TeV-scale bileptons, see-saw type II and lepton flavor violation in core-collapse supernova

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Abstract

Electrons and electron neutrinos in the inner core of the core-collapse supernova are highly degenerate and therefore numerous during a few seconds of explosion. In contrast, leptons of other flavors are non-degenerate and therefore relatively scarce. This is due to lepton flavor conservation. If this conservation law is broken by some non-standard interactions, electron neutrinos are converted to muon and tau-neutrinos, and electrons – to muons. This affects the supernova dynamics and the supernova neutrino signal in several ways. In particular, the total neutrino luminosity in the first second of the collapse is increased due to the larger free path of the non-electron neutrinos. This effect may have important consequences as the increase of the neutrino luminosity is known to facilitate the explosion. We consider an extension of the Standard Model by scalar bileptons (i.e. heavy scalars with lepton number 2) which mediate lepton flavor violation. It is shown that in case of TeV-mass bileptons the electron fermi gas is equilibrated with non-electron species inside the inner supernova core at a time-scale $\sim (1-100)$ ms. In particular, a scalar triplet which generates neutrino masses through the see-saw type II mechanism is considered. Non-observation of rare decays together with known data on neutrino mixing and neutrino masses restrict possible lepton flavor violation effects in this case. However a region in the parameter space of the model exists which fits all the experimental constraints and provides lepton flavor violation sufficient for observable effects in supernova.

1 Introduction

The present contribution is largely based on our recent paper [1]. In addition, here we present an extended discussion of the increase of the total neutrino luminosity during the first second of the collapse and possible consequences of this important effect for supernova dynamics (see section 5.3).

If the Standard Model (SM) is extended by a triplet of heavy scalar fields interacting with leptons and bosons of the SM, it becomes possible to generate Majorana neutrino masses through the see-saw type II mechanism [2]-[4]. Originally the mass of this triplet was assumed to be at a GUT scale; recently, however, TeV-scale see-saw became popular. In particular, it was argued that TeV-scale see-saw type II avoids electroweak hierarchy problem, encountered in GUT-scale see-saw, and meets the naturalness argument (see e.g. [5][6]). See-saw II signatures at the LHC (see e.g. [7][8] and references therein) and in low-energy experiments (see e.g. [6][9] and references therein) are extensively investigated.

In the present paper we point out that TeV-scale see-saw II affects the core-collapse supernova physics, namely the supernova neutrino signal and the dynamics of the explosion. This conclusion is valid in a certain area of the parameter space of the neutrino mixing matrix. The

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effect is due to the lepton flavor violating (LFV) four-fermion interactions, which are induced by the scalar exchange.

Scalar triplet, which enters the see-saw II, is a particular case of scalar bilepton, i.e. scalar which couples to a pair of leptons. Another type of bilepton is a doubly charged singlet scalar. If possessing TeV-scale mass, it may also be produced at LHC and influence the results of low-energy experiments. We include the analysis of its effect on the LFV in supernova in the present paper, because it mediates LFV interactions analogously to the doubly charged component of the scalar triplet.

It is known that LFV processes may alter drastically the conditions in the interiors of a core-collapse supernova [10][11]. We discuss several possible consequences of such alteration, the most intriguing one being the possibility to facilitate the explosion.

The paper is organized as follows. In section 2 we estimate the value of the effective four-fermion LFV coupling necessary to significantly affect the supernova physics. In section 3 we consider a simple extension of the SM with a doubly charged bilepton. Section 4 is devoted to see-saw type II extension of the SM. Influence of LFV inside the supernova core on supernova physics is discussed in section 5. The results are summarized in section 6.

2 Model independent considerations

An explosion of a non-thermonuclear supernova is a manifestations of a collapse of a progenitor star core. Although numerous studies have failed to reproduce an explosion of a core-collapse supernova, there exists a commonly accepted general picture of the collapse (which is referred to as "standard picture" in what follows; for the details see, for example, [12]). When the mass of the iron core of a massive star reaches the Chandrasekhar limit, the infall phase of the collapse starts. The core contracts due to the gravitational attraction. Some fraction of electrons is converted to electron neutrinos through the inverse beta processes. When the density of the inner part of the core reaches the nuclear density value, $\sim 3 \cdot 10^{14}$ g/cm^3, the infalling matter of the outer core bounces from it. A shock wave emerges, which starts to propagate outwards. A powerful shock could expel the stellar envelope and produce the supernova explosion. However, according to the detailed numerical calculations [13], the shock wave looses its energy and stalls after some tens of milliseconds. There exist several scenarios of *presumably* successful explosion (e.g. [14]), but neither of them is justified to date by detailed self-consistent calculations with neutrino transport.

During the collapse extreme values of density, temperature, electron and electron neutrino chemical potentials are reached. For the estimates we use the reference values of the supernova core parameters as presented in Table 1.

ρ	n_B	Y_e	Y_{ν_e}	Y_μ	Y_{ν_μ}, Y_{ν_τ}
$2 \cdot 10^{14}$ g/cm^3	$1.2 \cdot 10^{38}$ cm^{-3}	0.30	0.07	$\sim 10^{-5}$	$\sim 10^{-4}$

T	μ_e	μ_{ν_e}	μ_μ	$-\mu_{\nu_\mu}$	$-\mu_{\nu_\tau}$
10 MeV	200 MeV	160 MeV	40 MeV	~ 1 MeV	$\ll 1$ MeV

Table 1: Typical conditions in the inner supernova core ($m \lesssim 0.5 M_\odot$) during the first 50 ms after core bounce as obtained by us with the help of the open-code program Boom [15]. Only SM interactions are taken into account.

This data is obtained by us with the help of the open-code programme [15] (in which only SM interactions are taken into account), being in good agreement with the results of more sophisticated calculations, see e.g. [16][17]. The presented values are the mean values for the inner supernova core ($m \lesssim 0.5 M_\odot$); the central values are even greater. Note that in the stan-

dard picture of the collapse electrons and electron neutrinos are plentiful and highly degenerate during the first hundreds of milliseconds. Their total number roughly equals the total number of electrons in the core before collapse, and decreases slowly due to the leakage of neutrinos from the core. By contrast, there are very few non-electron neutrinos and muons. Non-electron neutrinos have negligible chemical potentials; they are created in pairs with their antiparticles, concentrations being proportional to T^3. Muons have chemical potential approximately equal to $\mu_e - \mu_{\nu_e}$, their concentration has an additional suppression factor $e^{-(m_\mu - \mu_\mu)/T}$. Lepton flavor conservation law prevents electrons and electron neutrinos to turn into other types of leptons.

This picture is modified if there exist LFV processes beyond the SM [10][11]. If electrons and electron neutrinos are converted into leptons of other flavors inside the inner core[1], the dynamics of the collapse and the supernova neutrino signal is different from the SM case. In particular, the energy possessed by leptons is redistributed between all neutrino flavors, and degenerate seas of ν_μ and ν_τ emerge. The following LFV reactions could be *in principle* relevant for supernova:[2]

$$\begin{aligned} e^-e^- &\to \mu^-\mu^-, \\ e^-\nu_e &\to \mu^-\nu_\mu, \\ \nu_e\nu_e &\to \nu_\mu\nu_\mu, \\ \nu_e\nu_e &\to \nu_\tau\nu_\tau, \end{aligned} \tag{1}$$

$$\begin{aligned} e^-e^- &\to e^-\mu^-, \\ e^-\nu_e &\to e^-\nu_{\mu,\tau}, \\ e^-\nu_e &\to \mu^-\nu_{e,\tau}, \\ \nu_e\nu_e &\to \nu_e\nu_{\mu,\tau}, \\ \nu_e\nu_e &\to \nu_\mu\nu_\tau, \end{aligned} \tag{2}$$

The reactions are divided into two groups. In the first group all individual lepton flavor numbers, L_e, L_μ and L_τ are changed by two units or unchanged at all (e.g. for the first reaction $\Delta L_e = -2,\ \Delta L_\mu = 2,\ \Delta L_\tau = 0$). In the second group at least some of the individual lepton flavor numbers are changed by one unit. As is discussed below, processes from the second group are tightly constrained by non-observation of rare decays and hardly can affect the supernova physics; in contrast, the processes from the first group can affect the supernova physics and at the same time fit the present experimental constraints.

Let us estimate the value of the cross sections of the LFV reactions potentially important for supernova core. For this purpose we demand that such reactions should lead to the thermal equilibrium between e and ν_e on the one hand and leptons of other flavor(-s) on the other hand at a certain timescale Δt (note that equilibration between charged leptons and neutrinos of the same flavor is established virtually instantly due to the ordinary weak interactions). We consider two reference values for Δt. The first one, $\Delta t_1 = 1$ ms, is characteristic for the dynamics of the supernova (e.g. the shock propagates through the core during several ms). The second one, Δt_2, should characterize the Kelvin-Helmholtz timescale of the cooling of a proto-neutron star (it is somewhat larger than the neutrino diffusion timescale, see section **??**), which determines the long-term supernova neutrino signal. Although the inner core is deleptonized due to the diffusion at a timescale ~ 5 s [18], we conservatively chose $\Delta t_2 = 300$ ms, as at greater times the numbers presented in Table 1 are significantly changed.

From the above-mentioned condition we find for the processes in which electrons are present in the initial state

$$\sigma \gtrsim 1/(\Delta t n_e c) \simeq 3 \cdot 10^{-48} (300 \text{ ms}/\Delta t) \text{cm}^2. \tag{3}$$

[1]Not to be confused with ordinary resonant neutrino flavor conversion due to the MSW-effect, which take place in the envelope. It affects the neutrino signal but do not affect the supernova dynamics.

[2]We consider processes in which only leptons take part.

If only neutrinos are involved in the process, the cross section should be several times greater, as the neutrino density is several times smaller. The estimated value appears to be rather small compared to ordinary weak cross sections. For example,

$$\sigma(\nu_e n \to e^- p) \simeq 2 \cdot 10^{-39} \text{ cm}^2 \tag{4}$$

for 160 MeV neutrinos [19].

Reactions (1) and (2) at low energies are described by effective four-fermion terms in the Lagrangian, the cross section being of order of $G^2_{\rm LFV}\mu^2_{e,\nu_e}$, where μ_{e,ν_e} stands for the chemical potential of electron and electron neutrino correspondingly (every reaction is characterized, in general, by its own $G_{\rm LFV}$). From (3) one gets a rough estimate for $G_{\rm LFV}$ necessary for large LFV in supernova. In case of electron-electron collisions it reads

$$G_{\rm LFV} \gtrsim 4 \cdot 10^{-4}\sqrt{300 \text{ ms}/\Delta t} \text{ TeV}^{-2}. \tag{5}$$

In case of neutrino-electron and neutrino-neutrino collisions the bound is somewhat larger due to the smaller neutrino density and chemical potential. Such bounds on $G_{\rm LFV} \sim \lambda^2/M^2$ indicate the mass M of intermediate heavy boson of order of several TeVs if the dimensionless coupling constant λ is of order of 0.1.

Are such values of $G_{\rm LFV}$ compatible with present experimental constraints? This question is accurately treated in the following two sections, here we provide a preliminary discussion. The answer is different for two groups of reactions, (1) and (2). In the former case, the best experimental limit stems from the non-observation of oscillations of muonium atom, $\mu^- e^+ \leftrightarrow \mu^+ e^-$[20], and reads $G_{\rm LFV} \lesssim 10^{-1}$ TeV^{-2}. Although muonium oscillations involve only charged leptons, due to the SU(2) symmetry the bound on the effective constant is valid for the first three processes in (1), including those which involve neutrinos. As for the fourth process in (1), it evades even this not very restrictive bound. Thus there is a room for LFV in supernova due to four reactions (1) with $|\Delta L_{e,\mu,\tau}| = 0, 2$.

As for reactions with $|\Delta L_l| = 1$ (2), they are tightly constrained by the rare lepton decays:

$$\begin{array}{lcl} \mu & \to & eee, \\ \mu & \to & e\gamma, \\ \tau & \to & lll. \end{array} \tag{6}$$

The constraints span from $G_{\rm LFV} \lesssim 10^{-2}$ TeV^{-2} (LFV decays of τ) to $G_{\rm LFV} \lesssim 10^{-5}$ TeV^{-2} ($\mu^- \to e^- e^- e^+$), see [21] and references therein. Therefore processes (2) can fit condition (3) only marginally or can not fit it at all. Thus they are likely to be irrelevant for LFV in supernova.

To conclude this section, we are interested in such an extension of the SM, in which four-fermion LFV processes with $|\Delta L_l| = 1$ are suppressed compared to the processes with $|\Delta L_l| = 2$. In the next section we give a simple but instructive example of such extension. In section 4 we show that see-saw type II model fits this requirements for certain values of neutrino masses and mixing parameters.

3 Doubly charged scalar singlet

Consider the SM extended by one doubly charged scalar Δ, singlet under SU(2). It can not be coupled to quarks due to its electric charge. The only possible coupling of such scalar to leptons reads

$$\mathcal{L}_\Delta = \sum_{l,l'} \lambda_{ll'} \overline{l^c_R} l'_R \Delta + h.c. \tag{7}$$

Here
l and l' are charged leptons, $l, l' = e, \mu, \tau$;

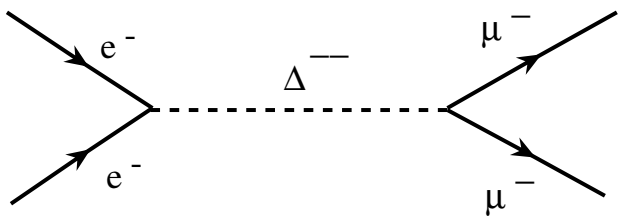

Figure 1: $ee \to \mu\mu$ LFV transition mediated by the doubly charged bilepton Δ^{--}

$\lambda_{ll'}$ is a matrix of coupling constants, summation over lepton flavors is implied.
C-conjugated right-handed fermions here and in what follows are defined as $\psi_R^c \equiv (\psi_R)^c = \gamma^2(\psi_R)^*$, standard representation of gamma-matrices being used.

Let us choose the coupling matrix to be proportional to unit matrix: $\lambda_{ll'} = \lambda\delta_{ll'}$. In this model low-energy processes with $|\Delta L_l| = 1$ are forbidden.[3]

The exchange of doubly charged bilepton gives rise only to one LFV reaction, interesting in the context of supernova core conditions (see Fig. 1):

$$e^-e^- \to \mu^-\mu^-. \tag{8}$$

The corresponding effective low-energy LFV term reads

$$\mathcal{L}_{LFV}^{eff} = \frac{\lambda^2}{M_\Delta^2}\overline{e_R^c}e_R(\overline{\mu_R^c}\mu_R)^\dagger, \tag{9}$$

which gives (see e.g. [22][23][24])

$$\sigma(ee \to \mu\mu) = \frac{\lambda^4}{M_\Delta^4}(1 - 2m_\mu^2/s)|t_{max} - t_{min}|/8\pi. \tag{10}$$

Note that one should take into account the combinatoric factors and the identity of fermions in the final state while calculating this cross section. In the center-of-mass reference frame $|t_{max} - t_{min}| = 4E_e\sqrt{E_e^2 - m_\mu^2}$. Taking $E_e \simeq \mu_e \sim 2m_\mu$ one obtains

$$\sigma(ee \to \mu\mu) \simeq 2 \cdot 10^{-42}\left(\frac{\lambda^4}{(M_\Delta/\mathrm{TeV})^4}\right)\ \mathrm{cm}^2. \tag{11}$$

Comparing this with the estimate (3) one finds that the supernova physics is modified if

$$\frac{\lambda^2}{(M_\Delta/\mathrm{TeV})^2} \gtrsim 10^{-3}\sqrt{300\ \mathrm{ms}/\Delta t}. \tag{12}$$

As was mentioned above, the best experimental bound on λ^2/M_Δ^2 follows from the non-observation of oscillations of muonium atom [20]. We infer this bound from the review paper [6] taking into account that the definition of coupling constants in [6] and in the present work differ by the factor $\sqrt{2}$:

$$\frac{\lambda^2}{(M_\Delta/\mathrm{TeV})^2} < 0.2, \quad 90\%\ \mathrm{CL}. \tag{13}$$

From inequalities (12) and (13) it follows that even for $\Delta t = 1$ ms there is a room for a singlet bilepton which affects the supernova physics and evades direct experimental constraints. If one takes $\lambda = 0.1$, then the bilepton mass should satisfy

$$220\ \mathrm{GeV} < M_\Delta \lesssim 3.2 \cdot (300\ \mathrm{ms}/\Delta t)^{-1/4}\ \mathrm{TeV}. \tag{14}$$

[3]Note that in general the coupling matrix $\Lambda \equiv ||\lambda_{ll'}||$ is not invariant under rotations of the charged lepton basis. It transforms according to $\Lambda' = U^T\Lambda U = \lambda U^T U$, where U is a matrix of a unitary rotation, $U^\dagger U = 1$. However, in case when U does not contain complex phases one gets $U^\dagger = U^T$ and $\Lambda' = \Lambda$. The role of complex phases is investigated in more detail in the next section.

4 Scalar triplet. See-saw type II.

4.1 Cross sections

Let us now discuss LFV mediated by a scalar triplet $\mathbf{\Delta}$. We are especially interested in the case when such triplet is responsible for the generation of Majorana neutrino masses through the see-saw type II mechanism [2]-[4]. In this mechanism scalar triplet additionally couples to the SM Higgs bosons, this coupling producing a vacuum expectation value for the neutral component of the triplet. The neutrino masses are proportional to this vev.

The see-saw type II Lagrangian contains two major ingredients, a scalar-lepton interaction,

$$\mathcal{L}_{ll\Delta} = \sum_{l,l'} \lambda_{ll'} \overline{L_l^c} i\tau_2 \Delta L_{l'} + h.c., \tag{15}$$

and a scalar potential, which in its minimal form reads

$$V = -M_H^2 H^\dagger H + f(H^\dagger H)^2 + M_\Delta^2 Tr(\Delta^\dagger \Delta) + \frac{1}{\sqrt{2}}(\tilde{\mu} H^T i\tau_2 \Delta^\dagger H + h.c.). \tag{16}$$

Here

$$\Delta \equiv \mathbf{\Delta\tau}/\sqrt{2} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \tag{17}$$

$L_l \equiv \begin{pmatrix} (\nu_l)_L \\ l_L \end{pmatrix}$ is a doublet of left-handed leptons of flavor $l = e, \mu, \tau$, H is a Higgs doublet, $\tilde{\mu}$ is a parameter with the dimension of mass. Note that the definition of coupling constant $\tilde{\mu}$ varies in different works by a numerical coefficient. Our definition coincides e.g. with that in [9].

Note that due to the anticommutation of the fermion fields 3×3 matrix $\Lambda \equiv ||\lambda_{ll'}||$ is symmetric,

$$\Lambda^T = \Lambda. \tag{18}$$

The vev of the neutral component of the triplet reads

$$\langle \Delta^0 \rangle = \frac{\tilde{\mu} v^2}{2\sqrt{2} M_\Delta^2}, \tag{19}$$

where $v \equiv \sqrt{2}\langle H^0 \rangle = 246$ GeV. Due to the triplet vev neutrinos acquire the Majorana mass according to

$$m = 2\langle \Delta^0 \rangle \Lambda, \tag{20}$$

where $m \equiv ||m_{ll'}||$ is a neutrino mass matrix in a flavor basis. One gets that in the see-saw type II model neutrino mass matrix m is proportional to the coupling matrix Λ. Therefore now Λ can not be exactly diagonal as in the previous section, because m has non-zero non-diagonal entries (we know this since neutrinos oscillate). However, current experimental data on neutrino masses and mixing are incomplete and do not fix the ratio of diagonal and non-diagonal elements. In particular, it may appear that m is approximately diagonal, and thus it is possible to achieve the suppression of the processes with $|\Delta L_l| = 1$ compared to the processes with $|\Delta L_l| = 2$. This issue is considered in detail in the next subsection.

Lagrangian (15) gives rise to all four reactions (1), potentially relevant for supernova. Their cross sections in the center of mass reference frame read

$$\sigma(ee \to \mu\mu) = (|\lambda_{ee}|^2 |\lambda_{\mu\mu}|^2 / M_\Delta^4)(1 - m_\mu^2/2E_e^2)\sqrt{1 - m_\mu^2/E_e^2}\; E_e^2/2\pi, \tag{21}$$

$$\sigma(e\nu_e \to \mu\nu_\mu) = (|\lambda_{ee}|^2 |\lambda_{\mu\mu}|^2 / M_\Delta^4)(1 - m_\mu^2/4E_e^2)^2 E_e^2/4\pi, \qquad E_e = E_{\nu_e}, \tag{22}$$

$$\sigma(\nu_e\nu_e \to \nu_\mu\nu_\mu) = (|\lambda_{ee}|^2 |\lambda_{\mu\mu}|^2 / M_\Delta^4) E_{\nu_e}^2/2\pi, \tag{23}$$

process	constraint on	bound, $\times(M_\Delta/\mathrm{TeV})^2$
$\mu^- \to e^+e^-e^-$	$\lvert\lambda_{e\mu}\lambda_{ee}\rvert$	$< 2.4 \cdot 10^{-5}$
$\tau^- \to e^+e^-e^-$	$\lvert\lambda_{e\tau}\lambda_{ee}\rvert$	$< 2.6 \cdot 10^{-2}$
$\tau^- \to \mu^+\mu^-\mu^-$	$\lvert\lambda_{\mu\tau}\lambda_{\mu\mu}\rvert$	$< 2.4 \cdot 10^{-2}$
$\tau^- \to \mu^+e^-e^-$	$\lvert\lambda_{\mu\tau}\lambda_{ee}\rvert$	$< 1.9 \cdot 10^{-2}$
$\tau^- \to e^+\mu^-\mu^-$	$\lvert\lambda_{e\tau}\lambda_{\mu\mu}\rvert$	$< 2.0 \cdot 10^{-2}$
$\mu \to e\gamma$	$\lvert\lambda^*_{e\mu}\lambda_{ee} + \lambda^*_{\mu\mu}\lambda_{e\mu} + \lambda^*_{\tau\mu}\lambda_{e\tau}\rvert$	$< 9.4 \cdot 10^{-3}$
$\mu^-e^+ \leftrightarrow \mu^+e^-$	$\lvert\lambda_{\mu\mu}\lambda_{ee}\rvert$	< 0.2

Table 2: Constraints from low energy experiments. We extract numerical values from the review paper [6] taking into account that the definitions of coupling constants in [6] and in the present work differ by the factor $\sqrt{2}$.

$$\sigma(\nu_e\nu_e \to \nu_\tau\nu_\tau) = (|\lambda_{ee}|^2|\lambda_{\tau\tau}|^2/M_\Delta^4)E_{\nu_e}^2/2\pi, \tag{24}$$

where neutrinos and electrons are assumed to be extremely relativistic.

Density and chemical potential of electrons are higher than those for electron neutrinos; also an additional factor 1/2 persists in eq. (22). As a consequence, the first reaction in (1) dominates over the second and the third ones provided the c.o.m. energy is considerably greater than the threshold $2m_\mu$. The conditions on effective coupling constants, *either* of which ensures the appreciable rate of LFV in supernova core, now read

$$\frac{|\lambda_{ee}\lambda_{\mu\mu}|}{M_\Delta^2} \gtrsim 10^{-3}\sqrt{300\ \mathrm{ms}/\Delta t}\ \frac{1}{\mathrm{TeV}^2}, \tag{25}$$

$$\frac{|\lambda_{ee}\lambda_{\tau\tau}|}{M_\Delta^2} \gtrsim 2.4 \cdot 10^{-3}\sqrt{300\ \mathrm{ms}/\Delta t}\ \frac{1}{\mathrm{TeV}^2}. \tag{26}$$

It is shown in the following subsection that $\lambda_{\mu\mu} \simeq \lambda_{\tau\tau}$ for the interesting set of neutrino parameters. Therefore it is always easier to satisfy condition (25) than condition (26); for this reason we concentrate on condition (25) in what follows.

If one assumes $\lambda_{ee} \simeq \lambda_{\mu\mu} = 0.1$ and takes $\Delta t = 300$ ms, then, analogously to the case considered in the previous section, supernova appears to be able to test $M_\Delta \lesssim 3.2$ TeV. To compare, the LHC can probe $M_\Delta \lesssim 1$ TeV [7].

4.2 Neutrino mass matrix, rare decays and experimental constraints on TeV-scale see-saw type II

LFV mediated by bileptons is constrained by non-observation of rare decays (6) and muonium conversion, see [6][22]. The widths of rare decays and the rate of conversion are proportional to $|\lambda_{l_1l_2}\lambda_{l_3l_4}/M_\Delta^2|^2$, where $l_1 - l_4$ stand for the corresponding leptons. Experimental bounds are presented in Table 2.

Evidently the most stringent constraint stems from $\mu \to eee$ decay. Therefore in the present contribution we focus on this constraint. It may be shown that if this constraint does not contradict the condition (25), then other constraints also do not contradict it [1].

The $\mu \to eee$ constraint does not contradict the condition (25) if

$$|\lambda_{\mu\mu}/\lambda_{e\mu}| = |m_{\mu\mu}/m_{e\mu}| \gtrsim 40\sqrt{300\ \mathrm{ms}/\Delta t}. \tag{27}$$

The neutrino mass matrix in the flavor basis, m, is obtained from the diagonal mass matrix through the transformation [25]

$$m = U^* \cdot \mathrm{diag}(m_1, m_2, m_3) \cdot U^\dagger, \tag{28}$$

where $U \equiv ||U_{li}||$ $(l = e, \mu, \tau,\ i = 1, 2, 3)$ is a PMNS neutrino mixing matrix:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times$$

$$\times \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1), \tag{29}$$

$$c_{ij} \equiv \cos\theta_{ij}\ , \quad s_{ij} \equiv \sin\theta_{ij}\ \ .$$

Note that eq.(28) implies $m^T = m$ in accordance with eqs.(18),(20). The explicit expressions for the entries of m may be found in [7]; for the sake of completeness we provide this expressions in our notations.[4]

$$\begin{array}{lcl} m_{ee} & = & a\ c_{13}^2 + s_{13}^2 m_3 e^{2i\delta} \\ m_{\mu\mu} & = & m_1 e^{-i\alpha_1}(s_{12}c_{23} + s_{13}e^{-i\delta}c_{12}s_{23})^2 + m_2 e^{-i\alpha_2}(c_{12}c_{23} - s_{13}e^{-i\delta}s_{12}s_{23})^2 + m_3 c_{13}^2 s_{23}^2 \\ m_{\tau\tau} & = & m_1 e^{-i\alpha_1}(s_{12}s_{23} - s_{13}e^{-i\delta}c_{12}c_{23})^2 + m_2 e^{-i\alpha_2}(c_{12}s_{23} + s_{13}e^{-i\delta}s_{12}c_{23})^2 + m_3 c_{13}^2 c_{23}^2 \\ m_{e\mu} & = & c_{13}[d s_{12}c_{12}c_{23} + s_{13}e^{i\delta}s_{23}(m_3 - a e^{-2i\delta})] \\ m_{e\tau} & = & c_{13}[-d s_{12}c_{12}s_{23} + s_{13}e^{i\delta}c_{23}(m_3 - a e^{-2i\delta})] \\ m_{\mu\tau} & = & s_{23}c_{23}(-b + c_{13}^2 m_3) - s_{13}d e^{-i\delta}s_{12}c_{12}(c_{23}^2 - s_{23}^2) + s_{13}^2 a e^{-2i\delta}s_{23}c_{23} \end{array} \tag{30}$$

Here we define the parameters with the dimension of mass

$$\begin{array}{lcl} a & \equiv & m_1 e^{-i\alpha_1}c_{12}^2 + m_2 e^{-i\alpha_2}s_{12}^2 \\ b & \equiv & m_1 e^{-i\alpha_1}s_{12}^2 + m_2 e^{-i\alpha_2}c_{12}^2 \\ d & \equiv & m_2 e^{-i\alpha_2} - m_1 e^{-i\alpha_1} \end{array} \tag{31}$$

If the masses m_1 and m_2 are quasi-degenerate, $m_1 \simeq m_2$, and Majorana phases α_1 and α_2 are approximately equal, $\alpha_1 \simeq \alpha_2$ (this case is of particular interest, as is shown below), then this parameters reduce to

$$a \simeq b \simeq e^{-i\alpha_1}m_1 \tag{32}$$

$$d \simeq e^{-i\alpha_1}(m_2 - m_1) \tag{33}$$

There are nine parameters in the mass matrix: three neutrino masses m_i, three mixing angles θ_{ij}, the Dirac phase δ and two Majorana phases $\alpha_{1,2}$. Experimentally measured (or constrained) quantities are [26]:

$$\theta_{12} \simeq 34^o, \quad \theta_{23} \simeq 45^o \text{ or } \theta_{23} \simeq 135^o, \quad \theta_{13} < 13^o, \tag{34}$$

$$\Delta m_{21}^2 \simeq 0.8 \cdot 10^{-4}\mathrm{eV}^2, \quad \Delta m_{32}^2 \simeq \pm 2.4 \cdot 10^{-3}\mathrm{eV}^2. \tag{35}$$

In addition, cosmological considerations [27] demonstrate that

$$m_1, m_2, m_3 < 0.35 \text{ eV}. \tag{36}$$

Unknown quantities are phases δ and $\alpha_{1,2}$, the mixing angle θ_{13}, the sign of Δm_{32}^2, the quadrant of θ_{23}, the absolute scale of neutrino masses. Depending on the sign of Δm_{32}^2 two patterns of neutrino masses are possible: normal mass hierarchy (NH), $m_1 < m_2 < m_3$, and inverted mass hierarchy (IH), $m_3 < m_1 < m_2$.

Formulas (30) allow to check whether condition (27) is valid for any specific choice of neutrino masses and mixing parameters. However, let us outline general trends of the dependence of $|m_{\mu\mu}/m_{e\mu}|$ on the neutrino masses, mixing angles and phases.

[4] It is worth noting that if the Majorana phase matrix would stay at the first place in the r.h.s. of eq.(29), then the absolute values of entries of m would not depend on Majorana phases. The correctness of the ordering of matrices in eq.(29) is justified by the diagonalization of lepton mass matrices [25].

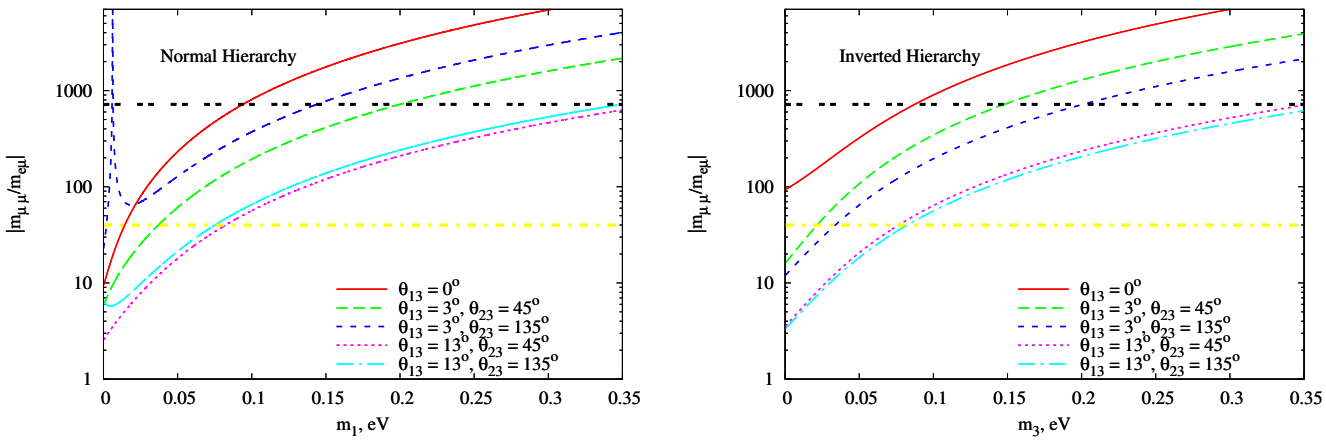

Figure 2: Ratio $|m_{\mu\mu}/m_{e\mu}|$ in case of exact CP-conservation ($\alpha_1 = \alpha_2 = \delta = 0$). If $\theta_{13} = 0$ then $|m_{\mu\mu}/m_{e\mu}|$ does not depend on the quadrant of θ_{23}. Two thick horizontal lines correspond to $|m_{\mu\mu}/m_{e\mu}| = 40$ and $|m_{\mu\mu}/m_{e\mu}| = 720$, which are minimal values sufficient for LFV in supernova core to proceed at scales 300 ms and 1 ms accordingly.

Generically

$$|m_{\mu\mu}| \sim \max\{m_2, m_3\}. \tag{37}$$

If one takes all three phases, $\alpha_1, \alpha_2, \delta$, randomly from the interval $[0, 2\pi]$, then in the majority of cases

$$|m_{e\mu}| \sim \max\{m_2, s_{13}m_3\}, \tag{38}$$

which implies $|m_{\mu\mu}/m_{e\mu}| \lesssim 5$.[5] Hence, in order to fit inequalities (27) one should first of all impose some bounds on the phases. Note that if all three phases vanish (which correspond to the exact CP-conservation in the neutrino sector), $m_{e\mu}$ reduces to

$$m_{e\mu} \simeq s_{12}c_{12}c_{23}\Delta m^2_{21}/(m_1 + m_2) + s_{13}s_{23}\Delta m^2_{32}/(m_2 + m_3). \tag{39}$$

The first term here is small compared to m_2 if $m_1 \simeq m_2$. The second term is small compared to m_3 if either $m_2 \simeq m_3$ or s_{13} is small. Therefore we come to a conclusion that small phases, large masses (which implies degeneracy of masses) and small θ_{13} favour LFV in the supernova core. This is analogous to the conclusions of ref.[28], in which collider and low-energy LFV signatures were considered.

We illustrate and refine this conclusions by means of plots depicted in Figs. 2-3. On Fig. 2 the ratio $|m_{\mu\mu}/m_{e\mu}|$ is plotted as a function of a minimal neutrino mass (m_1 for NH, m_3 for IH) for different values of θ_{13} and different choices of the quadrant of θ_{23}. All phases are taken to be zero, $\alpha_1 = \alpha_2 = \delta = 0$. One can see that in case of quasi-degeneracy of all three neutrino masses, $m_1 \simeq m_2 \simeq m_3 \gtrsim 0.1$ eV, condition (27) is fulfilled for any allowed value of θ_{13}. In case of a less stringent restriction, $m_1 \simeq m_2 \gtrsim 0.05$ eV, condition (27) with $\Delta t = 300$ ms is fulfilled for $\theta_{13} \lesssim 3^o$. Note that two terms in $m_{e\mu}$ have different signs either in case of NH, $c_{23} < 0$ or in case of IH, $c_{23} > 0$, see eq.(39). This explains, particulary, the discontinuity at $m_1 \simeq 0.006$ eV in case of normal hierarchy, $\theta_{13} = 3^o$, $\theta_{23} = 135^o$.

To what extent is it possible to relax the requirement $\alpha_1 = \alpha_2 = \delta = 0$? To answer this question we first note the following facts.

- If

$$\alpha_1 = \alpha_2, \tag{40}$$

[5] To get this bound one should take into account that $m_2 > 0.009$ eV and in case of normal hierarchy $m_3 > 0.05$ eV.

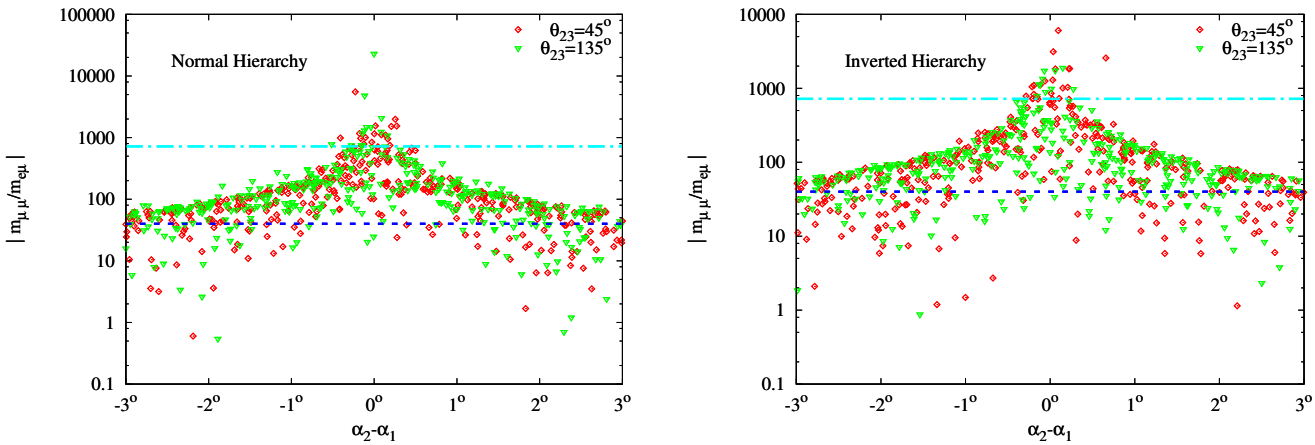

Figure 3: Scatter plots for the ratio $|m_{\mu\mu}/m_{e\mu}|$. Each point corresponds to a random choice of neutrino parameters in the following ranges: $m_{\min} \in [0.05\text{ eV}, 0.35\text{ eV}], \theta_{13} \in [0^o, 3^o], \delta \in [-90^o, 90^o], \alpha_1 \in [-2\delta - 3^o, -2\delta + 3^o], \alpha_2 \in [-2\delta - 3^o, -2\delta + 3^o]$. If the hierarchy is normal, then $m_{\min} = m_1$, otherwise $m_{\min} = m_3$

then the parameter d reduces to $e^{-i\alpha_1}(m_2 - m_1)$, as was pointed out above. If we for a moment put $\theta_{13} = 0$, then condition (40) ensures that $m_{e\mu}$ is generically small: $m_{e\mu} = c_{13}c_{12}s_{12}c_{23}e^{-i\alpha_1}(m_2 - m_1)\Delta m^2_{21}/(m_1 + m_2)$.

- If a more restrictive condition is superimposed,

$$\alpha_1 = \alpha_2 = -2\delta, \tag{41}$$

then $m_{e\mu}$ appears to be a sum of two generically small terms (with some relative phase factor) even for $\theta_{13} \neq 0$:

$$m_{e\mu} \simeq c_{12}s_{12}c_{23}e^{2i\delta}\Delta m^2_{21}/(m_1 + m_2) + s_{13}e^{i\delta}s_{23}\Delta m^2_{32}/(m_2 + m_3). \tag{42}$$

Condition (41) is more general than the requirement $\alpha_1 = \alpha_2 = \delta = 0$, and embraces the latter as a specific case. We illustrate allowed deviations from this condition on Fig.3. One can see that equalities (41) should be valid up to $\sim 3^0$ for $\Delta t = 300$ ms in case when $\theta_{13} \lesssim 3^0$ and $m_{\min} \gtrsim 0.05$ eV. One may further relax condition (41) imposing more restrictive requirements on the degeneracy of neutrino masses and smallness of θ_{13}. On the other hand the specific value of $\alpha_1 \simeq \alpha_2 \simeq -2\delta$, the quadrant of θ_{23} and the hierarchy of masses generically do not affect the value of $|m_{\mu\mu}/m_{e\mu}|$ significantly.

Now it is easy see that in the interesting range of neutrino mass-mixing parameters the last two diagonal elements of the matrix Λ (and of the matrix m) should be quasi-degenerate – a claim made in the end of section 4.1. This statement is verified numerically in [1].

As was mentioned above, as far as the condition (27) is fulfilled, not only $\mu \to eee$ constraint but all other constraints from the rare decays are satisfied. It is also worth mentioning that the $\tau \to \mu ee$ rare decay is the most useful one among $\tau \to lll$ decays for the purpose of constraining the impact of LFV on the supernova physics [1]. The importance of the $\tau \to \mu ee$ measurement for probing the see-saw type II model was emphasized in [29].

5 Possible consequences of LFV on supernova physics

Before discussing the influence of LFV on supernova physics, we first give a sketchy qualitative picture of a standard collapse (cf., e.g. [30]) in section 5.1. Then in subsection 5.2 we discuss

the collapse with LFV. The most intriguing effect of increase of neutrino luminosity due to LFV is discussed separately in subsection 5.3.

5.1 Standard collapse

There are several stages in the standard core-collapse scenario where different neutrino processes dominate: 1) neutronization in the transparent core at infall before bounce; 2) bounce, followed by shock-breakout with the most powerful neutrino burst; 3) accretion onto growing proto-neutron star with a stalled shock and possible shock revival; 4) Kelvin-Helmholtz stage of cooling of a hot proto-neutron star. The latter stage is characterized by a slow contraction of the hot core on a timescale of many seconds with the release of gravitational energy through neutrino emission.

Consider a polytropic equation of state

$$P = K\rho^{\gamma} \tag{43}$$

The fundamental critical value of γ is 4/3 in Newtonian theory. If the average $\gamma < 4/3$ over a stellar core, then this core is dynamically unstable. The only mass that may have a hydrostatic equilibrium for $\gamma = 4/3$ in Newtonian gravity is the Chandrasekhar mass

$$M_{\rm Ch} = 2.018(4\pi)\left(\frac{K}{\pi G}\right)^{3/2}. \tag{44}$$

This relation is derived in all classical textbooks on stellar structure. Physically, it is explained by a chain of simple arguments. The pressure P in general is the flux of momentum p, $P \sim pnv$, where n is the number density and v is the velocity of particles. For extremely relativistic (ER) particles $P \sim pnc$, and for degenerate electrons $p \sim p_{\rm Fermi} \sim 1/\text{distance} \sim n^{1/3}$. By definition of Y_e – the electron number density per baryon, $n = \rho Y_e/m_n$, where m_n is nucleon mass. This implies $P \propto n^{4/3} = (\rho Y_e/m_n)^{4/3}$ – which means $\gamma = 4/3$ in relation (43). (The definition of γ is $\partial \log P/\partial \log \rho|_S$.)

We see that $K \propto (Y_e/m_n)^{4/3}$ in (43). On the other hand, gravity requires the equilibrium pressure in the star center to be force/area $\propto G_N M^2/R^4 \propto G_N M^{2/3}\rho^{4/3}$. When equated with $P = K\rho^{4/3}$ this fixes $M_{\rm Ch}$ in (44). Since $K \propto (Y_e/m_n)^{4/3}$ one finds $M_{\rm Ch} \propto (Y_e/m_n)^2$. When the coefficients are substituted accurately, one gets

$$M_{\rm Ch} = 5.8 Y_e^2 M_\odot.$$

Curiously, since, in natural units $G_N \sim m_{Pl}^{-2}$, one may obtain an elegant expression

$$M_{\rm Ch} \sim \frac{m_{Pl}^3 Y_e^2}{m_n^2},$$

where m_{Pl} is Planck mass.

In general, instead of Y_e there should be a sum over all particles producing pressure (e.g. e and ν). And instead of the nucleon mass m_n there may be a mass of another particle if the latter dominates in producing gravity.

The contribution of baryons to the pressure is not significant until $\rho > \rho_n \approx 3\times 10^{14}$ (nuclear matter density). The equation of state during collapse is almost polytropic with γ slightly lower 4/3 due to small corrections from nuclei. The value of K, which depends on $Y_L = Y_e + Y_\nu$ and entropy S, is nearly constant since neutrinos are trapped.

Of course, at the bounce and later, in the center of the core the pressure is dominated by nuclear forces. However, the initial collapse begins when the pressure is dominated by electrons. The onset of the collapse in standard picture is determined either by the drop of

adiabatic exponent γ below 4/3 due to iron dissociation or due to neutronization (or both, and in both cases the collapse begins due to the lowering pressure when the star core contracts).

The dynamics of collapse in this case can be described by self-similar solutions [31] which show that a collapsing core separates into a homologous ($v \propto r$) central part, and an outer, supersonically infalling part. When the collapse is halted and bounces, a shock wave is produced at the sonic point. There is a self-similar solution with the shock as well [32].

A standard estimate for the free-fall collapse timescale [30] gives

$$t_f = \left(\frac{3}{8\pi G_N \rho}\right)^{1/2} \simeq 1.2\rho_{12}^{-1/2}\text{ms}, \tag{45}$$

where ρ_{12} is the average density in units of 10^{12} g/cm^3. Below we use notation ρ_k for density in units of 10^k g/cm^3.

This estimate is not very realistic because it assumes zero pressure. In reality the force produced by the pressure of ER degenerate leptons ($\sim n^{4/3}R^2$) and the gravity force ($\sim GM^2/R^2$) scale according to the same law ($\propto R^{-2}$). Therefore the contribution of the pressure force remains finite and non-negligible compared to the gravity force during the entire collapse. A better estimate for the collapse timescale t_c which agrees with numerical simulations can be derived from self-similar models [31, 32]:

$$t_c \simeq 6t_f. \tag{46}$$

The mass of the inner core turns out to be about 10% greater than the Chandrasekhar mass for the same value of K, and is thus proportional to the lepton fraction Y_L^2.

Another important timescale characterizes neutrino diffusion. Only when the diffusion is less rapid than the collapse will the lepton fraction be frozen for the duration of the collapse. The standard expression for the diffusion timescale is

$$t_d = \frac{3R^2}{c\ell_\nu} \simeq \frac{M^{2/3}\rho^{1/3}\sigma}{cm_n}. \tag{47}$$

If one takes the values corresponding to the inner core already after bounce, $M \simeq 0.6M_\odot,\ \rho \sim \rho_{14},\ \sigma \sim 10^{-39}$ cm^2, one gets

$$t_d \sim 10 \text{ s}. \tag{48}$$

Kelvin-Helmholtz time is a bit larger than the diffusion timescale because t_d must be multiplied by a factor reflecting the total energy storage over the energy in neutrinos.

Neutrino signal basically consists of three consecutive periods. The first one is the neutronization burst – a short ($\sim$ 10 ms) powerful outburst of neutrinos which emerges when the shock wave reaches the neutrino sphere. Neutronization burst takes place few ms after bounce, and contains almost exclusively electron neutrinos, which are created during neutronization of stellar matter. The second period coincides with the existence of stalled shock and lasts $\sim$ 1 s. The third period corresponds to the cooling of the proto-neutron star; it is characterized by an exponential decrease of neutrino luminosity and energy and lasts $\sim$ 10 s. At the second and the third stages neutrinos and antineutrinos of all flavors are in approximate equilibrium at the neutrino sphere (not inside the core!) and therefore are emitted approximately in equal quantities.

Neutrino oscillations take place *above* the neutrino sphere. In the layer $\sim$ 100 km above the neutrino sphere collective oscillations occur (see e.g. [33]). In the outer stellar envelope more familiar MSW resonant flavor conversion take place (see e.g. [34]). Oscillations modify the flavor content of the neutrino signal. In particular, they may convert a fraction of electron neutrinos of neutronization burst into non-electron neutrinos.

5.2 Collapse with LFV

The rate of LFV reactions (1) during the *infall* phase is negligible, with possible exception of the last few milliseconds. There are two reasons for this. First, the cross-sections (21)–(24) are small being proportional to the typical energies of e and ν_e (i.e., to chemical potentials of e and ν_e). Second, the rates $n_{e,\nu_e}\langle\sigma v\rangle$ are small when the concentrations n_e and n_{ν_e} are low. Moreover, the first two reactions in (1) have thresholds proportional to muon mass m_μ. The reactions (1) may become important the few ms before bounce when the densities in the center approach the nuclear matter density.

Let us estimate, for example, when $ee \to \mu\mu$ process becomes important. The reaction is possible when $E_e > m_\mu$. At low density an electron may obtain such high energy only due to an exponential factor like $\exp(-m_\mu/T)$, however T is always appreciably lower than m_μ. At higher densities electrons become degenerate, and we should estimate their chemical potential, taking into account that electrons are extremely relativistic:

$$\mu_e \approx 11(\rho_{10}Y_e)^{1/3}\ \text{MeV}. \tag{49}$$

We see that μ_e surpasses the muon mass at $\rho \approx 3\times 10^{13}$ g/cm^3 for $Y_e = 0.3$ and the process $ee \to \mu\mu$ sets in. However, if μ_e is only slightly above m_μ, only a small fraction of total electron density $n_e = \rho Y_e/m_n$ contributes to $n_e\langle\sigma v\rangle$ of this reaction: only electrons near and above the Fermi surface take part in the reaction.

Nevertheless, the process of transformation $ee \to \mu\mu$ starts decreasing the electron pressure already at $\rho_{13} \sim 1$, and becomes very important at $\rho_{13} \sim 10$, when the chemical potential of electrons is twice the muon mass and the bulk of electrons contributes to the reaction. At this stage we may safely use an estimate for the reaction rate

$$\frac{1}{t} \sim n_e\langle\sigma v\rangle \approx 300\rho_{14}Y_e\ \text{s}^{-1}, \tag{50}$$

where we take $E_e = 2m_\mu$, $\lambda = 0.1$, and $M_\Delta = 1$ TeV (this gives $\sigma \approx 2\times 10^{-46}\text{cm}^2$). Thus the timescale for the process $ee \to \mu\mu$ is about 10 ms a few ms before the bounce. Our computations with the BOOM code show that the density $\rho \sim 10^{13}$ g/cm^3 is achieved in the center about 5 to 7 ms before the bounce in good agreement with estimate (46), and there is 1 to 2 ms from $\rho \sim 10^{14}$ g/cm^3 until the bounce.

The disappearance of electrons in $ee \to \mu\mu$ reaction starts actively contributing to the decrease of electron pressure – hence to the dynamics of the collapse. Though the effect is rather small (in a couple of ms we lose only 10 to 20% of electrons) still all small deviations of γ from 4/3 may play a role. Moreover, the process of conversion will continue on later stages. It will lead to the decrease of electron (and electron neutrino) chemical potential and to the growth of entropy and temperature, since electrons are taken away well below the Fermi surface (cf. [35, 36].

The two-way LFV reaction $ee \rightleftharpoons \mu\mu$ leads eventually to the establishment of equality $\mu_\mu = \mu_e$ for the chemical potentials. Muons become moderately degenerate since $\mu_\mu - m_\mu$ becomes somewhat larger than T. When the equilibrium is already established, the muon gas has $\gamma \simeq 5/3$ ($P \sim pn_\mu v$ and $v = p/m_\mu$, hence $P \sim p^2 n_\mu/m_\mu \propto n_\mu^{5/3}/m_\mu$), which is harder than for ER leptons, although softer than for nuclear matter of high density. However, in the beginning of the transformation $ee \to \mu\mu$, when the reverse process is not yet effective, the pressure of the muon gas is much smaller than that of the electron gas due to the large mass (and therefore low momenta) of muons. Thus in the beginning of the $ee \to \mu\mu$ conversion, which may start several ms before the standard bounce, a decrease of P_e can not be compensated by the increase of P_μ, and we have an effect of reduction of total pressure similar to neutronization. This may lead to a faster collapse, hence to a stronger bounce.

There is an opposite conclusion in ref. [37]. They claim that the reduction of Y_e (accompanied by the reduction of P_e) leads to a weaker bounce. Note, however, that a *one-zone* collapse

model was used in [37]. It may be good for a uniform homologously contracting core where γ is close to 4/3. Even when electrons are transformed into trapped electron neutrinos the gamma for lepton gas is close to 4/3 (all leptons are extremely relativistic). In this framework they give a standard estimate of the mass of the homologous core (HC), which is just approximated by a Chandrasekhar mass $M_{\rm Ch}$: $M_{\rm HC} \simeq M_{\rm Ch} \simeq 5.8 Y_e^2 M_\odot$ (a bit more accurate estimate following [31, 30] is $M_{\rm HC} \simeq 1.1 M_{\rm Ch}$). However, if the core started collapsing and then Y_e drops fast enough and non-uniformly (e.g., faster in the center, as in our case), then the picture may be more complicated. Effective γ lower than 4/3 means faster collapse, hence a partial loss of homology. Quantitatively, the effect will be small if the process $ee \to \mu\mu$ is very slow. Anyway, here we see the qualitative difference with the standard case which may lead to the deformation of the homologous flow at infall and even to the formation of a second strong shock (because the innermost parts of the flow, those that are in the deepest potential well, have to accelerate due to the decrease of pressure). The modification of the shock wave naturally leads to the modification of the neutronization burst. We postpone the discussion of quantitative results for future research keeping in mind possible new dynamical phenomena.

Another place where the reduction of γ may be interesting is the shock after the bounce. There is a stage of more or less steady accretion through a stalled shock. When the pressure in the shock is dominated by leptons, and not by nuclear matter, the reduction of γ means higher compression (the density jump in any strong shock is $(\gamma+1)/(\gamma-1)$. However, when the free path corresponding to LFV processes (1) is much larger than a hundred of km this effect will not be noticeable.

The equilibrium between electrons and muons implies the equilibrium between electron and muon neutrinos which is established through charged current interactions. Also ν_e may be equilibrated with ν_μ and ν_τ directly through the LFV reactions (1). Anyhow, as a result the energy possessed by leptons is redistributed between two or three neutrino flavors, and degenerate seas of ν_μ and probably ν_τ emerge. The important consequences of such redistribution are discussed in the following subsection.

5.3 Increase of neutrino luminosity due to LFV and consequences for the delayed explosion scenario

How does the emergence of degenerate seas of non-electron neutrinos influence the neutrino transport? The key feature of ν_μ and ν_τ propagation in nuclear matter is that they do not take part in Charge Current (CC) interactions, in contrast with ν_e. The CC cross sections in the inner core are several times larger than Neutral Current (NC) cross sections. In the case of scattering on *free* nucleons

$$\sigma(\nu_e n \to ep) = 4\sigma(\nu_\mu n \to \nu_\mu n). \tag{51}$$

In the dense supernova matter this cross sections are modified (see e.g. [38]). Moreover, while calculating mean free paths one has to take into account Pauli blocking for final particles, which is different for electrons and neutrinos. Nevertheless one expects that the opacity of the inner core for non-electron neutrinos is considerably lower than for electron neutrinos. This means that the neutrino luminosity (total emitted energy per second) *increases* in case of LFV.

How does the increase of neutrino luminosity affect the explosion? Burrows&Goshy [39] and Murphy&Burrows [40] analyze this issue and conclude that such increase facilitates the explosion in the delayed explosion scenario. More specifically, they argue that for any given accretion rate $\dot{M}$ there exists a critical luminosity $L_{\nu_e\bar{\nu}_e}$. If the actual luminosity exceeds this critical luminosity, the supernova explodes. This is illustrated by Fig. 1 in [39] and Fig. 17 in [40]. In [39] the analysis is based on 1D models, why in [40] 2D models are explored. The comparison shows that the critical luminosity for 2D models is 70% the critical luminosity for 1D models.

Thus one may expect that LFV in the inner core tends to *facilitate* the delayed explosion.

To obtain a quantitative picture one has to make numerical simulations of the explosion in which ν_e comes to equilibrium with ν_μ (and, possibly, ν_τ) in the inner core. However to get a rough idea of what will happen one may use a simple trick which does not require considerable modification of the supernova simulation code. Namely, one may use a code with a standard physics but with Charged Current interactions artificially switched off for neutrinos in the inner core. This will emulate the situation when non-electron neutrinos dominate the transport in the inner core.

Remarkably, the relationship between the delayed explosion scenario and the decrease of neutrino opacities in the inner core was already studied by Burrows&Sawyer [41]. They do not consider any new physics. Instead, they take into account the density and spin correlations of nucleons and recalculate NC neutrino-nucleon scattering rates. They find that this scattering rates are suppressed by large factors around and above nuclear density, i.e. exactly in the inner supernova core. They conclude that supernova cores are more transparent than previously thought. Then they make some simplified protoneutron star cooling simulations with the artificially decreased neutrino opacities. They find that the early luminosities are not altered, but the luminosities after ~ 500 ms are increased by factors that range from 10% to 100%. A brief but instructive discussion of this effect and of consequences for the delayed explosion is presented in Section VII of ref. [41].

To conclude this section, we suggest that equilibration between ν_e and $\nu_{\mu,\tau}$ in the inner core due to New, Lepton Flavor Violating Physics may increase neutrino luminosity and thus facilitate the delayed explosion. To quantitatively test this idea it is necessary to make a numerical simulation with new physics included which lasts at least one second after bounce. However, to get a rough idea of what happens one may merely switch off Charged Current interactions in the inner core without introducing any new physics in the simulation.

6 Summary

We have shown that the lepton flavor violation, generated by the exchange of TeV-scale scalar bileptons, may drastically modify conditions inside a core-collapse supernova. An important example of such bileptons is a scalar triplet responsible for the neutrino mass generation in the see-saw type II scenario. In this scenario the matrix of scalar-lepton couplings is proportional to the neutrino mass matrix. The latter is currently partially fixed by neutrino oscillations and cosmological data. Also the effective four-fermion couplings, generated by the triplet, are restricted by low-energy experiments. We analyze the body of data and demonstrate that there exists an allowed region of the see-saw II parameter space in which LFV is strong enough to lead to the thermal equilibrium between electron and non-electron species inside the collapsing core. Roughly speaking, LFV affects the supernova physics if the matrix of scalar-lepton couplings is approximately proportional to the unit matrix, $\lambda_{ll'} \simeq \lambda\delta_{ll'}$, and the effective coupling λ^2/M_Δ^2 is greater than 10^{-3} TeV$^{-2} \simeq 10^{-4}G_F$. The former condition corresponds to neutrino masses greater than 0.05 eV and neutrino phases which satisfy the relations $|\alpha_1 + 2\delta|, |\alpha_2 + 2\delta| \ll \pi$.

Equilibration between electron and non-electron species is a striking feature, absent in the standard picture of the collapse. It starts to settle not earlier than few milliseconds before bounce, when the density exceeds 10^{13} g/cm^3. Such equilibration lifts the chemical potentials of muons and non-electron neutrinos higher than 100 MeV, somewhat decreases the chemical potentials of electrons and electron neutrinos, leads to the increase of temperature and entropy. In addition to Fermi seas of extremely degenerate electrons and electron neutrinos, which are present in the standard picture, seas of extremely degenerate non-electron neutrinos and moderately degenerate muons emerge in case of LFV. This may affect the supernova physics in various ways. An elaborated study should be carried out in order to accurately and thoroughly describe all possible consequences. In particular, collapse simulations with new interactions included are necessary. In the present work we restrict ourselves by just outlining various possibilities. The

most intriguing one is the increase of the neutrino luminosity in the first second of the collapse due to the larger free path of non-electron neutrinos compared to electron neutrinos. Such increase is known to facilitate the explosion in the delayed explosion scenario. Moreover, it can be read out from the neutrino signal of a future galactic supernova.

Other possible modifications of supernova dynamics are basically due to the decrease of pressure in the center of the core, where LFV reactions proceed effectively. Such decrease gives rise to a non-homologous stage of the collapse which may start few ms before the "standard" bounce. This may lead to modifications of the shock wave and even to additional shock waves. This, in turn, may lead to modifications of the neutronization burst and even to emergence of several neutronization bursts.

We emphasize once more time that all effects listed above should be verified by profound supernova calculations. Once such calculations are done, it is possible to use the results in various ways. On the one hand, one can try to construct a successful supernova explosion model taking into account lepton flavor violation. On the other hand, one can extract parameters of the see-saw type II model and other models which include bileptons (or restrict such parameters) from the already persistent or future astrophysical data.

As a finale remark we note that the discussed signatures of LFV in supernova are rather general and may be relevant in the context of various microscopic models, not necessarily those which include bileptons. In particular, the required LFV may be generated by the spin flips of Majorana neutrinos with large magnetic moment [42] or by the flavor changing neutral currents in SUSY with R-parity violation [43].

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