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Can broken rotational invariance be reconciled with inflation ?

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Abstract

Motivated by claims of broken rotational invariance in the WMAP data, a number of models have appeared in the literature which realize this effect through vector field(s) with a nonvanishing spatial vacuum expectation value. We discuss why many of such models have ghost instabilities.

I dedicate this talk to the memory of Lev Kofman. I would like to testify my gratitude towards Lev, for the wonderful opportunity of working with him and for the time spent together. Our interaction has had the deepest influence on me, and I feel fortunate to have known a person and a physicist of such caliber.

My initial interest in the subject discussed here was to understand how cosmological perturbations behave when one relaxes the assumption of isotropy and homogeneity of the background. This is in turn related to the question of "how do we really know that our universe is homogeneous and isotropic ?". For definiteness, I studied the simplest case of a Bianchi I background (essentially, a universe with different expansion rates in the different directions), which was undergoing isotropzation at the onset of inflation. I noticed that one gravity wave polarization had a large growth in the anisotropic regime, but I regarded this mostly as a mathematical curiosity. On the contrary, Lev immediately understood the physical origin of the effect, and its possible implications. The growth is rooted in the instability of Kasner spaces, studied in seminal papers by Belinsky, Khalatnikov and Lifshitz [1]. Lev was fascinated by this effect, due to its universality: it describes the behavior of a wide class of spaces which are contracting towards a singularity, and which is characterized by a transition between different Kasner geometries (each of them being unstable; these are the so called BKL oscillations). Lev realized that what we had computed was this same phenomenon for an expanding Kasner space, with the only difference that now the instability is shut off as the inflaton takes over and the geometry isotropizes. The growth of the tensor mode could have resulted in a large gravity wave signal; this may be visible at the largest scales, provided that inflation hadn't lasted too much. When I replied that this is less appealing than the standard inflationary predictions, as it introduces a dependence on initial conditions, Lev replied that this reminded him of the pre-inflationary epoch, in which cosmologists were used to the fact that what we observe would have been dependent on initial conditions, in a way that cannot be predicted from the underlying theory. It is now easy to take for granted the "umbrella" protection that inflation provides by decoupling

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the (possibly) pre-existing initial conditions from what is observed. Whether this is or is not the case is of course just matter of observations, and not of our theoretical taste. This was the last lesson that I have had the privileged of learning from Lev.

The main theme of the series of works that I discuss here is whether it is possible to reconcile one of the so called WMAP anomalies with the inflationary picture, and whether this could lead to new predictions. At present, the WMAP satellite [2] provides the best data on our universe at the largest observable scales. Over the years, a number of anomalies have been argued to be present in the WMAP data. If of cosmological origin, such anomalies would call for a drastic rethought of the inflationary picture. While the first of such claims regarded the very largest scales (specifically, an alignment between the largest multiples of the CMB temperature anisotropies decomposition [3]), for which the galactic cut and foreground removal may be an issue, later works discussed features at higher multipoles ($\equiv$ smaller scales). For instance, ref. [4] has claimed evidence of breaking of rotational invariance by considering multipoles up to $\ell = 400$. Specifically, it is claimed that the data suggest a primordial power spectrum with an angular dependence with respect to a "privileged direction" in the sky. The direction emerged from this study had no astrophysical relevance. Successive works [5] corrected this analysis; a greater ($\sim 9\sigma$) evidence for the anomaly appears, although the privileged direction is now found to almost coincide with the ecliptic axis, strengthening the case for a systematic origin.

A number of studies of the WMAP anomalies have put forward some parametrizations of these effects. In many cases, the parametrization has no underlying theoretical model, and, while it can serve as a useful template to study the data, does not improve our theoretical understanding of the anomalies. An improvement on this is offered by a number of works that actually propose concrete cosmological models that aim to reproduce such features. In a particularly interesting class of models, the statistical isotropy is broken by the nonvanishing spatial vev of some vector field(s); this can lead to either a small anisotropy in the inflationary expansion [6, 7, 8], or to an anisotropic mechanism of generation of the primordial perturbations [9]. All these models have some nonminimal ingredients that prevent the quick isotropization (in this case, the quick decrease of the vector vev) that typically takes place during inflation. Even if the later findings of [5] strongly disfavor a cosmological origin of this anomaly, there are still various motivations for studying such models. From a phenomenological point of view, the Planck satellite [10], whose CMB data are expected to be released in summer 2012, will provide a nontrivial check of the WMAP results, as, if of cosmological origin, the effect of broken rotational invariance will be even more significant in Planck (due to the greater accuracy at large ℓ). From a theoretical point of view, when taken at face value, the models [6, 7, 8, 9] seem to imply that it is actually not so difficult to break statistical isotropy, so that we should not take the latter as a strong theoretical prior; in this sense, even the term "anomaly" would appear to be unjustified: the anisotropy could simply be an extra parameter which can be easily obtained from the theory, and that the observations indicate to be zero or small.

A third motivation is that the original proposals [6, 7, 8, 9] did not perform a complete study of the cosmological perturbations. Such a complete study could result in new phenomenological predictions, that could allow to prove or falsify such models as the origin of the anomaly reported in [4, 5]. In a non-isotropic background, the scalar and tensor metric perturbations are coupled to each other already at the linearized level. This can result in a nonstandard (and, potentially, enhanced) gravity wave amplitude and in a sizable scalar-tensor correlation [11] (as this quantity vanishes in the standard case, this would be a distinctive signature of the anisotropy). This computation was actually the initial goal of our works. We however found that the models [6, 7, 8, 9] possess ghost instabilities, which, in our opinion, invalidate any phenomenological predictions which have been made for them.

We reported explicit and exhaustive computations for the model [7], and for the models [8, 9] in [12] and [13], respectively. We have made analogous computations for the model of [6], but we have not published them; they can be performed by the same methods of [12, 13]. For brevity reasons, we will not repeat them here. We rather perform a much simpler computation, that disregards the metric perturbations, but that clarifies why the instability arises [14].

We start the discussion by considering a massive vector on a homogeneous and isotropic background

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{M^2}{2} A_\mu A^\mu \right] . \tag{1}$$

We assume that A_μ has negligible vev, and we decompose its fluctuations as $A_\mu = \left(\alpha_0,\, \partial_i \alpha_L + \alpha_i^T\right)$. The transverse vector perturbation α_i^T, satisfying $\partial_i \alpha_i^T = 0$, contains two physical modes. These modes are well behaved, and decoupled from the α_0, α_L perturbations. We disregard them in the following. For $M^2 \neq 0$, the two perturbations α_0, α_L encode one additional degree of freedom, namely the longitudinal vector polarization. Indeed the mode α_0 is non-dynamical, since it appears without time derivatives in the action, and must be integrated out. Its equation of motion, after Fourier decomposition in the spatial directions, gives $\alpha_0 = \left[p^2 / \left(p^2 + M^2\right)\right] \dot{\alpha}_L$, where $p = k/a$ is the physical momentum of the mode, k the comoving momentum, and dot denotes time differentiation. Inserting this solution back into (1) we obtain the action for the dynamical mode:

$$S_{\rm longitudinal} = \int dt\, d^3k\, a^3\, \frac{p^2 M^2}{2} \left[\frac{|\dot{\alpha}_L|^2}{p^2 + M^2} - |\alpha_L|^2 \right] . \tag{2}$$

The longitudinal vector mode exists due to the mass term, so it is not a surprise that M^2 multiplies the kinetic term. We see that this mode is a ghost in the UV for $M^2 < 0$. Identical conclusions are reached by computing the vector field propagator, or by using the Stuckelberg formalism [12]. We stress the analogy with the massive gravity case, in which also the mass term controls the stability. Specifically, a ghost is found at the linearized level, unless the mass is precisely of the Fierz-Pauli type. This stability considerations apply even if the longitudinal mode is not a separate scalar field, but simply one of the polarizations of a massive vector or tensor.

We now show that, in the models [6, 8, 9], M^2 needs to have the wrong sign (we refer the reader to [14, 12] for the discussion of the model [7]). We assume for definiteness that the spatial vev of the vector is aligned along the x direction. This gives the line element

$$ds^2 = -dt^2 + a\,(t)^2\, dx^2 + b\,(t)^2 \left[dy^2 + dz^2\right] . \tag{3}$$

We introduce the two expansion rates $H_a \equiv \dot{a}/a$, $H_b \equiv \dot{b}/b$, and we define their average H and rescaled difference h through $H \equiv \frac{H_a + 2\,H_b}{3}$ and $h \equiv \frac{H_b - H_a}{3}$. The inflationary expansions that we consider below are characterized by constant or slowly evolving rates. For the models we are considering, $h/H = {\rm O}\left(B^2\right)$, where B is the rescaled vev of the vector field $\langle A_x \rangle \equiv M_p\, a\, B$ [6, 8]. Therefore, B must also be slowly rolling during the slow roll regime. We consider the phenomenologically relevant case of moderate anisotropy, $B < 1$. To achieve slow roll, consider the action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{F^2}{4} - V\left(A^2\right) + \frac{\xi}{2} R\, A^2 \right] . \tag{4}$$

where the vacuum energy leading to the cosmological expansion is included in V. Expanding the potential at quadratic order in A_μ, and comparing with eq. (1), this action leads to the mass term

$$M^2 = 2\,\frac{\partial V}{\partial A^2} - \xi\, R = 2\,\frac{\partial V}{\partial A^2} - 6\xi \left(2H^2 + h^2 + \dot{H}\right) . \tag{5}$$

The equations of motion for the rescaled vev B obtained from (4) is $\ddot{B} + 3\,H\,\dot{B} + \mathcal{Q} B = 0$, with

$$\mathcal{Q} \equiv 2\,\frac{\partial V}{\partial A^2} - 2\,H\,h - 5\,h^2 - 2\,\dot{h} + (1 - 6\,\xi) \left(2\,H^2 + h^2 + \dot{H}\right) . \tag{6}$$

Slow roll of B requires $\mathcal{Q} \ll H^2$ (since the $3H\dot{B}$ term provides a "friction" to the motion). Ref. [6] studied solutions with constant $H_{a,b}$ in absence of the $A^2 R$ term, $\xi = 0$. This requires $\mathcal{Q} = 0$, or, in other terms

$$\frac{\partial V}{\partial A^2} = -H^2 + H\,h + 2\,h^2 = -H_a\,H_b < 0\,. \tag{7}$$

This corresponds to a negative square mass in eq. (5). Ref. [8] achieves the slow roll with a sufficiently small $\partial V/\partial A^2$, and with $\xi = 1/6$. This also gives a negative square mass in eq. (5). The model of [9] is also characterized by the action (4), with $\xi = 1/6$ (the requirement in this case is related to the scale invariance of the resulting curvature perturbations), and so also possesses a ghost, even if the background solution is isotropic (we should actually say "even if the background anisotropy is very small"; strictly speaking, a nonvanishing vector vev - which needs to be the case also for [9] - results in a small anisotropy; therefore, also for this model, the relevant stability computation is the one reported in Sections IV and V of [13]).

The addition of metric perturbations significantly complicates the computation, but does not change the above conclusions. More precisely, one perturbation (which becomes the longitudinal mode, in the limit in which gravity is decoupled) goes from being a well behaved mode to a ghost close to horizon crossing. When this happens, the linearized solutions for the perturbations ($\delta g_{\mu\nu}$ included) diverge. For the model [8], for which more vectors are presents, we found that some perturbations behave in this way, while some others are ghost during the whole sub-horizon regime. We also found that, if a small and positive mass term is added to the model of [9] (such mass term is already present in [8]) the perturbations also diverge when the total mass M^2 vanishes.

The computations we have performed are valid at the linearized level. One may argue that the instability we have found manifests itself only at this level, but it is somewhat cured by nonlinear interactions. While one cannot a priori disregard this possibility, one should also have in mind that all the phenomenological predictions given in the literature for the above models are based on linearized computations. One should not trust such predictions, which are based on an approximate treatment of the linearized system of perturbations, while the compete linearized computations that we have performed show instead that the solutions actually diverge. Concerning nonlinear interactions, one may expect that they actually worsen the problem, as the coupling between a healthy field and a ghost leads to vacuum decay; therefore theories with ghost can only be considered as effective field theories, valid only up a scale set by the mass term.

In fact, all these theories require a cut-off which makes them invalid at high energies, irrespectively of the sign of the mass term. We can see this based on the behavior of massive vector fields at high energies. The models studied here have a gauge invariance that is broken in a hard way by the explicit mass term $M^2 A^2$ for the vector. It is well known that, in such cases, the interactions of the longitudinal bosons violate unitarity at a scale which is parametrically set by M , leading to a quantum theory out of control. For the present models, M is the Hubble rate or below, so that the entire sub-horizon regime may be ill-defined. Although we are aware of explicit computations of this problem only in Minkowski spacetime, we believe that it applies also to the inflationary case, if one has unbroken Lorentz invariance and transitions to a locally flat frame (moreover, during inflation the mass term for the vector is nearly constant, and the momentum is adiabatically varying in the sub horizon regime). While this problem is present for both signs of M^2, "curing" a theory which has a hard vector mass and a ghost is more problematic than curing a theory with only a hard vector mass. The most immediate UV completion of a theory with a hard mass is through a higgs mechanism. The mass would be then due to the vev of a scalar field that becomes dynamical above the scale M. In this way the theory remains under control also in the short wavelength regime, and one can apply all the standard computations valid for scalar fields during inflation. However, if M^2 needs to have the wrong sign, the scalar field in this UV completed theory needs to be a ghost. In fact, when one states that a theory with ghosts is only valid as an effective theory, one is assuming that a UV complete theory which is ghost-free exists. However, we are not aware of any explicit construction that realizes this.

Recently, some models [15, 16] have been proposed precisely to avoid the instabilities that we have discussed. Although more complicated than those discussed here, such models are interesting, since they are existence proofs that it is indeed possible to avoid the rapid inflationary isotropization in a controllable fashion. They also constitute the first complete realizations in which anisotropic signatures can be concretely computed. The model [15] is characterized by coupling between the kinetic term of the vector field and a function of the inflaton, $f(\phi) F^2$. For a suitable choice of f the model can admit a slow roll anisotropic inflationary solution. As there is no mass term for the vector, this model is free from ghost instabilities. It was first shown in [17] that the distinctive scalar-tensor correlation is much smaller than what could be estimated from the amount of anisotropy in the background expansion. The model [16] is instead characterized by external time-dependent functions that multiply the kinetic and the mass term of the vector. For a suitable time dependence, the vector field produces a scale invariant and statistically isotropic power spectrum. It would be interesting to extend this model replacing these external functions with functions of a dynamical scalar field.

I conclude with a note on the references. In this written report of my talk I concentrate only on works that I have coauthored plus works that were essential for the narrative. For a more comprehensive review, and for proper credit to the relevant literature, I refer the interested reader to the list of references of my two latest works [13, 17] on the subject.

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