|$. Hence, to the first order in the gradient expansion we have $\eta_*(\mathbf{x}_1) - \eta_*(\mathbf{x}_2) = -\mathbf{v}(\mathbf{x_1} - \mathbf{x}_2)$, and $\mathbf{v}$ is independent of $\mathbf{x}$. Therefore, the two-point function is

$$\langle \delta\theta(\mathbf{x}_1) \, \delta\theta(\mathbf{x}_2) \rangle = \int \frac{d^3k}{k} \frac{h^2}{16\pi^3 q^2} \mathrm{e}^{i\mathbf{q}(\mathbf{x}_1 - \mathbf{x}_2)} + c.c.$$

We now change the integration varible from $\mathbf{k}$ to $\mathbf{q}$, recall that the integration measure d^3k/k is Lorentz-invariant, and obtain

$$\langle \delta\theta(\mathbf{x}_1) \, \delta\theta(\mathbf{x}_2) \rangle = h^2 \int \frac{d^3q}{16\pi^3 q^3} \mathrm{e}^{i\mathbf{q}(\mathbf{x}_1 - \mathbf{x}_2)} + c.c.$$

This is precisely the two-point correlation function of the linear field (14).

The latter argument is straightforwardly generalized to multiple correlators: for a given realization of the random field $\eta_*(\mathbf{x})$, they are all expressed in terms of the two-point correlation function (29). In other words, the infrared effects are removed by the field redefinition,

$$\hat{\mathcal{A}}_{\mathbf{q}} = \mathrm{e}^{-ik\eta_*(0)} \sqrt{\frac{k}{q}} \hat{A}_{\mathbf{k}} \; , \tag{30}$$

where $\mathbf{k}$ and $\mathbf{q}$ are still related by (26). The operators $\hat{\mathcal{A}}_{\mathbf{q}}$, $\hat{\mathcal{A}}_{\mathbf{q}}^\dagger$ obey the standard commutational relations, while in our approximation, the field (28), written in terms of these operators, coincides with the linear field (14). We conclude that the infrared radial modes are, in fact, not particularly dangerous, as they do not grossly affect the properties of the field $\delta\theta$.

4 Statistical anisotropy

4.1 First order in h

Let us continue with the analysis at the first order in h. In this approximation, the non-trivial effect of the large wavelength perturbations $\delta\eta_*(\mathbf{x})$ on the perturbations of the phase, and hence on the resulting adiabatic perturbations, occurs for the first time at the second order in the gradient expansion, i.e., at the order $\partial_i\partial_j\eta_*$. Let us concentrate on the effect of the modes of $\delta\eta_*$ whose present wavelengths exceed the present Hubble size. We are dealing with one realization of the random field $\delta\eta_*$, hence at the second order of the gradient expansion, $\partial_i\partial_j\eta_*$ is merely a tensor, constant throughout the visible Universe. In this Section we calculate the statistical anisotropy associated with this tensor.

To this end, we make use of the perturbation theory in $\partial_i\partial_j\eta_*$. In the first place, we have to find the background, since the function (17) is no longer the solution to the field equation (5) at the second order in the gradient expansion. We write

$$\chi_c = \chi_c^{(1)} + \chi_c^{(2)} \ , \tag{31}$$

where

$$\chi_c^{(1)} = \frac{1}{h[\eta_*(\mathbf{x}) - \eta]} \ , \tag{32}$$

and $\chi_c^{(2)} = O(\partial_i\partial_j\eta_*)$. We plug the expression (31) into Eq. (5), linearize in $\partial_i\partial_j\eta_*$ and obtain the following equation for the correction,

$$\chi_c^{(2)\,\prime\prime} - \frac{6}{(\eta_* - \eta)^2}\chi_c^{(2)} = -\frac{\partial_i\partial_i\eta_*}{h(\eta_* - \eta)^2} \ , \tag{33}$$

where we again neglect terms of order $\mathbf{v}^2$. Clearly, the correction to the background depends only on the scalar $\partial_i\partial_i\eta_*$, so it does not yield statistical anisotropy. Nevertheless, we keep this correction in what follows.

The general solution to Eq. (33) is

$$\chi_c^{(2)} = C_1(\eta_* - \eta)^3 + \frac{C_2}{(\eta_* - \eta)^2} + \frac{1}{6h}\partial_i\partial_i\eta_* \ .$$

The first term in the right hand side is irrelevant in the super-"horizon" regime, the second term is merely a shift of η_* in the leading order expression (32), so the non-trivial correction is given by the third term. The combination entering Eq. (7) for the perturbations of the imaginary part is now given by (we consistently work at the linear order in $\partial_i\partial_j\eta_*$)

$$2h^2\chi_c^2 = \frac{2}{(\eta_*(\mathbf{x}) - \eta)^2} + \frac{2}{3}\frac{\partial_i\partial_i\eta_*}{\eta_* - \eta} \ . \tag{34}$$

Note that we can set $\eta_* = \text{const}$ in the second term, since we neglect corrections of order $\partial_i\partial_j\eta_* \cdot \partial_k\eta_*$.

Let us now obtain the solution to Eq. (7) to the first order in $\partial_i\partial_j\eta_*$. The initial condition is still given by Eq. (10). We search for the solution in the following form,

$$\delta\chi_2^{(-)}(\mathbf{k},\mathbf{x},\eta) = \mathrm{e}^{i\mathbf{k}\mathbf{x} - ik\eta_*(\mathbf{x}) - i\mathbf{k}\mathbf{v}[\eta_*(\mathbf{x})-\eta]} \cdot \Big[F(q,\eta_*(\mathbf{x}) - \eta) + F^{(2)}(q,\eta_*(\mathbf{x}) - \eta)\Big] \ ,$$

where the leading term F is again defined by (12) and $F^{(2)}$ is proportional to $\partial_i\partial_j\eta_*$. We expand Eq. (7) in $\partial_i\partial_j\eta_*$ and obtain to the linear order

$$F^{(2)\,\prime\prime} + k^2F^{(2)} - \frac{2}{\xi^2}F^{(2)} = \partial_i\partial_i\eta_* \cdot S + k_ik_j\,\partial_i\partial_j\eta_* \cdot T \ , \tag{35}$$

where $\xi = \eta_* - \eta$,

$$S = -ikF + \frac{\partial F(k,\xi)}{\partial \xi} + \frac{2}{3\xi}F \ , \tag{36}$$

$$T = -2F\xi - 2i\frac{\partial F(k,\xi)}{\partial k} = -\frac{2}{k^2\xi}\mathrm{e}^{ik\xi} \ . \tag{37}$$

After calculating the right hand side of Eq. (35) we set $\mathbf{q} = \mathbf{k}$ and $\mathbf{v} = 0$, since we neglect the terms of order $\partial_i\partial_j\eta_* \cdot \partial_k\eta_*$. Note that the last term in (36) comes from the correction to the background, see Eq. (34), and that the last expression in (37) is obtained by using the explicit form of the Hankel function $H_{3/2}^{(1)}$.

The soultion $F^{(2)}$ should vanish as $\eta_* - \eta \to \infty$. Hence, it is given in terms of the retarded Green's function (recall that $\xi' > \xi$ corresponds to $\eta' < \eta$)

$$G(\xi, \xi') = \frac{\pi\sqrt{\xi\xi'}}{2} \cdot \Theta(\xi' - \xi) \cdot \left[J_{3/2}(k\xi)N_{3/2}(k\xi') - N_{3/2}(k\xi)J_{3/2}(k\xi')\right] \; , \tag{38}$$

where $J_{3/2}$ and $N_{3/2}$ are the Bessel functions. Namely,

$$F^{(2)}(\xi) = \int_{\xi}^{\infty} d\xi' \; G(\xi, \xi') \left[\partial_i\partial_i\eta_* \cdot S(\xi') + k_i k_j \; \partial_i\partial_j\eta_* \cdot T(\xi')\right] \; . \tag{39}$$

We are interested in the behavior of this solution in the super-"horizon" regime, $k\xi \to 0$. Since the most singular behavior of S and T at small ξ is ξ^{-2}, the first term in square brackets in (38) gives finite contribution to the integral (39) as $\xi \to 0$, while the second term yields $F^{(2)}(\xi) \propto \xi^{-1}$. Hence, the behavior of the correction $F^{(2)}(\xi)$ at small ξ is the same as that of the leading term $F(\xi)$, so the phase perturbations $\delta\theta$, the correction included, are constant in time in the super-"horizon" regime. This is a consistency check of our approach.

Since we treat $\partial_i\partial_j\eta_*$ as constant over the visible Universe, the power spectrum of the phase perturbations contains a part linear[5] in $\partial_i\partial_j\eta_*$. This linear part comes from the interference of $F^{(2)}$ and F. In the super-"horizon" regime, F is pure imaginary, see (13). Hence, we are interested in the behavior of the imaginary part of $F^{(2)}$ as $\xi \to 0$. This behavior is straightforward to obtain. Namely, for T-term in (39) we write

$$\begin{aligned}
\mathrm{Im}\; F^{(2)\,T}|_{\xi\to 0} &= -\frac{\pi\sqrt{\xi}}{2} N_{3/2}(k\xi) \cdot \int_0^{\infty} d\xi' \sqrt{\xi'} J_{3/2}(k\xi') \left(-\frac{2\sin k\xi'}{k^2\xi'}\right) \cdot k_i k_j \; \partial_i\partial_j\eta_* \\
&= -\frac{\pi}{2}\frac{1}{k^2\xi} \cdot \frac{k_i k_j}{k^2}\partial_i\partial_j\eta_* \; .
\end{aligned}$$

Performing similar calculation for S-term in (39), we obtain that in the super-"horizon" regime

$$F + F^{(2)} = \frac{i}{q(\eta_* - \eta)} \left(1 - \frac{\pi}{2k} \cdot \frac{k_i k_j}{k^2}\partial_i\partial_j\eta_* + \frac{\pi}{6k}\partial_i\partial_i\eta_*\right) \; .$$

The two non-trivial terms in parenthesis give the correction to the power spectrum of the phase perturbations due to the radial modes whose whavelengths exceed the present Hubble size. The same correction is characteristic of the adiabatic perturbations, so we have finally

$$\mathcal{P}_\zeta = A_\zeta \left[1 - \frac{\pi}{k} \cdot \frac{k_i k_j}{k^2} \left(\partial_i\partial_j\eta_* - \frac{1}{3}\delta_{ij}\partial_k\partial_k\eta_*\right)\right] \; , \tag{40}$$

where the adiabatic amplitude A_ζ is independent of k within our aproximations. Notably, the angular average of the correction vanishes, so we are dealing with the statistical anisotropy proper.

Neither the magnitude nor the exact form of the tensor $\partial_i\partial_j\eta_* - (1/3)\delta_{ij}\partial_k\partial_k\eta_*$ can be unambiguously predicted because of the cosmic variance. To estimate the strength of the statistical anisotropy, let us consider the variance

$$\langle(\partial\partial\eta_*)^2\rangle \equiv \langle\left(\partial_i\partial_j\eta_* - \frac{1}{3}\delta_{ij}\partial_k\partial_k\eta_*\right) \cdot \left(\partial_i\partial_j\eta_* - \frac{1}{3}\delta_{ij}\partial_k\partial_k\eta_*\right)\rangle_{k\lesssim H_0} \; ,$$

where the notation reflects the fact that we take into account only those modes whose present wavelengths exceed the present Hubble size. We make use of (19) and obtain

$$\langle(\partial\partial\eta_*)^2\rangle = \frac{9h^2}{16\pi^2}\int_{k\lesssim H_0} \frac{dk}{k^5} \cdot \frac{2k^4}{3} \simeq \frac{3h^2}{4\pi^2}H_0^2 \; . \tag{41}$$

In this way we arrive at the first non-trivial term in (4). Higher orders in the gradient expansion give contributions to the statistical anisotropy which are suppressed by extra factors of H_0/k.

[5]Had we considered the perturbations $\delta\eta_*$ whose wavelengths are much shorter than the present Hubble size, it would be appropriate to perform ensemble averaging. In that case the linear part would average out.

4.2 Order h^2: contribution of deep infrared modes

Let us now turn to the statistical anisotropy at the second order in h. The major contribution at this order is proportional to $\partial_i \eta_* \partial_j \eta_* \equiv v_i v_j$. Indeed, the overall time shift $\eta_*(0)$ is irrelevant, while the terms involivng higher derivative combinations like $\partial_i \partial_j \eta_* \partial_i \eta_*$ are suppressed by powers of H_0/k, cf. (40), (41). Furthermore, the "velocity" $\mathbf{v}$ is enhanced, albeit only logarithmically, by the deep infrared effects, see (20). Hence, to extract the major contribution to the statistical anisotropy at the order h^2, we use the two terms of the derivative expansion, explicitly written in (21).

Once the terms of order $\mathbf{v}^2$ are not neglected, the function (24) is no longer a solution to the field equation (5). Instead, the solution is (keeping two terms only in the gradient expansion)

$$\chi_c = \frac{1}{h\gamma \left[\eta_*(0) - \eta - \mathbf{v}\mathbf{x}\right]} \; . \tag{42}$$

So, it is appropriate to study the solutions to Eq. (7) in this background. It is straightforward to see that the solution that obeys the initial condition (10) is

$$\delta\chi_2^{(-)}(\mathbf{k}, \mathbf{x}, \eta) = \mathrm{e}^{iq_{||}\gamma(x_{||}+v\eta) + i\mathbf{q}^T\mathbf{x}^T - iq\gamma\eta_*(0)} \cdot F\left[q, \gamma(\eta_*(0) - \eta - \mathbf{v}\mathbf{x})\right] \; , \tag{43}$$

where the Lorentz-boosted momentum is, as usual, $q_{||} = \gamma(k_{||} + kv)$, $\mathbf{q}^T = \mathbf{k}^T$, $q = \gamma(k + k_{||}v)$, $\gamma = (1 - v^2)^{-1/2}$, and notations $||$ and T refer to components which are parallel and normal to $\mathbf{v}$, respectively. The interpretation of this solution is again that it is the Lorentz-boosted solution (11).

According to the scenario discussed in this paper, the phase perturbations $\delta\theta$ freeze out at the hypersurface

$$\eta = \eta_*(0) - \mathbf{v}\mathbf{x} \equiv \eta_*(0) - vx_{||}$$

and then stay constant in the cosmic time η. It follows from (42) and (43) that at this hypersurface and later, the perturbations of the phase are

$$\delta\theta(\mathbf{x}) = \int \frac{d^3q}{\sqrt{q}} \frac{h}{4\pi^{3/2} q} \; \mathrm{e}^{i\gamma^{-1}q_{||}x_{||} + i\mathbf{q}^T\mathbf{x}^T} \hat{\mathcal{A}}_{\mathbf{q}} + h.c. \; , \tag{44}$$

where the operators $\hat{\mathcal{A}}_{\mathbf{q}}$ are defined by (30) and obey the standard commutational relations, and we have omitted irrelevant constant phase factor. We see that to the order $\mathbf{v}^2$, the effect of the deep infrared modes does not disappear: it is encoded in the factor γ^{-1} in the first term in the exponent. It is clear that the momentum of perturbation labeled by $\mathbf{q}$ is actually equal to

$$\mathbf{p} = (\gamma^{-1}q_{||}, \mathbf{q}^T) \; .$$

Accordingly, the power spectrum (omitting the correction discussed in Section 4.1) is given by

$$\mathcal{P}_{\delta\theta}(\mathbf{p}) \frac{d^3p}{4\pi p^3} = \frac{h^2}{16\pi^3} \frac{\gamma d^3p}{[(\gamma p_{||})^2 + (\mathbf{p}^T)^2]^{3/2}} \; .$$

The same power spectrum charactersizes the adiabatic perturbations in our scenario, and changing the notation $\mathbf{p} \to \mathbf{k}$ we obtain finally

$$\mathcal{P}_\zeta(\mathbf{k}) = A_\zeta \frac{p^3}{[(\gamma k_{||})^2 + (\mathbf{k}^T)^2]^{3/2}} = A_\zeta \left(1 - \frac{3}{2}\frac{(\mathbf{k}\mathbf{v})^2}{k^2}\right) \; .$$

Hence, we have arrived at the last term in (4). Again, neither the direction of $\mathbf{v}$ nor its length can be unambiguously calculated because of cosmic variance; recall, however, that the value of $|\mathbf{v}|$, and hence the parameter c_2 in (4), is logarithmically enhanced due to the infrared effects, see (20).

In the case of the pseudo-Nambu–Goldstone mechanism of conversion of the phase perturbations into the adiabatic ones, both terms in the statistical ansotropy (4) are small because of the bound (15). On the other hand, the effect may be stronger in the case of the modulated decay mechanism.

5 Conclusion

We conclude by making a few remarks.

First, our mechanism of the generation of the adiabatic perturbations can work in any cosmological scenario that solves the horizon problem of the hot Big Bang theory, including inflation, bouncing/cyclic scenario, pre-Big Bang, etc. In some of these scenarios (e.g., bouncing Universe), the assumption that the phase perturbations are superhorizon in conventional sense by the end of the conformal rolling stage may be non-trivial. It would be of interest to study also the opposite case, in which the phase evolves for some time after the end of conformal rolling.

Second, we concentrated in this paper on the effect of infrared radial modes, and employed the derivative expansion. The expressions like (28), which we obtained in this way, must be used with caution, however. Bold usage of (28) would yield, e.g., non-vanishing equal-time commutator $[\theta(\mathbf{x}), \theta(\mathbf{y})]$, which would obviously be a wrong result. The point is that the formula (28) is valid in the approximation $\mathbf{v} = \text{const}$; with this understanding, the equal-time commutator vanishes, as it should.

Finally, the non-linearity of the field equation gives rise to the intrinsic non-Gaussianity of the phase perturbations and, as a result, adiabatic perturbations. The non-Gaussianity emerges at the order $O(h^2)$, and may be sizeable for large enough values of the coupling h. The form of the non-Gaussianity is rather peculiar in our scenario. Unlike in many other cases, the three-point correlation function vanishes, while the four-point correlation function of $\delta\theta$ (and hence of adiabatic perturbations) involves the two-point correlator of the independent Gaussian field $\delta\eta_*$. In view of the result of Section 3, correlation functions of $\delta\theta$ should be infrared-finite, at least at the order $O(h^2)$. This result shows also that the analysis of the non-Gaussianity requires methods beyond the gradient expansion. We leave this important issue for the future.

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References

[1] E. Komatsu *et al.*, arXiv:1001.4538.

[2] A. A. Starobinsky, JETP Lett. **30** 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. **30**, 719 (1979)] Phys. Lett. B **91**, 99 (1980);
A. H. Guth, Phys. Rev. D **23**, 347 (1981);
A. D. Linde, Phys. Lett. B **108**, (1982) 389; Phys. Lett. B **129** 177 (1983);
A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[3] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, (1981) 532 [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)];
S. W. Hawking, Phys. Lett. B **115**, 295 (1982);
A. A. Starobinsky, Phys. Lett. B **117**, 175 (1982);
A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982);
J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[4] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 (2001) [arXiv:hep-th/0103239];

J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt and N. Turok, Phys. Rev. D **65**, 086007 (2002) [arXiv:hep-th/0108187].

[5] P. Creminelli, M. A. Luty, A. Nicolis and L. Senatore, JHEP **0612** (2006) 080 [arXiv:hep-th/0606090].

[6] J. L. Lehners, P. McFadden, N. Turok and P. J. Steinhardt, Phys. Rev. D **76**, 103501 (2007) [arXiv:hep-th/0702153];
E. I. Buchbinder, J. Khoury and B. A. Ovrut, Phys. Rev. D **76**, 123503 (2007) [arXiv:hep-th/0702154];
P. Creminelli and L. Senatore, JCAP **0711**, 010 (2007) [arXiv:hep-th/0702165].

[7] A. Notari and A. Riotto, Nucl. Phys. B **644**, 371 (2002) [arXiv:hep-th/0205019];
F. Di Marco, F. Finelli and R. Brandenberger, Phys. Rev. D **67**, 063512 (2003) [arXiv:astro-ph/0211276].

[8] D. Wands, Phys. Rev. D **60** (1999) 023507 [arXiv:gr-qc/9809062];
F. Finelli and R. Brandenberger, Phys. Rev. D **65** (2002) 103522 [arXiv:hep-th/0112249];
L. E. Allen and D. Wands, Phys. Rev. D **70** (2004) 063515 [arXiv:astro-ph/0404441];
R. H. Brandenberger, arXiv:1003.1745.

[9] S. Mukohyama, JCAP **0906** (2009) 001 [arXiv:0904.2190 [hep-th]].

[10] P. Creminelli, A. Nicolis and E. Trincherini, arXiv:1007.0027.

[11] V. A. Rubakov, JCAP **0909** (2009) 030 [arXiv:0906.3693 [hep-th]].

[12] J. K. Erickson, D. H. Wesley, P. J. Steinhardt and N. Turok, Phys. Rev. D **69** (2004) 063514 [arXiv:hep-th/0312009];
D. Garfinkle, W. C. Lim, F. Pretorius and P. J. Steinhardt, Phys. Rev. D **78** (2008) 083537 [arXiv:0808.0542 [hep-th]].

[13] J. L. Lehners, Phys. Rept. **465** (2008) 223 [arXiv:0806.1245 [astro-ph]].

[14] V. Rubakov and M. Osipov, arXiv:1007.3417.

[15] L. Ackerman, S. M. Carroll and M. B. Wise, Phys. Rev. D **75** (2007) 083502 [Erratum-ibid. D **80** (2009) 069901] [arXiv:astro-ph/0701357];
A. R. Pullen and M. Kamionkowski, Phys. Rev. D **76** (2007) 103529 [arXiv:0709.1144 [astro-ph]].

[16] M. A. Watanabe, S. Kanno and J. Soda, Phys. Rev. Lett. **102** (2009) 191302 [arXiv:0902.2833 [hep-th]]; arXiv:1003.0056 [astro-ph.CO];
T. R. Dulaney and M. I. Gresham, Phys. Rev. D **81** (2010) 103532 [arXiv:1001.2301];
A. E. Gumrukcuoglu, B. Himmetoglu and M. Peloso, Phys. Rev. D **81** (2010) 063528 [arXiv:1001.4088 [astro-ph.CO]].

[17] A. E. Gumrukcuoglu, C. R. Contaldi and M. Peloso, arXiv:astro-ph/0608405; JCAP **0711** (2007) 005 [arXiv:0707.4179 [astro-ph]].

[18] D. Polarski and A. A. Starobinsky, Class. Quant. Grav. **13** (1996) 377 [arXiv:gr-qc/9504030].

[19] A. D. Linde and V. F. Mukhanov, Phys. Rev. D **56** (1997) 535 [arXiv:astro-ph/9610219];
K. Enqvist and M. S. Sloth, Nucl. Phys. B **626** (2002) 395 [arXiv:hep-ph/0109214];
D. H. Lyth and D. Wands, Phys. Lett. B **524** (2002) 5 [arXiv:hep-ph/0110002];
T. Moroi and T. Takahashi, Phys. Lett. B **522** (2001) 215 [Erratum-ibid. B **539** (2002) 303] [arXiv:hep-ph/0110096].

[20] K. Dimopoulos, D. H. Lyth, A. Notari and A. Riotto, JHEP **0307**, 053 (2003) [arXiv:hep-ph/0304050].

[21] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **69** (2004) 023505 [arXiv:astro-ph/0303591];
L. Kofman, arXiv:astro-ph/0303614.

[22] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **69** (2004) 083505 [arXiv:astro-ph/0305548].

[23] F. Vernizzi, Phys. Rev. D **69** (2004) 083526 [arXiv:astro-ph/0311167].