Tunneling cosmological state and the origin of SM Higgs inflation

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Abstract

A path integral formulation for the tunneling cosmological state is suggested, which admits a consistent renormalization and renormalization group (RG) improvement in particle physics applications of quantum cosmology with heavy massive quantum fields. This formulation is applied to the inflationary cosmology driven by the Standard Model (SM) Higgs boson playing the role of an inflaton with a strong non-minimal coupling to gravity. In this way a complete cosmological scenario is obtained, which embraces the formation of initial conditions for the inflationary background in the form of a sharp probability peak in the distribution of the inflaton field and the ongoing generation of the CMB spectrum on this background. The status of the no-boundary and tunneling states is also discussed in cosmology driven by massless fields conformally coupled to gravity.

1 Introduction

At the dawn of inflation theory two prescriptions for the quantum state of the Universe were seriously considered as a source of initial conditions for inflation. These are the so-called no-boundary [1] and tunneling [2, 3] cosmological wavefunctions (see also [5] for a general review), whose semiclassical amplitudes are roughly inversely proportional to one another. In the model of chaotic inflation driven in the slow-roll approximation by the inflaton field φ with the potential $V(\varphi)$ these amplitudes read as $|\varPsi_\pm(\varphi)| \simeq \exp(\mp S_E(\varphi)/2)$, where $+/-$ label, respectively, the no-boundary/tunneling wavefunctions. Qualitatively they both describe the nucleation of a (quasi)deSitter spacetime from the the Euclidean half-instanton as depicted on Fig.1. Here, $S_E(\varphi)$ is the Euclidean Einstein action of the full de Sitter instanton S^4 with the effective cosmological constant given by the value of the inflaton field $\Lambda_{\rm eff} = V(\varphi)/M_{\rm P}^2$,

$$S_E(\varphi) \simeq -\frac{24\pi^2 M_{\rm P}^4}{V(\varphi)}, \tag{1}$$

in units of the reduced Planck mass $M_{\rm P}^2 = 1/8\pi G$ ($\hbar = 1 = c$). The no-boundary state was originally formulated as a path integral over Euclidean four-geometries; the tunneling state in the form of a path integral over Lorentzian metrics was presented in [3, 4], and both wavefunctions were also obtained as solutions of the minisuperspace Wheeler–DeWitt equation.

The no-boundary and tunneling states lead to opposite physical conclusions. In particular, in view of the negative value of the Euclidean de Sitter action the no-boundary state strongly enhances the contribution of empty universes with $V(\varphi) = 0$ in the full quantum state and, thus, leads to the very counterintuitive conclusion that infinitely large universes are infinitely more probable than those of a finite size – a property which underlies the once very popular but now nearly forgotten big-fix mechanism of S. Coleman [6]. On the other hand, the tunneling state

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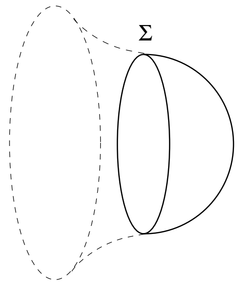

Figure 1: Nucleation of the Lorentzian signature (quasi)deSitter spacetime denoted by dashed lines from the hemisphere of the Euclidean gravitational instanton at the nucleation 3-surface Σ.

favors big values of $V(\varphi)$ capable of generating inflationary scenarios. Thus, it would seem that the tunneling prescription is physically more preferable than the no-boundary one. However, the status of the tunneling prescription turns out to be not so simple and even controversial.

Naive attempts to go beyond the minisuperspace approximation lead to unnormalizable states in the sector of spatially inhomogeneous degrees of freedom for matter and metric and invalidate, in particular, the usual Wick rotation from the Lorentzian to the Euclidean spacetime. This problem was partly overcome by imposing the normalizability condition on the matter part of the solution of the Wheeler–DeWitt equation [7], but the situation remained controversial for the following reason.

Modulo the issue of quantum interference between the "contracting" and "expanding" branches of the cosmological wavefunction discussed, for example, in [7, 5, 8, 9], the amplitudes of the no-boundary and tunneling branches of such a semiclassical solution take the form

$$\big|\,\Psi_\pm\big(\varphi,\Phi(\mathbf{x})\big)\big| = \exp\left(\mp\frac{1}{2}S_E(\varphi)\right)\big|\,\Psi_{\rm matter}\big(\varphi,\Phi(\mathbf{x})\big)\big|\,, \tag{2}$$

where $\Phi(\mathbf{x})$ is a set of matter fields separate from the spatially homogeneous inflaton, and $\Psi_{\rm matter}\big(\varphi,\Phi(\mathbf{x})\big)$ is their normalizable (quasi-Gaussian) part in the full wavefunction – in essence representing the Euclidean de Sitter invariant vacuum of linearized fields $\Phi(\mathbf{x})$ on the quasi-de Sitter background with $\Lambda_{\rm eff} = V(\varphi)/M_{\rm P}^2$. Quantum averaging over $\Phi(\mathbf{x})$ then leads to the following quantum distribution of the inflaton field

$$\rho_\pm^{\rm 1-loop}(\varphi) = \int d\big[\,\Phi(\mathbf{x})\,\big]\,\big|\,\Psi_\pm\big(\varphi,\Phi(\mathbf{x})\big)\big|^2 = \exp\Big(\mp S_E(\varphi) - S_E^{\rm 1-loop}(\varphi)\Big)\ , \tag{3}$$

where $S_E^{\rm 1-loop}(\varphi) = (1/2){\rm Tr}\ln(\delta^2 S_E[\,\varphi,\Phi\,]/\delta\Phi(x)\,\delta\Phi(y))$ is the contribution of the UV divergent one-loop effective action [10, 11, 12].[1] With the aid of this algorithm a sharp probability peak was obtained in the *tunneling* distribution $\rho_-^{\rm 1-loop}(\varphi)$ for the model with a strong non-minimal coupling of the inflaton to gravity [10, 13, 14]. This peak was interpreted as generating the quantum scale of inflation – the initial condition for its inflationary scenario. Quite remarkably, for accidental reasons this result was free from the usual UV renormalization ambiguity. It did not require application of the renormalization scheme of absorbing the UV divergences into the redefinition of the coupling constants in the tree-level action $S_E(\varphi)$.

However, beyond the one-loop approximation and for other physical correlators the situation changes, and one has to implement a UV renormalization in full. But with the $\mp S_E(\varphi)$ ambiguity in (3) this renormalization would be different for the tunneling and no-boundary states.

[1] For the tunneling state this equation might be regarded as a result of fine-tuning, because in order to guarantee this equation the basis functions of the operator $\delta^2 S_E/\delta\Phi\,\delta\Phi$ in contrast to the no-boundary case should not be regular at the pole of the hemisphere of Fig. 1 – a natural selection criterion for the Euclidean de Sitter invariant vacuum within the no-boundary construction. I am grateful to V.A.Rubakov for the discussion of this point.

For instance, an asymptotically free theory in the no-boundary case (associated with the usual Wick rotation to the Euclidean spacetime) will not be asymptotically free in the tunneling case. The tunneling versus no-boundary gravitational modification of the theory will contradict basic field-theoretical results in flat spacetime. This strongly invalidates a naive construction of the tunneling state of the above type. In particular, it does not allow one to go beyond the one-loop approximation in the model of non-minimally coupled inflaton and perform its renormalization group (RG) improvement.

Here we suggest a solution of this problem by formulating a new path integral prescription for the tunneling state of the Universe. This formulation is based on a recently suggested construction of the cosmological density matrix [15] which describes a microcanonical ensemble of cosmological models [16]. The statistical sum of this ensemble was calculated in a spatially closed model with a generic set of scalar, spinor, and vector fields conformally coupled to gravity. It was obtained in the saddle-point approximation dominated by the contribution of the thermal cosmological instantons of topology $S^3 \times S^1$. These instantons also include the vacuum S^4 topology treated as a limiting case of the compactified time dimension S^1 in $S^3 \times S^1$ being ripped in the transition from $S^3 \times S^1$ to S^4. This limiting case exactly recovers the Hartle–Hawking state of [1], so that the whole construction of [15, 16] can be considered as a generalization of the vacuum no-boundary state to the quasi-thermal no-boundary ensemble. The basic physical conclusion for this ensemble was that it exists in a bounded range of values of the effective cosmological constant, that it is capable of generating a big-boost scenario of the cosmological acceleration [18] and that its vacuum Hartle–Hawking member does not really contribute because it is suppressed by the infinite *positive* value of its action. This is a genuine effect of the conformal anomaly of quantum fields [19, 20], which qualitatively changes the tree-level action (1).

Below we shall show that the above path integral actually has another saddle point corresponding to the negative value of the lapse function $N < 0$, which is gauge-inequivalent to $N > 0$ [21]. In the case of heavy massive quantum fields driving inflation, this leads to the inversion of the sign of the action in the exponential of the statistical sum and, therefore, deserves the label "tunneling". In this tunneling state the thermal part vanishes and its instanton turns out to be a purely vacuum one. Finally, this construction no longer suffers from the above mentioned controversy with the renormalization. A full quantum effective action within the gradient and curvature expansion is supposed to be calculated and renormalized by the usual set of counterterms on the background of a generic metric. Then the result should be analytically continued to $N < 0$ and taken at the *tunneling* saddle point of the path integral over the lapse function N. We will also show that for cosmology driven by conformal field theory, in contrast to the one generated by massive fields, the tunneling state is forbidden at the dynamical level.

Below we shall apply this construction to a cosmological model for which the Lagrangian of the graviton-inflaton sector reads

$$\boldsymbol{L}(g_{\mu\nu}, \varPhi) = \frac{1}{2}\left(M_{\rm P}^2 + \xi|\varPhi|^2\right)R - \frac{1}{2}|\nabla\varPhi|^2 - V(|\Phi|), \tag{4}$$

$$V(|\Phi|) = \frac{\lambda}{4}(|\varPhi|^2 - v^2)^2, \quad |\varPhi|^2 = \varPhi^\dagger\varPhi, \tag{5}$$

where $\varPhi$ is the Standard Model (SM) Higgs boson, whose expectation value plays the role of an inflaton and which is assumed here to possess a strong non-minimal curvature coupling with $\xi \gg 1$. Here, as above, $M_{\rm P}$ is a reduced Planck mass, λ is a quartic self-coupling of $\varPhi$, and v is an electroweak (EW) symmetry breaking scale.

The early motivation for this model with a GUT type boson $\varPhi$ [22, 23] was to avoid an exceedingly small quartic coupling λ by invoking a non-minimal coupling with a large ξ. This was later substantiated by the hope to generate the no-boundary/tunneling initial conditions for inflation [13, 14]. This theory but with the SM Higgs boson $\varPhi$ instead of the abstract GUT setup of [13, 14] was suggested in [24], extended in [25] to the one-loop level and considered regarding its reheating mechanism in [26]. The RG improvement in this model has predicted CMB parameters – the amplitude of the power spectrum and its spectral index – compatible with WMAP observations in a finite range of values of the Higgs mass, which is close to the widely accepted range dictated by the EW vacuum stability and perturbation theory bounds [27, 28, 29, 30, 31, 32].

The purpose of our paper is to extend the results of [30, 31] by suggesting that this model does not only have WMAP-compatible CMB perturbations, but can also generate the initial conditions for the inflationary background upon which these perturbations propagate. These initial conditions are realized in the form of a *sharp probability peak* in the tunneling distribution function of the inflaton.

2 Tunneling cosmological wavefunction within the path integral formulation

The microcanonical density matrix in quantum cosmology was suggested in [16] as a formal projector on the subspace of physical states satisfying the system of the Wheeler-DeWitt equations $\hat{H}_\mu(\varphi, \partial/i\partial\varphi)\,\rho(\varphi,\varphi_-)=0$,

$$\hat{\rho} \sim \text{``}\Big(\prod_\mu \delta(\hat{H}_\mu)\Big)\text{''} \tag{6}$$

where $\hat{H}_\mu$ denotes the operator realization of the full set of the gravitational Hamiltonian and momentum constraints, $H_\mu(q,p)$, the condensed index signifying a collection of discrete labels along with continuous spatial coordinates, $\mu = (\perp, a, \mathbf{x})$, $a = 1,2,3$. The phase space variables (q,p) include the collection of spatial metric coefficients and matter fields $q = (g_{ab}(\mathbf{x}), \phi(\mathbf{x}))$ (denoted also by φ when used as arguments of the density matrix kernel) and their conjugated momenta p.

The justification for (6) as the density matrix of a *microcanonical* ensemble in spatially closed cosmology was put forward in [16] based on the analogy with an unconstrained system having a conserved Hamiltonian $\hat{H}$. The microcanonical state with a fixed energy E for such a system is given by the density matrix $\hat{\rho} \sim \delta(\hat{H} - E)$. A major distinction of (6) from this case is that spatially closed cosmology does not have freely specifiable constants of motion like the energy or other global charges. Rather it has as constants of motion the Hamiltonian and momentum constraints H_μ, all having a particular value — zero. Therefore, the expression (6) can be considered as the analogue of equipartition – a natural candidate for the microcanonical quantum state of the *closed* Universe.

Perturbatively (at least within the semiclassical loop expansion) the kernel of this projector can be written down as a phase-space path integral of the canonically quantized gravity theory

$$\rho(\varphi_+, \varphi_-) = e^{\varGamma} \int\limits_{q(t_\pm)=\varphi_\pm} D[\,q,p,N\,]\; \exp\left[\,i\int_{t_-}^{t_+} dt\,(p\,\dot{q} - N^\mu H_\mu)\right]. \tag{7}$$

Here N^μ are the Lagrange multipliers dual to the constraints – lapse and shift functions $N^\mu = (N(\mathbf{x}), N^a(\mathbf{x}))$, and the functional integration runs over the histories interpolating between the configurations $\varphi_\pm$ which are the arguments of the density matrix kernel. The range of integration over N^μ is of course real because this integration over the Lagrange multipliers is designed in order to generate delta functions of constraints. The Hamiltonian action in the exponential is the integral over the coordinate time t which is just the ordering parameter ranging between arbitrary initial and final values $t_\pm$, the result being entirely independent of their choice. The integration measure $D[\,q,p,N\,]$, of course, includes the Faddeev-Popov gauge-fixing procedure which renders the whole integral gauge and time-parametrization independent.

After integration over canonical momenta the path integral above takes the Lagrangian form of the integral over the configuration space coordinates q and the lapse and shift functions N^μ. Taken together they comprise the full set of spacetime metric with the Lorentzian signature $g_{\mu\nu}^{\rm Lorentzian}$ and matter fields ϕ ,

$$ds^2 = -N^2_{\rm Lorentzian}dt^2 + g_{ab}(dx^a + N^a dt)(dx^b + N^b dt), \tag{8}$$

in terms of which the Lagrangian form of the classical action reads as $S[\,g_{\mu\nu}^{\rm Lorentzian},\phi\,]$. One more notational step consists in the observation that this Lorentzian metric can be viewed as the Euclidean one, $g_{\mu\nu}^{\rm Euclidean}$, with the imaginary value of the Euclidean lapse function

$$ds^2 = N^2_{\rm Euclidean}dt^2 + g_{ab}(dx^a + N^a dt)\,(dx^b + N^b dt), \tag{9}$$

$$N_{\rm Lorentzian} = -iN_{\rm Euclidean}, \tag{10}$$

so that the Euclidean theory action is related to the original Lorentzian action $S[\,g_{\mu\nu}^{\rm Lorentzian},\phi\,]$ by a typical equation

$$iS[\,g_{\mu\nu}^{\rm Lorentzian},\phi\,] = -S_E[\,g_{\mu\nu}^{\rm Euclidean},\phi\,]. \tag{11}$$

Here the imaginary factor arises from the square root of the metric determinant in the Lagrangian, which in the ADM form reads of course as $g^{1/2} = N(\det g_{ab})^{1/2}$. Note that the analytic continuation from the Lorentzian to the Euclidean picture takes place in the complex plane of the lapse function rather than in the complex plane of time (time variable is the same in both pictures), though of course it is equivalent to the usual Wick rotation $t_{\rm Euclidean} = it_{\rm Lorentzian}$.

With these notations the density matrix (7) takes the form of the Euclidean quantum gravity path integral

$$\rho(\,\varphi_+,\varphi_-\,) = e^{\varGamma}\int\limits_{q(t_\pm)=\,\varphi_\pm} D[\,g_{\mu\nu},\phi\,]\,e^{-S_E[\,g_{\mu\nu},\phi\,]}. \tag{12}$$

However, in view of (10) the range of integration over the Euclidean lapse $N \equiv N_{\rm Euclidean}$ (in what follows we will omit the Euclidean label for brevity) belongs to the imaginary axis

$$-i\infty < N < i\infty, \tag{13}$$

and in the Lagrangian density of the Euclidean action the choice of the branch for the square root of the metric determinant is specified as $g^{1/2} = N(\det g_{ab})^{1/2}$. These conventions will be important in what follows.

The topology of spacetime configurations which are integrated over in (12) is $R^1 \times S^3$ as depicted on the upper part of Fig.2. This topology of the spacetime bulk interpolating between the hypersurfaces Σ and Σ' reflects the mixed nature of the density matrix and establishes entanglement correlations between φ and φ'. These configurations however include as a limiting case the disconnected bulk obtained by pinching and ripping the spacetime bridge between Σ and Σ' (see lower part of Fig.2). This is associated with the contribution which factorizes into the direct product of pure states of the Hartle-Hawking type shown on Fig.1.

The normalization factor $\exp\varGamma$ in (12) follows from the density matrix normalization ${\rm tr}\hat\rho = 1$ and determines the main object of interest – the statistical sum of the model. The trace operation implies integration over the diagonal elements of the density matrix, so that the statistical sum takes the form of the path integral

$$e^{-\varGamma} = \int\limits_{\rm periodic} D[\,g_{\mu\nu},\phi\,]\,e^{-S_E[\,g_{\mu\nu},\phi\,]} \tag{14}$$

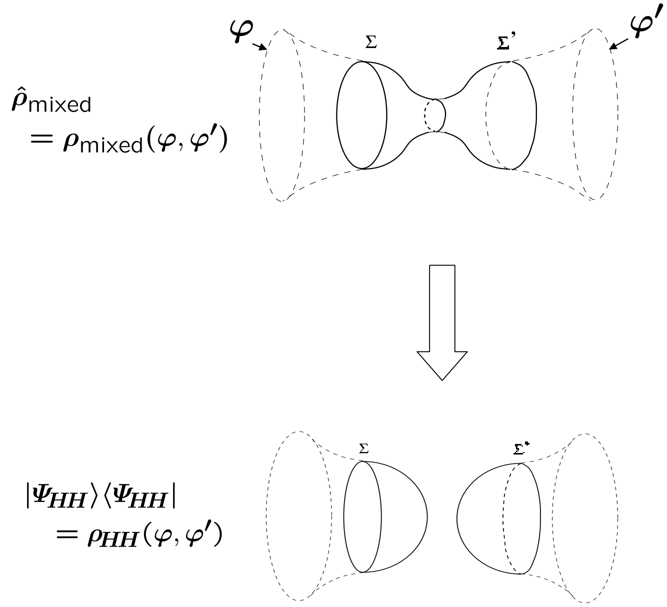

Figure 2: The topology of spacetime configurations underlying mixed states and pure states (of the Hartle-Hawking type) in the density matrix.

over the periodic configurations whose spacetime topology $R^1 \times S^3$ follows from the identification of the boundary surfaces Σ and Σ'. For contributions of the entangled states this leads to to the topology $S^1 \times S^3$ depicted on the upper part of Fig.3, whereas the pure state contribution yields the topology of S^4.

The further calculation of the statistical sum can be based on disentangling the minisuperspace sector from the full configuration space.

$$g_{\mu\nu}, \phi \to a(\tau), N(\tau), \varPhi(x), \quad \varPhi(x) = (\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), ...), \tag{15}$$

$$ds^2 = N^2(\tau)\, d\tau^2 + a^2(\tau)\, d^2\Omega^{(3)}, \tag{16}$$

Then the path integral can be cast into the form of an integral over a minisuperspace lapse function $N(\tau)$ and scale factor $a(\tau)$ of a spatially closed Euclidean FRW metric,

$$e^{-\varGamma} = \int D[\,a, N\,]\; e^{-S_{\rm eff}[\,a,\,N\,]}, \tag{17}$$

$$e^{-S_{\rm eff}[\,a,\,N]} = \int D\varPhi(x)\, e^{-S_E[\,a,\,N;\,\varPhi(x)\,]}\;. \tag{18}$$

Here, $S_{\rm eff}[\,a,\,N\,]$ is the Euclidean effective action of all inhomogeneous "matter" fields $\varPhi(x) = \varPhi(\tau, \mathbf{x})$ (which include also metric perturbations $h_{\mu\nu}$) on the minisuperspace background of the FRW metric and $S_E[a, N; \varPhi(x)] \equiv S_E[\,g_{\mu\nu}, \phi\,]$ is the original Euclidean action rewritten in terms of this minisuperspace decomposition.

The convenience of writing the path integral (17) in the Euclidean form follows from the needs of the semiclassical approximation. In this approximation, it is dominated by the contribution of a saddle point, $\varGamma_0 = S_{\rm eff}[\,a_0, N_0\,]$, where $a_0 = a_0(\tau)$ and $N_0 = N_0(\tau)$ solve the equation of motion for $S_{\rm eff}[\,a, N\,]$

$$\frac{\delta S_{\rm eff}[\,a_0, N_0\,]}{\delta N_0(\tau)} = 0 \tag{19}$$

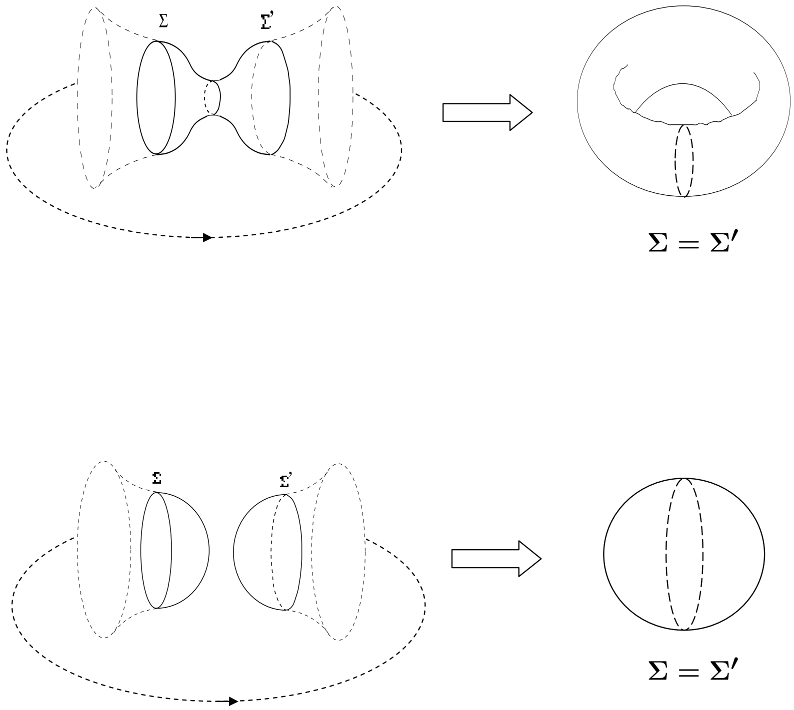

Figure 3: Transition from the density matrix to its statistical sum for entangled and pure states.

and satisfy periodicity conditions dictated by the definition of the statistical sum. Such periodic solutions exist with a real Euclidean N rather than in the Lorentzian domain with the imaginary lapse. This means that the contour of integration over N along the imaginary axis (13) should be deformed into the complex plane to traverse the real axis at some $N_0 \neq 0$ corresponding to the Euclidean solution of the equations of motion for the minisuperspace action as it is depicted in Fig.4.

The residual one-dimensional diffeomorphism invariance of this action (which is gauged out by the gauge-fixing procedure implicit in the integration measure $D[\,a,N\,]$) allows one to fix the ambiguity in the choice of N_0. There remains only a double-fold freedom in this choice. This freedom is of either positive, $N_0 > 0$, or negative, $N_0 < 0$, values of the lapse, because, on the one hand, all values in each of these equivalence classes are gauge equivalent and, on the other hand, no continuous family of nondegenerate diffeomorphisms exists relating these classes to one another. Without loss of generality one can choose as representatives of these classes $N_0 = \pm 1$ and label the relevant solutions and on-shell actions, respectively, as $a_\pm(\tau)$ and

$$\varGamma_\pm = S_{\rm eff}[\,a_\pm(\tau),\pm 1\,]\;. \tag{20}$$

Gauge inequivalence of these two cases, $\varGamma_- \neq \varGamma_+$, is obvious because, for example, all local contributions to the effective action are odd functionals of N, $S_{\rm local}[\,a,N\,] = -S_{\rm local}[\,a,-N\,]$. Thus we can heuristically identify the statistical sums $\varGamma_\pm$ correspondingly with the "no-boundary" and "tunneling" prescriptions for the quantum state of the Universe,

$$\exp(-\varGamma_{\rm no-boundary/tunnel}) = e^{-\varGamma_\pm}. \tag{21}$$

This result shows that for both prescriptions a full quantum effective action as a whole sits in the exponential of the partition function without any splitting into the minisuperspace and

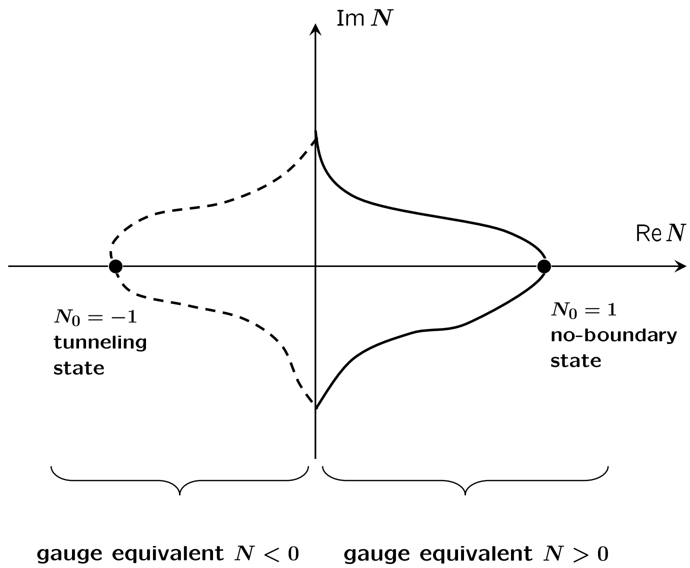

Figure 4: Integration contours in the complex lapse plane, passing through the "no-boundary" and "tunneling" saddle points of the integral.

matter contributions weighted by different sign factors like in (3). This means that the usual renormalization scheme is applicable to the calculation of (20) – generally covariant UV counterterms should be calculated on the background of a generic metric and afterwards evaluated at the FRW metric with $N = \pm 1$, depending on the choice of either the no-boundary or tunneling prescription.

The success in applications of these algorithms depends on the extent of our calculational skills in obtaining the effective action $S_{\rm eff}[\,a,\,N]$. The latter can be efficiently calculated in two different cases – for massless quantum fields conformally coupled to gravity and in the opposite case of very heavy massive fields. In the first case the presence of local conformal symmetry and its violation by exactly calculable anomaly allows one to find $S_{\rm eff}[\,a,\,N]$ as a functional of a generic FRW metric $(a(\tau),\,N(\tau))$, whereas in the second case $S_{\rm eff}[\,a,\,N]$ is known as a restriction to the minisuperspace background of a universal inverse mass (or gradient and curvature) expansion.

3 CFT driven cosmology: new status of the no-boundary state

Here we present the application of (19)-(21) to the no-boundary state in the gravitational theory with a matter sector dominated by a large number of free (linear) fields conformally coupled to gravity – conformal field theory (CFT)

$$S_E[\,g_{\mu\nu},\phi\,] = -\frac{1}{16\pi G}\int d^4x\,g^{1/2}\,\Big(R-2\Lambda\Big) + S_{CFT}[\,g_{\mu\nu},\phi\,]. \tag{22}$$

The effective action in such a system is dominated by the quantum action of these conformal fields which simply outnumber the non-conformal fields (including the graviton). This quantum effective action, in its turn, is exactly calculable by the conformal transformation converting (16) into the static Einstein metric with $a = \mathrm{const}$. In units of the Planck mass $m_P = (3\pi/4G)^{1/2}$

the action reads [15]

$$S_{\rm eff}[\,a,N\,]=m_P^2\oint d\tau\,N\left\{-aa'^2-a+\frac{\Lambda}{3}a^3+B\left(\frac{a'^2}{a}-\frac{a'^4}{6a}\right)+\frac{B}{2a}\right\}+F(\eta), \quad (23)$$

$$F(\eta)=\pm\sum_\omega \ln\left(1\mp e^{-\omega\eta}\right),\quad \eta=\oint\frac{d\tau N}{a}, \quad (24)$$

where $a'\equiv da/Nd\tau$. The first three terms in curly brackets of (23) represent the Einstein action with a primordial (but renormalized by quantum corrections) cosmological constant $\Lambda\equiv 3H^2$ (H is the corresponding Hubble constant), the B-terms correspond to the contribution of the conformal anomaly and the contribution of the vacuum (Casimir) energy ($B/2a$) of conformal fields on a static Einstein spacetime. $F(\eta)$ is the free energy of these fields – a typical boson or fermion sum over field oscillators with energies ω on a unit 3-sphere, η playing the role of the inverse temperature — an overall circumference of the toroidal instanton measured in units of the conformal time. The constant $B=3\beta/4m_P^2$ is determined by the coefficient β of the topological Gauss-Bonnet invariant $E=R_{\mu\nu\alpha\gamma}^2-4R_{\mu\nu}^2+R^2$ in the overall conformal anomaly of quantum fields

$$g_{\mu\nu}\frac{\delta S_{\rm eff}^{CFT}}{\delta g_{\mu\nu}}=\frac{1}{4(4\pi)^2}g^{1/2}\left(\alpha\Box R+\beta E+\gamma C_{\mu\nu\alpha\beta}^2\right) \quad (25)$$

$$e^{-S_{\rm eff}^{CFT}[\,g_{\mu\nu}]}=\int D[\,\phi\,]\,e^{-S_{CFT}[\,g_{\mu\nu},\phi\,]}. \quad (26)$$

Here $S_{\rm eff}^{CFT}[\,g_{\mu\nu}]$ is the effective action of quantum conformal fields in the external gravitational field and $C_{\mu\nu\alpha\beta}^2$ is the Weyl tensor squared term.

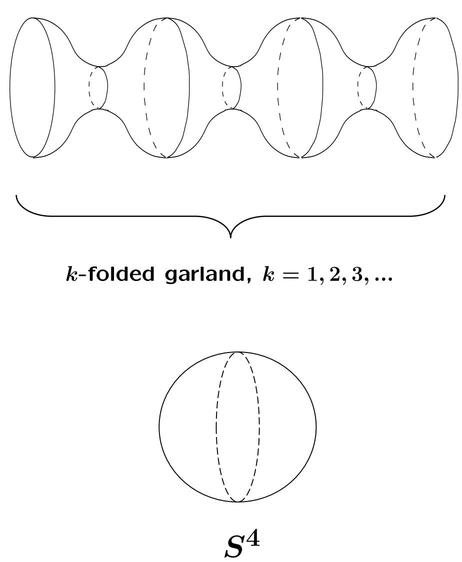

Figure 5: Garland-type periodic instantons with an oscillating scale factor (it is assumed that left and right boundaries of each k-folded garland are identified) and the Hartle-Hawking instanton S^4.

As shown in [15, 16, 17] the solutions of the effective equation (19) in this model give rise to the set of periodic garland-type instantons with oscillating scale factor of the topology $S^1\times S^3$ (that can be regarded as the thermal version of the Hartle-Hawking instantons) and the vacuum

Hartle-Hawking instantons with the S^4-topology (see Fig.5). The effective Friedmann equation (19)

$$-\frac{a'^2}{a^2}+\frac{1}{a^2}-B\left(\frac{a'^4}{2a^4}-\frac{a'^2}{a^4}\right)=\frac{\Lambda}{3}+\frac{C}{a^4}, \tag{27}$$

$$C=\frac{B}{2}+\frac{1}{m_P^2}\frac{dF(\eta)}{d\eta}=\frac{B}{2}+\frac{1}{m_P^2}\sum_\omega\frac{\omega}{e^{\omega\eta}\mp 1}, \quad \eta=\oint\frac{d\tau}{a}. \tag{28}$$

is modified by the anomalous B-term and the radiation term C/a^4 with the constant C characterizing the sum of the Casimir energy and the energy of the gas of thermally excited particles with the inverse temperature η – the instanton period in units of the conformal time. The latter is given in (28) by the integral over the full period of τ or the $2k$-multiple of the integral between the two neighboring turning points of the scale factor history $a(\tau)$, $\dot a(\tau_\pm)=0$. This k-fold nature implies that in the periodic solution the scale factor oscillates k times between its maximum and minimum values $a_\pm=a(\tau_\pm)$, $a_-\leq a(\tau)\leq a_+$,

$$a_\pm^2=\frac{1}{2H^2}\left(1\pm\sqrt{1-4H^2C}\right). \tag{29}$$

Thus, the period of the solutions is determined as a function of G and Λ from the second of Eqs.(28) and is not freely specifiable. This is the artifact of a microcanonical ensemble (see [16]) with only two freely specifiable dimensional parameters — the renormalized gravitational and renormalized cosmological constants.

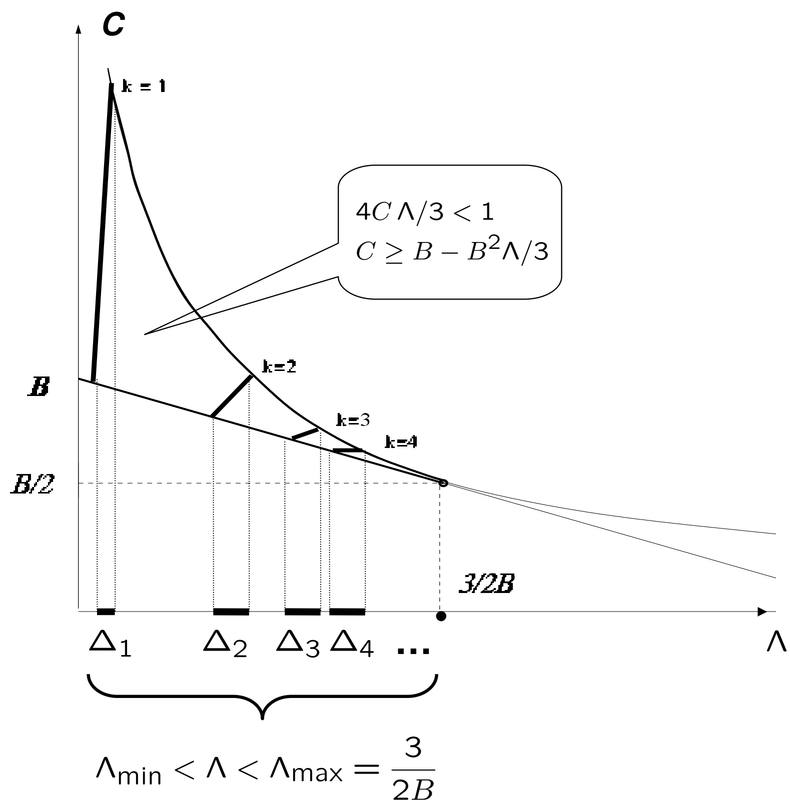

Figure 6: Band structure of cosmological constant spectrum for the thermal no-boundary state in the CFT driven cosmology.

As shown in [15], the $S^3\times S^1$ garland-type instantons exist only in the limited range of the cosmological constant $\Lambda = 3H^2$

$$0 < \Lambda_{\rm min} < \Lambda < \Lambda_{\rm max} = \frac{3}{2B}. \tag{30}$$

In this range they form an infinite $k = 0, 1, 2, ...$ sequence of one-parameter families depicted in Fig.6 in the two-dimensional plane of the cosmological constant Λ and the amount of radiation constant C (including together with the energy of the radiation gas, see (28), also the Casimir energy constant $B/2$.) These families interpolate between the two boundaries of a curvilinear triangle of the instanton domain in the (Λ, C)-plane – the lower straight line boundary $C = B - B^2\Lambda/3$ and the upper hyperbolic boundary $C = 3/4\Lambda$. Therefore the spectrum of admissible values of Λ inside (30) has a band structure – the bands Δ_k being formed by projections of these families onto the Λ-axes. Their sequence at $k \to \infty$ accumulates at the corner of this triangle — the upper bound of the range (30).

This bound represents a new quantum gravity scale which tends to infinity when one switches the quantum effects off, $\beta \to 0$. The lower bound $\Lambda_{\rm min}$ — the lowest point of $k = 1$ family — can be obtained numerically for any field content of the model. For a large number of conformal fields N, and therefore a large $\beta \sim N$, the lower bound is of the order $\Lambda_{\rm min} \sim 1/\beta G$. Thus the restriction (30) suggests a kind of $1/N$ solution of the cosmological constant problem, because specifying a sufficiently high number of conformal fields one can achieve a primordial value of Λ well below the Planck scale where the effective theory applies, but high enough to generate a sufficiently long inflationary stage. Also this restriction can be potentially considered as a selection criterion for the landscape of string vacua [15, 16].

The solutions of the system (27)-(28) include also the vacuum Hartle-Hawking instantons with no radiation $dF/d\eta = 0$. They represent the Euclidean de Sitter spacetime with the effective Hubble factor

$$H^2_{\rm eff} = \frac{1 - \sqrt{1 - 2BH^2}}{B}, \tag{31}$$

corresponding to the degenerate case when a torus $S^3 \times S^1$ gets ripped at the vanishing value of the scale factor $a_- = 0$ and topologically becomes a 4-sphere S^4 — a purely vacuum contribution to the statistical sum. The vacuum nature of these instantons follows from the fact that their conformal time period in (28) is divergent at τ_- in view of $a_- = 0$ and generates zero effective temperature $\sim 1/\eta$ with $F(\eta) = 0$. Such solutions exist for all $\Lambda \leq 3/2B$ (the horizontal segment at $C = B/2$ in Fig.6), but they are ruled out in the statistical sum by their infinite *positive* effective action. This property is due to the contribution of the conformal anomaly (cf. $1/a$-dependence in the kinetic B-terms of the effective action (23))[2]. Hence the tree-level predictions of the theory with a negative Euclidean action are drastically changed by the effect of the conformal anomaly.

3.1 Tunneling state

The situation with the tunneling state for CFT cosmology is more complicated. To begin with, the attempt to calculate the effective action for $N > 0$ and then analytically continue to negative N with $\eta < 0$ is not straightforward, because these two domains are separated by the imaginary axes densely filled by branch points of the logarithm in $F(\eta)$ at $\omega\eta = (2n + 1)i\pi$ for all integer n and all discrete spectrum values of field oscillator frequencies ω.[3]

[2]Note that on the vacuum solution of (27) $a'^2(\tau_-) = 1$, and the integrand of (23) tends to $+\infty$ at τ_- with $a \to 0$.

[3]The UV regularization by the cutoff on the maximum value of ω opens a small bridge in the vicinity of zero between the two half-planes of the complex N, but the result of analytic continuation through this bridge is far from obvious.

The alternative approach could consist of a direct calculation and renormalization of the effective action with N and η negative, that is the transition $\eta \to -\eta$ in (23) *before* the renormalization. Note that the origin of the conformal anomaly part of the action (23)

$$S_{\rm anomaly}[\,a,N\,]=m_P^2 B \oint d\tau\, N \left(\frac{a'^2}{a}-\frac{a'^4}{6a}\right) \tag{32}$$

and its *finite* Casimir energy term $\eta\, m_P^2 B/2$ is due to the covariant regularization and renormalization of quartic, quadratic and logarithmic divergences in the formal vacuum energy part of the action $\eta \sum_\omega (\omega/2)$. This can be written down as the relation

$$\left[\eta \sum_\omega \frac{\omega}{2}\right]^{\rm renorm} = \frac{m_P^2 B}{2}\,\eta + S_{\rm anomaly}[\,a,N\,], \tag{33}$$

signifying that the anomalous part of the action arises as a tail to the counterterm subtraction of UV divergences in $\eta \sum_\omega (\omega/2)$ and therefore is strongly correlated in sign with the latter. Naively it would seem that the inversion of $\eta \to -\eta$ in (27) would reverse the sign of the anomalous part and therefore infinitely enhance the contribution of the S^4-instanton in the tunneling case (opposite to the no-boundary situation above). The jump to this conclusion is however misleading, because after the inversion $\eta \to -\eta$ the free energy $F(-\eta)$ with $\eta > 0$ also becomes UV divergent, $F(-\eta)\,|^{\,\rm div} = \eta \sum_\omega \omega\,|^{\,\rm div}$, and therefore it requires the additional counterterm which actually restores the same no-boundary sign of the anomalous term. To see this we have a chain of simple transformations (valid modulo imaginary *field-independent* part) for the quantum part of the tunneling effective action

$$\begin{aligned}\left[-\eta \sum_\omega \frac{\omega}{2} + F(-\eta)\right]^{\rm renorm} &= \left[\eta \sum_\omega \frac{\omega}{2} + F(\eta)\right]^{\rm renorm}\\ &= S_{\rm anomaly}[\,a,N\,] + \frac{m_P^2 B}{2}\,\eta + F(\eta), \quad \eta, N > 0.\end{aligned} \tag{34}$$

This means that only the classical Einstein part of the full action has an opposite sign, whereas the quantum part is the same as in the no-boundary case. Effectively, this means that in the equation of motion (27)-(28) only the constant C changes its sign $C \to -C$. This change implies due to (29) that $a_-^2 < 0$, so that the scale factor of the Euclidean solution runs between $a = 0$ and a_+ and corresponds to the S^4 case of the vacuum Hartle-Hawking instanton[4]. However, in view of (34) above the total effective action diverges to $+\infty$, as in the vacuum no-boundary case, and rules out the tunneling contribution. Thus we come to a conclusion that in the CFT driven cosmology the tunneling state is dynamically forbidden.

4 Heavy massive fields: no-boundary and tunneling states

For heavy massive quantum fields the situation is more favorable for the tunneling state. Here we consider both no-boundary and tunneling case simultaneously because their formalisms are very close to one another. Now the Euclidean effective action universally has a gradient and curvature expansion in inverse powers of the mass parameters. It takes the form

$$S_{\rm eff}[g_{\mu\nu}] = \int d^4x\, g^{1/2} \left(M_{\rm P}^2 \Lambda - \frac{M_{\rm P}^2}{2}\, R(g_{\mu\nu}) + ...\right), \tag{35}$$

where we disregard the terms of higher orders in the curvature and derivatives of the mean values of matter fields. Here the cosmological term and the (reduced) Planck mass squared

[4] Its conformal time period η diverges, so that $F(\eta) = 0$ and $C = 0$, and the equation of motion yields exact de Sitter space solution with the effective Hubble constant (31).

$M_{\rm P}^2 = 1/8\pi G$ can be considered as functions of these mean values and treated as constants in the approximation of slowly varying fields. This effective action does not contain the thermal part characteristic of the statistical ensemble [15] because for heavy quanta the radiation bath is not excited. This is justified by the fact that the effective temperature of this bath turns out to be vanishing.

In fact, the minisuperspace action functional for (35) reads in units of $m_{\rm P}^2 = 3\pi/4G = 6\pi^2 M_{\rm P}^2$ as

$$S_{\rm eff}[\,a, N\,] = m_{\rm P}^2 \int d\tau\, N(-aa'^2 - a + H^2 a^3), \tag{36}$$

where $a' \equiv da/N d\tau$, and we use the notation for the cosmological constant $\Lambda = 3H^2$ in terms of the effective Hubble factor H. Then the saddle point for the path integral (17) – the stationary configuration with respect to variations of the lapse function, $\delta S_{\rm eff}[\,a, N\,]/\delta N = 0$, – satisfies the Euclidean Friedmann equation

$$a'^2 = 1 - H^2 a^2. \tag{37}$$

It has one turning point at $a_+ = 1/H$ below which the real solution interpolates between $a_- \equiv a(0) = 0$ and a_+. In the gauge $N = \pm 1$ for both no-boundary/tunneling cases this solution describes the Euclidean de Sitter metric, that is, one hemisphere of S^4 (depicted on Fig.1),

$$a_\pm(\tau) = \frac{1}{H}\sin(H\tau). \tag{38}$$

After the bounce from the equatorial section of the maximal scale factor a_+, this solution spans at the contraction phase the rest of the full four-sphere. Thus, this solution is not periodic and according to the discussion above describes a purely vacuum contribution to the statistical sum (17). Similarly to the case of conformal fields the effective temperature of this state is determined by the inverse of the full period of the instanton solution measured in units of the conformal time η. Therefore, for (38) it vanishes because this period between the poles of this spherical instanton is divergent. This justifies the absence of the thermal part in (35).

Thus, with $N = \pm 1$ the no-boundary and tunneling on-shell actions (20) read

$$\varGamma_\pm = \mp \frac{8\pi^2 M_{\rm P}^2}{H^2} \tag{39}$$

and the object of major interest here – the tunneling partition function in the space of positive values of $H^2 = \Lambda/3$ – is given by

$$\rho_{\rm tunnel}(\Lambda) = \exp\left(-\frac{24\pi^2 M_{\rm P}^2}{\Lambda}\right), \quad \Lambda > 0. \tag{40}$$

It coincides with the semiclassical tunneling wavefunction of the Universe [2], $|\varPsi_{\rm tunnel}|^2 \simeq \exp(-8\pi^2 M_{\rm P}^2/H^2)$, derived from the Wheeler–DeWitt equation in the tree-level approximation.

At the turning point a_+, the solution (38) can be analytically continued to the Lorentzian regime, $a_{\rm L}(t) = a(\pi/2H + it)$. The scale factor then expands as

$$a_{\rm L}(t) = \frac{1}{H}\cosh(Ht)\ , \tag{41}$$

which can be interpreted as representing the distributions of scale factors in the quantum ensemble (after decoherence) of de Sitter models distributed according to (40), see Fig. 1. Note that the attempt to extend this ensemble to negative Λ fails, because the equation (37) with $H^2 < 0$ does not have turning points with nucleating real Lorentzian geometries. Moreover, virtual cosmological models with Euclidean signature are also forbidden in the tunneling state because their positive Euclidean action diverges to infinity, so that $\rho_{\rm tunnel}(\Lambda) = 0$ for $\Lambda < 0$.

5 Quantum origin of the Universe with the SM Higgs-inflaton non-minimally coupled to curvature

The partition function of the above type can serve as a source of initial conditions for inflation only when the cosmological constant $\Lambda = 3H^2$ becomes a composite field capable of a decay at the exit from inflation. Usually this is a scalar inflaton field whose quantum mean value φ is nearly constant in the slow roll regime, and its effective potential $V(\varphi)$ plays the role of the cosmological constant driving the inflation. When the contribution of the inflaton gradients is small, the above formalism remains applicable also with the inclusion of this field whose ultimate effect reduces to the generation of the effective cosmological constant $\Lambda = V(\varphi)/M_{\rm P}^2$ and the effective Planck mass.

These constants are the coefficients of the zeroth and first order terms of the effective action expanded in powers of the curvature, and they incorporate radiative corrections due to all quantum fields in the path integral (18). Now there is no mismatch between the signs of the tree-level and loop parts of the partition function. Therefore, one can apply the usual renormalization and, if necessary, the renormalization group (RG) improvement to obtain the full effective action $S_{\rm eff}[g_{\mu\nu},\varphi]$ and then repeat the procedure of the previous section. In the slow roll approximation the effective action has the general form

$$S_{\rm eff}[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2} \Big(V(\varphi) - U(\varphi)\, R(g_{\mu\nu}) + \frac{1}{2}\, G(\varphi)\, (\nabla\varphi)^2 + ... \Big), \tag{42}$$

where $V(\varphi)$, $U(\varphi)$ and $G(\varphi)$ are the coefficients of the derivative expansion, and we disregard the contribution of higher-derivative operators. With the slowly varying inflaton the coefficients $V(\varphi)$ and $U(\varphi)$ play the role of the effective cosmological and Planck mass constants, so that one can identify in (35) and (36) the effective $M_{\rm P}^2 = m_{\rm P}^2/6\pi^2$ and H^2, respectively, with $2U(\varphi)$ and $V(\varphi)/6U(\varphi)$. Therefore, the tunneling partition function (40) becomes the following distribution of the field φ

$$\rho_{\rm tunnel}(\varphi) = \exp\left(-\frac{24\pi^2 M_{\rm P}^4}{\hat V(\varphi)} \right), \tag{43}$$

$$\hat V(\varphi) = \left(\frac{M_{\rm P}^2}{2} \right)^2 \frac{V(\varphi)}{U^2(\varphi)}, \tag{44}$$

where $\hat V(\varphi)$ in fact coincides with the potential in the Einstein frame of the action (42) [30, 31].

Now we apply this formalism to the model (4) of inflation driven by the SM Higgs inflaton $\varphi = (\Phi^\dagger\Phi)^{1/2}$. As shown in [30, 31], the one-loop RG improved action in this model has for large φ the form (42) with the coefficient functions

$$V(\varphi) = \frac{\lambda(t)}{4}\, Z^4(t)\, \varphi^4, \tag{45}$$

$$U(\varphi) = \frac{1}{2}\Big(M_{\rm P}^2 + \xi(t)\, Z^2(t)\, \varphi^2 \Big), \tag{46}$$

$$G(\varphi) = Z^2(t), \tag{47}$$

determined in terms of the running couplings $\lambda(t)$ and $\xi(t)$, and the field renormalization $Z(t)$. They incorporate a summation of powers of logarithms and belong to the solution of the RG equations which at the inflationary stage with a large $\varphi \sim M_{\rm P}/\sqrt{\xi}$ and large $\xi \gg 1$ read as (see [30, 31] for details)

$$\frac{d\lambda}{dt} = \frac{\boldsymbol{A}}{16\pi^2}\lambda - 4\gamma\lambda, \tag{48}$$

$$\frac{d\xi}{dt} = \frac{6\lambda}{16\pi^2}\xi - 2\gamma\xi \tag{49}$$

and $dZ/dt = \gamma Z$. Here, γ is the anomalous dimension of the Higgs field, the running scale $t = \ln(\varphi/M_t)$ is normalized at the top quark mass $\mu = M_t$, and $\boldsymbol{A} = \boldsymbol{A}(t)$ is the running parameter of the *anomalous scaling.* This quantity was introduced in [10] as the pre-logarithm coefficient of the overall effective potential of all SM physical particles and Goldstone modes. Due to their quartic, gauge and Yukawa couplings with φ, they acquire masses $m(\varphi) \sim \varphi$ and for large φ give rise to the asymptotic behavior of the Coleman-Weinberg potential,

$$V^{1-\mathrm{loop}}(\varphi) = \sum_{\mathrm{particles}} (\pm 1)\, \frac{m^4(\varphi)}{64\pi^2} \ln \frac{m^2(\varphi)}{\mu^2} \simeq \frac{\lambda \boldsymbol{A}}{128\pi^2}\, \varphi^4 \ln \frac{\varphi^2}{\mu^2}, \tag{50}$$

which can serve as a definition of $\boldsymbol{A}$.

The importance of this quantity and its modification due to the RG running of the non-minimal coupling $\xi(t)$,

$$\boldsymbol{A_I} = \boldsymbol{A} - 12\lambda \tag{51}$$

($\boldsymbol{A_I}$ gives the running of the ratio λ/ξ^2, $16\pi^2(d/dt)(\lambda/\xi^2) = \boldsymbol{A_I}(\lambda/\xi^2)$), is that for $\xi \gg 1$ mainly these parameters determine the quantum inflationary dynamics [14, 33] and yield the parameters of the CMB generated during inflation [25]. In particular, the value of φ at the beginning of the inflationary stage of duration N in units of the e-folding number turns out to be [25]

$$\varphi^2 = -\frac{64\pi^2 M_{\mathrm{P}}^2}{\xi \boldsymbol{A_I}(t_{\mathrm{end}})}(1 - e^x), \tag{52}$$

$$x \equiv \frac{N \boldsymbol{A_I}(t_{\mathrm{end}})}{48\pi^2}, \tag{53}$$

where a parameter x has been introduced which directly involves $\boldsymbol{A_I}(t_{\mathrm{end}})$ taken at the end of inflation, $t_{\mathrm{end}} = \ln(\varphi_{\mathrm{end}}/M_t)$, $\varphi_{\mathrm{end}} \simeq 2M_{\mathrm{P}}/\sqrt{3\xi}$. This parameter also enters simple algorithms for the CMB power spectrum $\Delta^2_\zeta(k)$ and its spectral index $n_s(k)$. As shown in [30, 31], the application of these algorithms under the observational constraints $\Delta^2_\zeta(k_0) \simeq 2.5 \times 10^{-9}$ and $0.94 < n_s(k_0) < 0.99$ (the combined WMAP+BAO+SN data at the pivot point $k_0 = 0.002$ Mpc^{-1} corresponding to $N \simeq 60$ [34]) gives the CMB-compatible range of the Higgs mass $135.6\ \mathrm{GeV} \lesssim M_{\mathrm{H}} \lesssim 184.5\ \mathrm{GeV}$, both bounds being determined by the lower bound on the CMB spectral index.

Now we want to show that, in addition to the good agreement of the spectrum of cosmological perturbations with the CMB data, this model can also describe the mechanism of generating the cosmological *background* itself upon which these perturbations exist. This mechanism consists in the formation of the initial conditions for inflation in the form of a sharp probability peak in the distribution function (43) at some appropriate value of the inflaton field φ_0 with which the Universe as a whole starts its evolution [21]. The shape and the magnitude of the potential (44) depicted in Fig.1 for several values of the Higgs mass clearly indicates the existence of such a peak.

Indeed, the negative of the inverse potential damps to zero after exponentiation the probability of those values of φ at which $\hat{V}(\varphi) = 0$ and, vice versa, enhances the probability at the positive maxima of the potential. The pattern of this behavior with growing Higgs mass M_{H} is as follows. As is known, for low M_{H} the SM has a domain of unstable EW vacuum, characterized by negative values of running $\lambda(t)$ at certain energy scales. Thus we begin with the EW vacuum instability threshold [35, 36] which exists in this gravitating SM at $M_{\mathrm{H}}^{\mathrm{inst}} \approx 134.27$ GeV [30, 31] and which is slightly lower than the CMB compatible range of the Higgs mass ($M_{\mathrm{H}}^{\mathrm{inst}}$ is chosen in Fig. 8 and for the lowest curve in Fig. 7). The potential $\hat{V}(\varphi)$ drops to zero at $t_{\mathrm{inst}} \simeq 41.6$, $\varphi_{\mathrm{inst}} \sim 80 M_{\mathrm{P}}$, and forms a false vacuum [30, 31] separated from the EW vacuum by a large peak at $t \simeq 34$. Correspondingly, the probability of creation of the Universe with

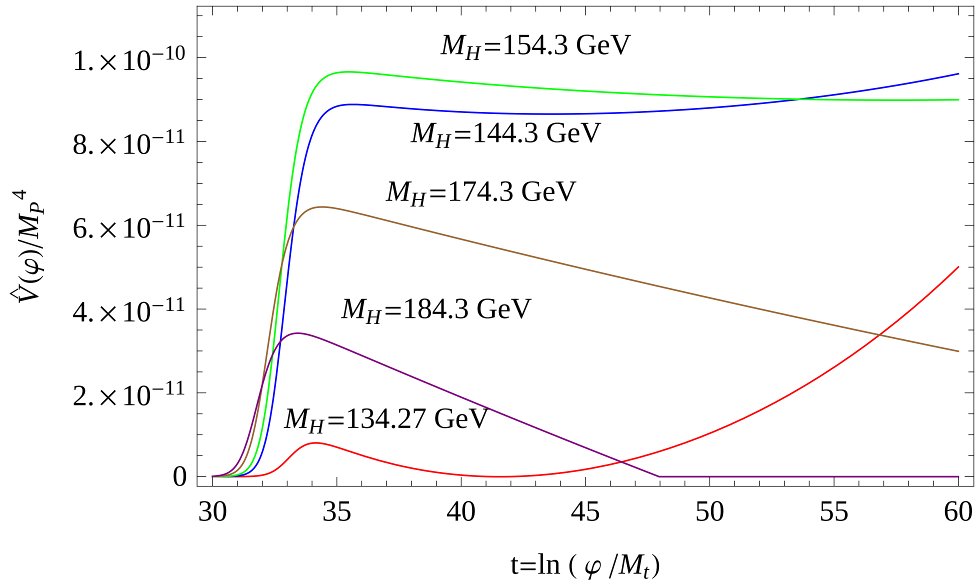

Figure 7: The succession of effective potential graphs above the EW vacuum instability threshold $M_{\rm H}^{\rm inst} = 134.27$ GeV up to $M_{\rm H} = 184.3$ GeV showing the occurrence of a metastable vacuum followed for high $M_{\rm H}$ by the formation of a negative slope branch. Local peaks of $\hat{V}$ situated at $t = 34 \div 35$ grow with $M_{\rm H}$ for $M_{\rm H} \lesssim 160$ GeV and start decreasing for larger $M_{\rm H}$ [30].

the initial value of the inflaton field at the EW scale $\varphi = v$ and at the instability scale $\varphi_{\rm inst}$ is damped to zero, while the most probable value belongs to this peak. The inflationary stage of the formation of the pivotal $N = 60$ CMB perturbation (from the moment $t_{\rm in}$ of the first horizon crossing until the end of inflation $t_{\rm end}$), which is marked by dashed lines in Fig.8, lies to the left of this peak. This conforms to the requirement of the chronological succession of the initial conditions for inflation and the formation of the CMB spectra.

The above case is, however, below the CMB-compatible range of $M_{\rm H}$ and was presented here only for illustrative purposes[5]. An important situation occurs at higher Higgs masses from the lower CMB bound on $M_{\rm H} \simeq 135.6$ GeV until about 160 GeV. Here we get a family of a metastable vacua with $\hat{V} > 0$. An example is the plot for the lower CMB bound $M_{\rm H} = 135.62$ GeV depicted in Fig. 9. Despite the shallowness of this vacuum its small maximum generates via (43) a sharp probability peak for the initial inflaton field. This follows from an extremely small value of $\hat{V}/M_{\rm P}^4 \sim 10^{-11}$, the reciprocal of which generates a rapidly changing exponential of (43). The location of the peak again precedes the inflationary stage for a pivotal $N = 60$ CMB perturbation (also marked by dashed lines in Fig. 9).

For even larger $M_{\rm H}$ these metastable vacua get replaced by a negative slope of the potential which interminably decreases to zero at large t (at least within the perturbation theory range of the model), see Fig. 1. Therefore, for large $M_{\rm H}$ close to the upper CMB bound 185 GeV, the probability peak of (43) gets separated from the non-perturbative domain of large over-Planckian scales due to a fast drop of $\hat{V} \sim \lambda/\xi^2$ to zero. This, in turn, follows from the fact that $\xi(t)$ grows much faster than $\lambda(t)$ when they both start approaching their Landau pole [30].

The location φ_0 of the probability peak and its quantum width can be found in analytical form, and their derivation shows the crucial role of the running $\boldsymbol{A_I}(t)$ for the formation of initial conditions for inflation. Indeed, the exponential of the tunneling distribution (43) for

[5] Another interesting range of $M_{\rm H}$ is below the instability threshold $M_{\rm H}^{\rm inst}$ where $\hat{V}$ becomes negative in the "true" high energy vacuum. As mentioned in the previous section, the tunneling state rules out aperiodic solutions of effective equations with $H^2 < 0$, which cannot contribute to the quantum ensemble of expanding Lorentzian signature models. Therefore, this range is semiclassically ruled out not only by the instability arguments, but also contradicts the tunneling prescription.

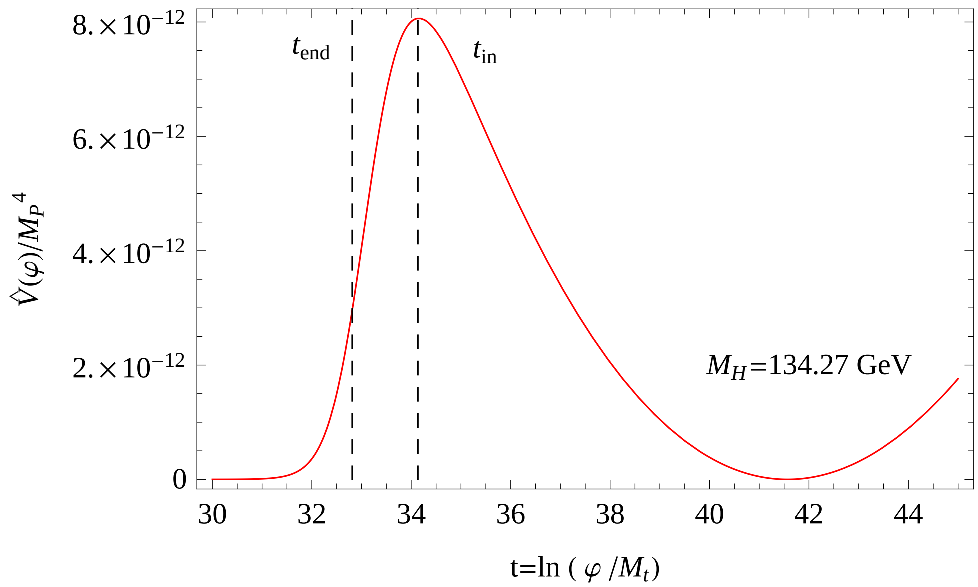

Figure 8: The effective potential for the instability threshold $M_{\rm H}^{\rm inst} = 134.27$ GeV. A false vacuum occurs at the instability scale $t_{\rm inst} \simeq 41.6$, $\varphi \sim 80 M_{\rm P}$. An inflationary domain for a $N = 60$ CMB perturbation (ruled out by the WMAP bounds) is marked by dashed lines [30].

$M_{\rm P}^2/\xi\varphi^2 \ll 1$ reads as $\varGamma_-(\varphi) = 24\pi^2 M_{\rm P}^4/\hat{V}(\varphi)$ and in view of the RG equations (48)–(49) has an extremum satisfying the equation

$$\varphi\frac{d\varGamma}{d\varphi} = \frac{d\varGamma}{dt} = -\frac{6\xi^2}{\lambda}\left(\boldsymbol{A_I} + \frac{64\pi^2 M_{\rm P}^2}{\xi Z^2\varphi^2}\right) = 0, \tag{54}$$

where we again neglect higher order terms in $M_{\rm P}^2/\xi Z^2\varphi^2$ and $\boldsymbol{A_I}/64\pi^2$ (extending beyond the one-loop order). Here, $\boldsymbol{A_I}$ is the anomalous scaling introduced in (50) and (51) – the quantity that should be negative for the existence of the solution for the probability peak,

$$\varphi_0^2 = -\frac{64\pi^2 M_{\rm P}^2}{\xi \boldsymbol{A_I} Z^2}\bigg|_{t=t_0}. \tag{55}$$

As shown in [30, 31], this quantity is indeed negative. In the CMB-compatible range of $M_{\rm H}$ its running starts from the range $-36 \lesssim \boldsymbol{A_I}(0) \lesssim -23$ at the EW scale and reaches small but still negative values in the range $-11 \lesssim \boldsymbol{A_I}(t_{\rm end}) \lesssim -2$ at the inflation scale. Also, the running of $\boldsymbol{A_I}(t)$ and $Z(t)$ is very slow – the quantities belonging to the two-loop order – and the duration of inflation is very short $t_0 \sim t_{\rm in} \simeq t_{\rm end} + 2$ [30, 31]. Therefore, $\boldsymbol{A_I}(t_0) \simeq \boldsymbol{A_I}(t_{\rm end})$, and these estimates apply also to $\boldsymbol{A_I}(t_0)$. As a result, the second derivative of the tunneling on-shell action is positive and very large, $d^2\varGamma_-/dt^2 \simeq -(12\xi^2/\lambda)\boldsymbol{A_I} \gg 1$, which gives an extremely small value of the quantum width of the probability peak,

$$\frac{\Delta\varphi^2}{\varphi_0^2} = -\frac{\lambda}{12\xi^2}\frac{1}{\boldsymbol{A_I}}\bigg|_{t=t_0} \sim 10^{-10}. \tag{56}$$

This width is about $(24\pi^2/|\boldsymbol{A_I}|)^{1/2}$ times – one order of magnitude – higher than the CMB perturbation at the pivotal wavelength $k^{-1} = 500$ Mpc (which we choose to correspond to $N = 60$). The point $\varphi_{\rm in}$ of the horizon crossing of this perturbation (and other CMB waves with different N's) easily follows from equation (52) which in view of $\boldsymbol{A_I}(t_0) \simeq \boldsymbol{A_I}(t_{\rm end})$ takes the form

$$\frac{\varphi_{\rm in}^2}{\varphi_0^2} = 1 - \exp\left(-N\frac{|\boldsymbol{A_I}(t_{\rm end})|}{48\pi^2}\right). \tag{57}$$

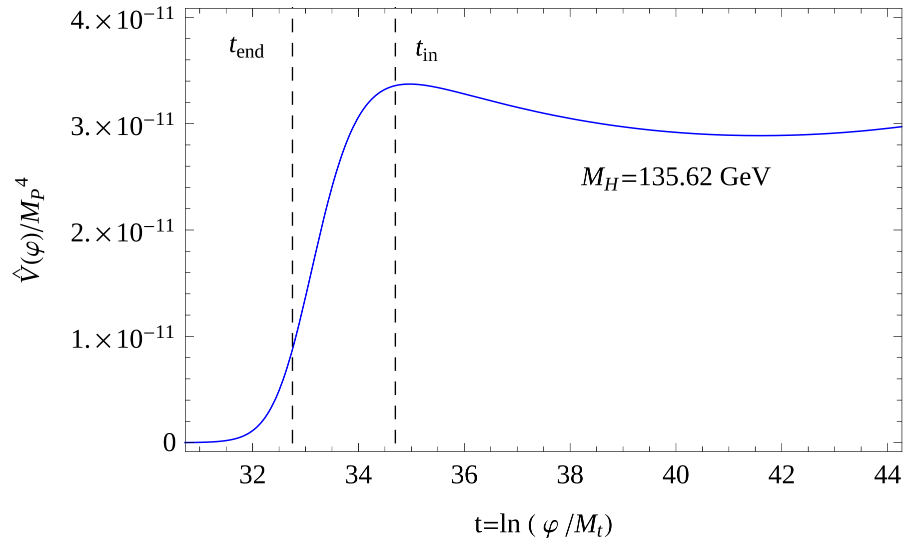

Figure 9: Inflaton potential at the lowest CMB compatible value of $M_{\rm H}$ with a metastable vacuum at $t \simeq 42$ [30].

It indicates that for wavelengths longer than the pivotal one the instant of horizon crossing approaches the moment of “creation” of the Universe, but always stays chronologically succeeding it, as it must.

6 Conclusions and discussion

In this paper we have constructed the tunneling quantum state of the Universe based on the path integral for the microcanonical ensemble in cosmology. For heavy massive quantum fields this state exists in the unbounded positive range of the effective cosmological constant, unlike the no-boundary state for massless conformally coupled fields (CFT driven cosmology), discussed in [15, 16] whose ensemble is bounded by the reciprocated coefficient of the topological term in the overall conformal anomaly. Also, in contrast to the no-boundary case, the tunneling state turns out to be a radiation-free vacuum one.

The status of the tunneling versus no-boundary states is rather involved. In fact, the formal Euclidean path integral (17) is a transformed version of the microcanonical path integral over Lorentzian metrics, so that its lapse function integration runs along the imaginary axis from $-{\rm i}\infty$ to $+{\rm i}\infty$ [16]. The absence of periodic solutions for stationary points of (17) with the Lorentzian signature makes one to distort the contour of integration over N into a complex plane, so that it traverses the real axis at the points $N = +1$ or $N = -1$ which give rise to no-boundary or tunneling states. One can show that the no-boundary thermal part of the statistical sum of [15] is not analytic in the full complex plane of N. The $N \gtrless 0$ domains are separated by the infinite sequence of its poles densely filling the imaginary axes of N. Therefore, the contour of integration passing through both points $N = \pm 1$ is impossible, and the no-boundary and tunneling states cannot be directly obtained by analytic continuation from one another[6]. They represent alternative solutions (quantum states) of the Wheeler-DeWitt equation.

According to the discussion of Sect.3.1 the tunneling state for a CFT driven cosmology can be alternatively defined without the analytic continuation from the domain of the positive

[6]In the case of the vacuum no-boundary state when the vanishing thermal part of the effective action cannot present an obstacle to analytic continuation in the complex plane of N the situation stays the same. Indeed, any integration contour from $-i\infty$ to $+i\infty$ crosses the real axes an odd number of times, so that the contribution of only one such crossing survives, because any two (gauge-equivalent) saddle points traversed in opposite directions give contributions canceling one another.

lapse, but turns out to be suppressed by an infinite positive action of the relevant vacuum S^4-instanton. This dynamical suppression casts certain doubt on the existence of a tunneling state in quantum cosmology at all, because in realistic cosmology the inclusion of any single massless conformal particle (like photon) might destroy it. However, we prefer to be not so categorical with this statement, because the cutoff regularization, mentioned in Sect.3.1, still opens the bridge for analytic continuation between the domains of positive and negative lapse, the result of this continuation still remaining an open issue.

For heavy quantum fields the path-integral formulation of the tunneling state admits a consistent renormalization scheme and a RG resummation which is very efficient in cosmology according to a series of recent papers [27, 28, 29, 30, 31, 32]. For this reason we have applied the obtained tunneling distribution to a recently considered model of inflation driven by the SM Higgs boson non-minimally coupled to curvature – the case of the model whose dynamics is dominated by heavy massive particles. In this way a complete cosmological scenario was obtained, embracing the formation of initial conditions for the inflationary background (in the form of a sharp probability peak in the inflaton field distribution) and the ongoing generation of the CMB perturbations on this background. As was shown in [30, 31], the comparison of the CMB amplitude and the spectral index with the WMAP observations impose bounds on the allowed range of the Higgs mass. These bounds turn out to be remarkably consistent with the widely accepted EW vacuum stability and perturbation theory restrictions. Interestingly, the behavior of the running anomalous scaling $\boldsymbol{A_I}(t) < 0$, being crucially important for these bounds, also guarantees the existence of the obtained probability peak [21]. The quantum width of this peak is one order of magnitude higher than the amplitude of the CMB spectrum at the pivotal wavelength, which could entail interesting observational consequences. Unfortunately, this quantum width is hardly measurable directly because it corresponds to an infinite wavelength perturbation (a formal limit of $N \to \infty$ in (57)), but indirect effects of this quantum trembling of the cosmological background deserve further study.

To summarize, the obtained results bring to life a convincing unification of quantum cosmology with the particle phenomenology of the SM, inflation theory, and CMB observations. They support the hypothesis that an appropriately extended Standard Model [37, 38] can be a consistent quantum field theory all the way up to quantum gravity and perhaps explain the fundamentals of all major phenomena in early and late cosmology.

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