

Cosmological constraint on the Higgs boson mass in the SM minimally coupled to the gravity

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Abstract

The critical value for the mass of Higgs field in the Standard Model at the minimal interaction with the gravity is derived $m_H^{\text{crit}} = 153 \pm 3$ GeV by the constraint of decoupling the Higgs boson from the inflation of early Universe: the mass greater than critical one makes the Higgs particle *not able* to produce the inflation, that involves an *arbitrary* spectrum of inhomogeneity in the Higgs field fluctuations.

1 Introduction

At present, in cosmology the inflation stage has become the commonly recognized model for the evolution of early Universe [1, 2, 3, 4, 5]. In the simplest scenario consistent with the observed anisotropy of cosmic microwave background radiation (CMBR) [6, 7], the supernova data [8, 9, 10] and large-scale structure of Universe (LSS) [11], a scalar field of inflaton should possess some specific properties: an almost flat potential of self-action with the energy density of the order of $(10^{16}\text{GeV})^4$.

In this respect, recently the possibility of producing the inflation stage by the Higgs boson with a non-minimal coupling to the gravitation has been studied [12, 13, 14, 15, 16]. Then, in addition to the Einstein–Hilbert Lagrangian of gravitational field

$$L_{\text{EH}} = -\frac{1}{16\pi G}R,$$

the interaction Lagrangian has included the term of the form $\xi\Phi^\dagger\Phi R$, where Φ denotes the Higgs field, R is the Ricci scalar, G is the gravitational constant, and ξ is the coupling constant. So, an appropriate conformal transformation introduces an effective field minimally coupled to the gravity, while a relevant effective potential has got a flat plateau with an altitude regulated by the parameter ξ . The realistic model suggests $\xi \gg 1$, and in this respect, a new dynamical scale is introduced by M_{Pl}/ξ [17, 18], where the Planck mass M_{Pl} is defined by the gravitational constant G as $M_{\text{Pl}}^2 = 1/G$. The new scale determines the altitude of potential plateau and produces the threshold for changing a regime of coupling at ultraviolet virtualities. In this way, some definite constraints on the mass of Higgs boson have been derived. A lower bound for the Higgs boson mass is determined by the cosmology data (mainly, a slope of primary spectrum for the inhomogeneity of matter distribution), while the upper bound is set by a reasonable weakness of the self-coupling.

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Similarly, a modified model of induced gravity with the Higgs boson non-minimally coupled to the gravity was considered in [19, 20]. In that approach, the Einstein–Hilbert term with the bare gravitational constant is excluded from the primary action. This formulation leads to an essential change of cosmological dynamics due to a varying gravitational constant. In this way, the inflationary evolution in the model results in the fact that the observed contrast of energy density in the Universe gives the strict preference for extremely large values of Higgs boson mass though below the Planck mass by four or five orders of magnitude.

As for the Higgs field nonminimally coupled to the gravity, it can successfully produce the inflation compatible with the observed properties of Universe. However, in that case one should introduce the additional coupling constant of Higgs field with the gravity at a specific value of such the parameter. Then, the mechanism of inflation caused by the Higgs field nonminimally coupled to the gravity essentially differs from the mechanism with the Higgs field minimally coupled to the gravity, so that we consider the minimal version in detail with no further reference to the nonminimal version beyond the Introduction.

In this paper we consider conditions of developing the inflation stage produced by the Higgs field of Standard Model (SM) minimally coupled to the gravity¹, i.e. in the case of $\xi = 0$, when the renormalizable potential of Higgs scalar is determined by two parameters: a vacuum expectation value fixed by the Fermi constant G_F in the weak interaction, and a mass not yet measured experimentally. In theoretical models treating the inflation of Universe due to the scalar field, the mass of inflaton should get a value at the scale of 10^{13} GeV in order to agree with the observed spectra of inhomogeneities in both CMBR and LSS. In contrast, the mass of Higgs boson is not greater than several hundreds GeV as follows from SM with account of loop corrections including the Higgs boson (the current status of Higgs particle physics see in review [21], while the probable fate of SM is discussed in [22]). In addition, the observations particularly require an extremely small constant of self-action for the scalar field producing the fluctuations transformed into the inhomogeneity of matter and anisotropy of cosmic microwave background radiation. So, we will use this argumentation in our motivation to theoretically forbid the inflation produced by the Higgs field minimally coupled to the gravity.

Thus, the main conclusion could be drawn as follows: the scenario, when the Higgs boson alone generates the Universe inflation consistent with the modern cosmological observations, is *experimentally forbidden*. However, irrespectively to the observational data, the Higgs boson is an ordinary scalar field, hence, in the framework of classical gravitation theory, the field can be *able* to produce the inflation regime. We argue for the situation when *the inflation generated by the Higgs boson could not develop in principle, if the mass of Higgs scalar in SM exceeds a critical value*. Such the critical value of Higgs boson mass defines the decoupling of Higgs boson from the inflation, at all.

Definitely, in the framework of quantum field theory in the curved spacetime, when the gravity is treated as a classical theory, there is a critical value of the Hubble constant, which sets the end of inflation regime depending on the parameters of inflaton. At lower values of the Hubble constant, a transition to a reheating of the Universe occurs due to generating various quanta of both the inflaton as well as other matter fields. The stage of reheating is properly called the moment of “Big Bang”. However, the critical value of Hubble constant and, hence, the corresponding energy density could be quite great, so that the gravity would not allow the classical description, i.e. quantum fluctuations in a metric would be essential, and the theory enters the scope beyond the validity of inflationary theory. Thus, a border of quantum gravity in cosmology could actually determine the decoupling of Higgs boson from the inflation regime.

Essential quantum fluctuations should be inevitably introduced, if the classical gravitational action S with account of term due to the relevant inflaton potential becomes comparable with the period of quantum mechanical amplitude Ψ taken in the classical limit, $\Psi \sim \exp\{iS\}$. The

¹In principle, the coupling ξ could deviate from zero by a small value, which can be neglected in the consideration, say, at $|\xi| \ll 1$.

action in cosmology is related to the Hubble rate H for the Universe expansion, hence, the curvature of space-time. The curvature of Planckian scale is beyond the classical description². However, it turns out formally, that the Higgs boson with a rather large mass would classically produce the inflation at the Planckian curvature of space-time, i.e. at the stage, when the quantum description of gravity cannot be ignored, hence, the inflationary regime cannot be induced.

In the present paper, we estimate the critical value for the Higgs boson mass by requirement that the Higgs field cannot produce the inflation regime at early stages of the Universe evolution. The decoupling mass of the Higgs particle is quite actual for modern experimental searches of Higgs boson at colliders [21, 22].

Some other aspects of Higgs particle physics as concerns for the inflation, basically for various fluctuations, were considered in [24].

We have tried to treat rather a complex problem to distinguish between two fine possibilities, when

1. the scalar field is able to produce the inflation of universe, hence, the spectrum of its fluctuations is definite, since it is formed during the inflation (though the parameters of such the inflation would be in a sharp conflict with the observed properties of our Universe, so that such the inflation should be *forbidden experimentally*, if only it is not accompanied by a further stage of true inflation due to the specific field of inflaton);
2. the scalar field is not able to produce the inflation of universe, since such the inflation is *forbidden theoretically* by some critical properties of field self-action, hence, the spectrum of field fluctuations becomes arbitrary, that causes the lose of predictability of field theory in this case.

The first of above possibilities occurs if the value of self-action coupling λ for the Higgs particle minimally coupled to the gravity is below the critical value³ $\lambda_c = \frac{1}{6}$, while the second possibility occurs if the coupling λ exceeds the critical value. Thus, the interval of $0 < \lambda < \lambda_c$ corresponds to the consistent Higgs field theory in the curved space-time, while $\lambda > \lambda_c$ causes arbitrary amplitudes of inhomogeneous fluctuations of Higgs field at the early stages of Universe evolution, in principle, that should be excluded *theoretically*.

This exclusion is similar to the known limit on the Higgs boson mass due to the Landau pole involving arbitrary quantum amplitudes in processes with the Higgs field at high virtualities, while the pole is not essential at much smaller virtualities. So, the inflation caused by the Higgs field should be accompanied by the further inflation due to the inflaton, that makes the fluctuations of Higgs matter practically invisible at late times. Nevertheless, those fluctuations should be definite at all times after the quantum gravity era. Then, we provide with the constraint $\lambda < \lambda_c$ instead of weaker $\lambda(\mu) < \infty$ given by the exclusion of Landau pole at high virtualities μ .

Thus, one could expect the subcritical values of Higgs boson mass $m_H < m_H^{\text{crit}} = 153 \pm 3$ GeV.

²This fact was originally recognized in [4], where A.Linde used it to set the constraint on the scalar field self-coupling constant λ by the order of magnitude, as was recently rederived in [23], $\lambda \ll 10^{-2}$. In the present paper we get an exact value for the critical value of self-coupling constant, but the order of magnitude result.

³Extremely small values of constant λ , when the inflation generated by the Higgs boson further develops due to switching into the regime driven by the specific inflaton, are excluded experimentally: $\lambda > 0.11$ (see discussion in [23]).

2 The Higgs boson as the inflaton

Let us consider the model of the Higgs boson in the gauge setting the real field $\Phi = \phi/\sqrt{2}$ with the minimal coupling to the gravity and potential⁴

$$V = \lambda(\phi^2 - v^2)^2/4,$$

where the vacuum expectation value $v = 1/\sqrt{\sqrt{2}G_F} \approx 246.2$ GeV is known experimentally. Then, the formula for the mass of Higgs field is ordinary given by

$$m^2 = 2\lambda v^2. \quad (1)$$

The Einstein–Hilbert action of gravity is classically defined by scalar curvature R

$$S_g = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R, \quad (2)$$

while the cosmology in the case of spatial homogeneity is described by the metric with a time-dependent scale factor $a(t)$,

$$ds^2 = dt^2 - a^2(t) d\mathbf{r}^2. \quad (3)$$

In the inflation regime, the metric can be well approximated by de Sitter one in the standard cosmological form, wherein the 3-dimensional space is flat and an observer is posed in the center point

$$ds^2 \approx dt^2 - e^{2Ht} d\mathbf{r}^2, \quad (4)$$

where $H = d \ln a / dt = \dot{a}/a$ is the Hubble constant, which value to the end of inflation can be *strictly* related with the constant λ defining the self-coupling for the Higgs field.

The appropriate framework of quasiattractor approach is systematically simple: the motion can be straightforwardly treated in terms of autonomous differential equations with a parametric attractor, whose critical points slowly drift with the Hubble constant⁵ [25, 26, 27].

Indeed, in terms of dimensionless variables defined as

$$x = \frac{\kappa}{\sqrt{6}} \frac{\dot{\phi}}{H}, \quad y = \sqrt[4]{\frac{\lambda}{12}} \frac{\kappa \phi}{\sqrt{\kappa H}}, \quad z = \frac{\sqrt[4]{3\lambda}}{\sqrt{\kappa H}}, \quad (5)$$

giving the fractions of kinetic energy x^2 and potential energy y^4 for the energy budget of scalar field: $x^2 + y^4 = 1$ at $\kappa^2 = 8\pi G$, while z introduces the parametric dependence in the field equations

$$x' = -3x^3 + 3x + 2y^3z, \quad y' = -\frac{3}{2}x^2y - xz, \quad (6)$$

wherein the evolution, i.e. the differentiation denoted by prime, is calculated with respect to e-folding defined by $N = \ln a_{\text{end}} - \ln a$, so that the parameter z evolves according to

$$z' = -\frac{3}{2}x^2z. \quad (7)$$

There are stable critical points⁶ for (6) (see [26, 27]) at

$$z^4 < \frac{3}{4}. \quad (8)$$

⁴We assume, that a nonzero cosmological constant can be surely neglected during the inflation.

⁵The scale factor runs as the exponent of e-folding N by definition $a \sim \exp\{-N\}$, while the Hubble constant gets a slow driftage logarithmic in the scale factor, more exactly, linear in e-folding for the quartic self-action of inflaton, $H - H_* \sim N$.

⁶Critical points are defined by condition $x' = y' = 0$, while the stability takes place when linear perturbations in differential equations near the critical points decline to zero.

The existence of critical points is caused by specific “friction term” in equations. The magnitude of friction is given by the Hubble constant, so that the kinetic energy of inflaton is suppressed with respect to the potential, and the field slowly rolls down to the minimum of potential. At large amount of e-folding $N \gg 1$, the quasiattractor is equivalent to the slow-roll approximation in the leading order of $1/N$ -expansion [28]. However, in contrast to the slow-rolling in the $1/N$ -expansion, the quasiattractor allows us to get the *strict* description for the final stage of inflation due to the *exact* determination of critical points for the parametric attractor. Reasonably, the inflation finishes at such value of Hubble rate, when a condition on the existence of stable critical points invalidates and the attractor becomes unstable, i.e. it disappears. Then, from the condition of (8) we get

$$2\pi GH_{\text{end}}^2 = \lambda. \quad (9)$$

To the end of inflation evolving with the parametric attractor we get $z_{\text{end}}^4 = 3/4$, $x_{\text{end}}^2 = 2/3$ and $y_{\text{end}}^4 = 1/3$, hence, $H_{\text{end}}^2 = 2\pi G\lambda\phi_{\text{end}}^4$, so that equivalently to (9) we get

$$2\pi G\phi_{\text{end}}^2 = 1. \quad (10)$$

Thus, the inflation produced by the Higgs boson stops at the Planckian scale of field value.

From (9) we see that if $\lambda \sim 1$, the value of H_{end} is about the Planck scale. This result repeats the arguments of [4, 23] as mentioned above. Therefore, the heavy Higgs boson formally corresponds to the inflationary Hubble rate about the Planckian scale of energy, where effects of quantum gravity cannot be ignored, hence, the inflation dynamics cannot develop, since the curvature of space-time gets the Planckian values.

3 A border of quantum gravity in cosmology

De Sitter metric (4) straightforwardly determines the scalar curvature standing in the action of classical gravitational field, $R = -12H^2$.

In the calculation of gravitational action, it is worth to note, that the coordinate r takes values in the region from zero to the horizon $r_H = 1/H$ and the integration in time t is limited by the interval from the negative infinity to a moment, which can be put to zero with no lose of generality of consideration (in fact, to a moment of inflation end). Notice, that the specified coordinate system covers only a half of de Sitter manifold⁷, therefore, the action can be doubled, in principle, but this would incorporate a part of the manifold causally independent of the cosmological observer. Finally, we get

$$S_g = \frac{1}{3GH^2}. \quad (11)$$

Similarly, we add the contribution of action for the matter approximated by the form

$$S_m \approx - \int d^4x \sqrt{-g} V, \quad (12)$$

where V is the matter potential, so that in the framework of inflation regime we neglect the kinetic term of inflaton in comparison with the potential. The contribution of potential is determined by the Einstein equations

$$V \approx \frac{3H^2}{8\pi G}.$$

Finally, the matter action equals

$$S_m = -\frac{1}{6GH^2}, \quad (13)$$

⁷The other half of manifold can be associated with the exponentially contracting universe, in contrast to the case of expanding universe, we consider.

yielding the sum $S = S_g + S_m$ equal to

$$S = \frac{1}{6GH^2}. \quad (14)$$

We have just got the action by making use of de Sitter metric. For the sake of generality, we have performed exact calculations in the case of matter with a state parameter w equal to the ratio of pressure p to energy density ρ : $p = w\rho$. In the range of $-1 < w < 1$, the integration in time runs along a finite interval with the scale factor spanning the region from a cosmic singularity to the moment defined by $a = 1$. Then, *the action of matter and gravity takes the same value of (14) independent of the state parameter w* . This fact is important, since it points to the stability of cosmological action versus the matter content. In addition, the spacetime to the end of inflation produced by the Higgs boson becomes to essentially differ from de Sitter spacetime: the fractions of kinetic and potential energies get values equal to $\frac{2}{3}$ and $\frac{1}{3}$, correspondingly, that gives $w = \frac{1}{3}$ specific for a radiation, i.e. a light-like or ultra-relativistic matter. Nevertheless, the estimate of (14) is rather universal, it does not significantly vary with changes in the expansion regime.

The quantum mechanical amplitude Ψ in the classical limit takes the form of $\Psi \sim \exp\{iS\}$, therefore, in order to separate the quantum regime from the classical behavior, one should compare the action with the period $\delta S = 2\pi$. Then, we get the constraint on H , when the quantum gravity effects become essential

$$12\pi GH^2 > 1. \quad (15)$$

The confidence level of such the constraint is discussed in section 5.

4 The mass of decoupling the Higgs boson from the inflation

For the case of inflation produced by the Higgs scalar, the relation of Hubble constant at the end of inflation with the self-action constant λ results in the supercritical region:

$$\lambda > \frac{1}{6}. \quad (16)$$

Thus, at the tree level the critical value for the Higgs mass takes the form

$$m_H^{\text{crit}} = \frac{v}{\sqrt{3}}. \quad (17)$$

Substituting the experimental inputs, we get $m_H^{\text{crit}} = 142.3$ GeV.

5 The confidence level of decoupling constraint

The decoupling mass obtained from the theoretical forbidding the inflation produced by the Higgs boson is based on the breaking the classical description due to quantum fluctuations, which make the inflation impossible. So, in order to estimate the confidence level of such the bound derived above, let us consider, for instance, a harmonic oscillator with the Hamiltonian

$$H = \frac{1}{2}(Q^2 + P^2) \hbar\omega, \quad (18)$$

wherein Q is the coordinate, while P is its canonically conjugated momentum. The state maximally close to the classical system with the energy $E = \hbar\omega(n + \frac{1}{2})$ is the coherent state with the minimized fluctuation of coordinate and momentum at any time of evolution

$$(\delta Q)_c^2 = (\delta P)_c^2 = \frac{1}{2}, \quad (19)$$

while the stationary quantum state with the definite energy of n quanta gives essential fluctuations

$$(\delta Q)_q^2 = (\delta P)_q^2 = n + \frac{1}{2}. \quad (20)$$

Therefore, one could estimate the relevance of the state assignment to the nonclassical system by evaluating

$$\chi^2 = \frac{(\delta Q)_q^2}{(\delta Q)_c^2} - \chi_0^2 = 2n, \quad (21)$$

wherein we put $\chi_0^2 = 1$ in order to match the vacuum to completely the quantum state.

The same criterium can be derived by considering the time evolution of average coordinate in the coherent state. So, from

$$Q(t) = Q_0 \cos \omega t + P_0 \sin \omega t, \quad (22)$$

one gets

$$\langle Q(t) \rangle = 0, \quad \langle Q^2(t) \rangle = \frac{1}{2}(Q_0^2 + P_0^2) = n, \quad (23)$$

so that the fluctuation is equal to

$$(\delta Q)_t^2 = n, \quad (24)$$

that yields

$$\chi^2 = \frac{(\delta Q)_t^2}{(\delta Q)_c^2} = 2n. \quad (25)$$

Then, we can estimate the hypothesis that the system essentially requires to carefully take into account important quantum fluctuation by the χ^2 -probability depending on the number of quanta in the system. The evaluation of n is system-dependent. We put

$$n = \frac{S}{2\pi}, \quad (26)$$

wherein S is the action of the cosmological system with the inflaton. The bound $m_H > m_H^{\text{crit}}$ is equivalent to $n < 1$ and, hence, $\chi^2 < 2$, so that the system is essentially quantum within the 2σ confidence level, i.e. with the probability of 90%.

Note, that changing the determination for the number of quanta in (26) by $n = S/\pi$ or $n = S/4\pi$ would result in the respective modification of confidence level for the cosmological bound on the Higgs boson mass: 99% or 68%, correspondingly. Analogously, the shift $m_H^{\text{crit}} \mapsto \sqrt{2}m_H^{\text{crit}}$ would decrease the confidence level of such the estimate with (26) to the value of 68%.

6 A renormalization group improvement

The above consideration has been based on the leading approximation of effective action, while quantum loop corrections would both modify the potential at large fields relevant to the inflation and renormalize the physical parameters of Lagrangian for the Higgs scalar, i.e. the field normalization, mass and coupling constant λ . These effects could be effectively taken into account by making use of renormalization group in SM [24] with an appropriate choice of renormalization point at the inflation stage, so that the whole effect would be reduced to the running of $\lambda(\mu)$ with the scale μ . Similar strategy has been explored in [16, 15] for the Higgs field non-minimally coupled to the gravity. So, in estimates we fit a pole mass of Higgs particle, determining the running mass $m(\mu)$ at the scale of t -quark mass, with other parameters of SM at the same scale $\mu = m_t$ to reach the critical value of λ in (16) at the scale of the order of the Planck mass. Then, the variation of final result due to the renormalization group can be estimated by comparing one- and two-loop calculations, which points to the uncertainty caused

by the choice of final scale in the running. Another source of uncertainties is connected to the empirical accuracy in the measurement of SM parameters at the starting scale of renormalization group evolution. So, the one-loop renormalization group results in the decoupling mass of Higgs particle equal to 153 GeV, while the two-loop evolution approximately gives the value of 150 GeV at $m_t = 171$ GeV. A complete analysis of uncertainties caused by variation of different parameters in the calculations by means of renormalization group will be given elsewhere.

Then, the renormalization group improvement of estimate results in the critical value for the Higgs boson mass $m_H^{\text{crit}} \approx 153$ GeV with uncertainty of 3 GeV. The difference between estimates at the tree level and due to the two-loop renormalization is significant, but it is rather moderate, so that we can draw the conclusion that the higher order corrections are still under control.

7 Final remarks

It is worth to note, that having estimated the inflation parameters we have neglected terms quadratic in the Higgs field, which is correct, if the vacuum expectation value is much less than the Planck mass, i.e. at $v \ll M_{Pl}$ (this condition is safely valid for the Higgs boson in SM). Therefore, the estimation of (17) is valid for any scalar Higgs boson with a small vacuum expectation value in gauge theories including grand unified theories (GUT). In addition, a grand unification could change both the running of gauge coupling constants and set of quantum fields active in the running. Then, the estimate obtained due to the renormalization group improvement would slightly move, though the value of such displacement should not sizably exceed the calculation uncertainty given above. In respect of GUT with the SU(5) symmetry we mention the modified induced gravity scenario with the Higgs field non-minimally coupled to the gravity as studied in [20].

Combining the known bound on the Higgs boson mass from the stability of its potential with the subcritical region following from the cosmological consideration, we get the expected interval of scalar particle mass in the form

$$129 \text{ GeV} < m_H < 153 \text{ GeV},$$

with the accuracy of 3 GeV on the upper bound and 1 GeV on the lower bound.

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