Quantum corrections in supersymmetric theories with the higher covariant derivative regularization

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Abstract

The structure of quantum corrections in supersymmetric theories regularized by higher covariant derivatives is investigated. We present the result of calculation of a two-loop β-function for the most general renormalizable $N = 1$ supersymmetric theory. All integrals, needed for obtaining this function, are integrals of total derivatives and can be easily calculated. For $N = 1$ supersymmetric electrodynamics we prove that contributions of planar diagrams with a single matter loop to the β-function are given by integrals of a total derivative in all orders.

1 Introduction

Quantum corrections in supersymmetric theories were studied for a long time. For a general renormalizable $N = 1$ supersymmetric theory a β-function and an anomalous dimension were calculated up to the four-loop approximation [1, 2]. Most calculations were made with a regularization by the dimensional reduction [3], which is a special modification of the dimensional regularization. The matter is that the dimensional regularization [4] breaks the supersymmetry and is not convenient for calculations in supersymmetric theories. However, it is well known that the dimensional reduction is not self-consistent [5]. Ways, allowing to avoid such problems, are discussed in the literature [6]. Other regularizations are sometimes applied for calculations in supersymmetric theories. For example, in Ref. [7] two-loop β-function of the N=1 supersymmetric Yang–Mills theory was calculated with the differential renormalization [8].

A self-consistent regularization, which does not break the supersymmetry, is the higher covariant derivative regularization [9], which was generalized to the supersymmetric case in Ref. [10] (another variant was proposed in Ref. [11]). However, using this regularization is rather technically complicated. The first calculation of quantum corrections for the (non-supersymmetric) Yang–Mills theory, regularized by higher covariant derivatives, was made in Ref. [12]. Taking into account corrections, made in subsequent papers [13], the result for the β-function appeared to be the same as the well-known result, obtained with the dimensional regularization [14]. In principle, it is possible to prove that in the one-loop approximation calculations with the higher covariant derivative regularization always agree with the results of calculations with the dimensional regularization [15]. Some calculations in the one-loop and two-loop approximations were made for various theories [16, 17] with a variant of the higher covariant derivative regularization, proposed in [18]. The structure of the corresponding integrals was discussed in Ref. [17].

Application of the higher derivative regularization to calculation of quantum corrections in the N=1 supersymmetric electrodynamics (SQED) in two and three loops [19] reveals an

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interesting feature of quantum corrections: loop integrals, defining a β-function, are integrals of total derivatives. (In Ref. [19] three-loop integrals, defining a β-function, were reduced to integrals of total derivatives using integration by parts.) This allows to calculate one of the loop integrals analytically and obtain the NSVZ β-function, which relates a β-function in n-th loop with a β-function and anomalous dimensions in the previous loops. Due to this, application of the higher covariant regularization is sometimes very convenient in the supersymmetric case. The fact that the integrals, appearing with the higher covariant derivative regularization, in the limit of zero external momentum become integrals of total derivatives, seems to be a general feature of all supersymmetric theories. This was verified in the two-loop approximation in [20] for a general renormalizable $N=1$ supersymmetric theory. A similar result, obtained with a different (simpler) version of the higher derivative regularization, is presented in this paper.

Arguments in favour of the factorization of integrands into total derivatives for $N=1$ SQED, confirmed by an explicit two-loop calculation, were presented in [21]. A partial explanation of this fact was made in [22] by substituting solution of the Ward identity into the Schwinger–Dyson equation for $N=1$ SQED, regularized by higher derivatives.[1] In particular, using this method it is possible to extract a contribution, giving the exact NSVZ β-function in all orders. However, there is also another contribution. In $N=1$ SQED this contribution is nontrivial starting from the three-loop approximation. In the three-loop approximation it is given by an integral of a total derivative and is equal to 0. It was conjectured in [22] that this occurs in all loops. A partial four-loop verification of this statement was made in Ref. [24]. In this paper we verify that in $N=1$ SQED this statement is true for the planar diagrams with a single matter loop.

The paper is organized as follows:

In Sec. 2 we recall basic information about $N=1$ supersymmetric theories and their regularization by higher covariant derivatives. In Sec. 3 we describe two-loop calculation of a β-function for a general renormalizable $N=1$ supersymmetric Yang–Mills theory, regularized by higher covariant derivatives. The result of a similar calculation for the $N=1$ SQED in the three-loop approximation is presented in Sec. 4. In both cases the integrals, defining the β-functions, are integrals of total derivatives. A partial explanation of this feature is given in Sec. 5, where for $N=1$ SQED we prove that such a factorization takes place for a sum of all planar diagrams with a single matter loop. The results are briefly discussed in the Conclusion.

2 N=1 supersymmetric Yang–Mills theory and the higher covariant derivative regularization

N=1 supersymmetric Yang-Mills theory with matter in the massless case is described by the action

$$S=\frac{1}{2e^2}\mathrm{Re}\,\mathrm{tr}\int d^4x\,d^2\theta\,W_a C^{ab} W_b+\frac{1}{4}\int d^4x\,d^4\theta\,(\phi^*)^i(e^{2V})_i{}^j\phi_j+\Big(\frac{1}{6}\int d^4x\,d^2\theta\,\lambda^{ijk}\phi_i\phi_j\phi_k+\mathrm{h.c.}\Big), \qquad (1)$$

where ϕ_i are chiral scalar matter superfields, V is a real scalar gauge superfield, and the supersymmetric gauge field stress tensor is given by $W_a=\bar D^2(e^{-2V}D_a e^{2V})/8$. The action is invariant under the gauge transformations

$$\phi\to e^{i\Lambda}\phi;\qquad e^{2V}\to e^{i\Lambda^+}e^{2V}e^{-i\Lambda}, \qquad (2)$$

if

[1] In both these papers the total derivatives were written in a more beautiful form than in [19]. Certainly, results of [19] can be also reformulated in this form. The corresponding expressions can be found in [23].

$$(T^A)_m{}^i\lambda^{mjk} + (T^A)_m{}^j\lambda^{imk} + (T^A)_m{}^k\lambda^{ijm} = 0, \qquad (3)$$

where Λ is an arbitrary chiral superfield.

For calculation of quantum corrections it is convenient to use the background field method. In the supersymmetric case it can be formulated as follows [25, 26]: Let us make the substitution

$$e^{2V} \to e^{2V'} \equiv e^{\mathbf{\Omega}^+} e^{2V} e^{\mathbf{\Omega}}, \qquad (4)$$

in action (1), where $\mathbf{\Omega}$ is a background superfield. Then the theory is invariant under the background gauge transformations

$$\phi \to e^{i\Lambda}\phi; \quad V \to e^{iK} V e^{-iK}; \quad e^{\mathbf{\Omega}} \to e^{iK} e^{\mathbf{\Omega}} e^{-i\Lambda}; \quad e^{\mathbf{\Omega}^+} \to e^{i\Lambda^+} e^{\mathbf{\Omega}^+} e^{-iK}, \qquad (5)$$

where K is an arbitrary real superfield. This invariance allows to set $\mathbf{\Omega} = \mathbf{\Omega}^+ = \mathbf{V}$. It is convenient to choose a regularization and a gauge fixing term so that invariance (5) is unbroken. First, we fix a gauge by adding

$$S_{\text{gf}} = -\frac{1}{32e^2}\,\text{tr}\int d^4x\, d^4\theta \left(V\boldsymbol{D}^2\bar{\boldsymbol{D}}^2 V + V\bar{\boldsymbol{D}}^2\boldsymbol{D}^2 V\right) \qquad (6)$$

to the action. The corresponding Faddeev–Popov and Nielsen–Kallosh ghost Lagrangians are constructed by the standard way. For regularization we add the terms

$$S_\Lambda = \frac{1}{2e^2}\text{tr}\,\text{Re}\int d^4x\, d^4\theta\, V\frac{(\boldsymbol{D}_\mu^2)^{n+1}}{\Lambda^{2n}}V + \frac{1}{4}\int d^4x\, d^4\theta\, (\phi^*)^i \Big[e^{\mathbf{\Omega}^+}\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}e^{\mathbf{\Omega}}\Big]_i{}^j\phi_j, \qquad (7)$$

where $\boldsymbol{D}_\alpha$ is the background covariant derivative.[2] (Because the considered theory contains a nontrivial superpotential, it is also necessary to introduce the higher covariant derivative term for the matter superfields.)

The regularized theory is evidently invariant under the background gauge transformations. The regularization, described above, is rather simple, but breaks the BRST-invariance of the action. That is why it is necessary to use a special subtraction scheme, which restore the Slavnov–Taylor identities in each order of the perturbation theory [27]. For the supersymmetric case such a scheme was constructed in Ref. [28].

It is well-known [29] that the higher covariant derivative term does not remove divergences in the one-loop approximation. In order to regularize the remaining one-loop divergences, it is necessary to introduce Pauli-Villars determinants into the generating functional

$$Z[J,\mathbf{V}] = \int D\mu \prod_I \Big(\det PV(V,\mathbf{V},M_I)\Big)^{c_I}\exp\Big\{i(S + S_\Lambda + S_{\text{gf}} + S_{\text{gh}} + \text{Sources})\Big\}, \qquad (8)$$

where $\sum c_I = 1$ and $\sum\limits_I c_I M_I^2 = 0$. It is useful to present the Pauli–Villars determinants in the following form:

$$\prod_I \Big(\int D\phi_I^* D\phi_I e^{iS_I}\Big)^{-c_I}, \qquad (9)$$

where S_I is the action for the Pauli–Villars fields,[3]

[2] Other choices of higher derivative terms are also possible. For example, a different variant of the higher covariant derivative regularization was considered in Ref. [20].

[3] Note that this action differs from the one, used in [19], because here the quotient of the coefficients in the kinetic term and in the mass term does not contain the factor Z. Using terminology of Ref. [30], one can say that here we calculate the canonical coupling α_c, while in Ref. [19] we calculated the holomorphic coupling α_h. Certainly, after the renormalization the effective action does not depend on the definitions. However, the definitions used here are much more convenient.

$$S_I = \frac{1}{4}\int d^4x\, d^4\theta\, (\Phi_I^*)^i \Big[e^{\mathbf{\Omega}^+}\Big(1+\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}\Big)e^{\mathbf{\Omega}}\Big]_i{}^j (\Phi_I)_j + \Big(\frac{1}{4}\int d^4x\, d^2\theta\, M_I^{ij}(\Phi_I)_i(\Phi_I)_j + \text{h.c.}\Big).$$

(A regularized one-loop diagram with cubic matter vertex is finite. Therefore, it is not necessary to include cubic terms into the Pauli–Villars Lagrangian.) The mass term should satisfy

$$(T^A)_k{}^i M^{kj} + (T^A)_k{}^j M^{ki} = 0. \tag{10}$$

Also we assume that $M^{ij}M^*_{jk} = M^2\delta^i_k$ and $M^{ij} = a^{ij}\Lambda$, where a^{ij} are constants. (Thus, there is the only dimensionful parameter Λ.)

The generating functional for connected Green functions and the effective action are defined by the standard way. Terms in the effective action, corresponding to the renormalized two-point Green function of the gauge superfield, have the form

$$\Gamma^{(2)}_V = -\frac{1}{8\pi}\text{tr}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \mathbf{V}(-p)\,\partial^2\Pi_{1/2}\mathbf{V}(p)\, d^{-1}(\alpha,\lambda,\mu/p). \tag{11}$$

where α is a renormalized coupling constant, and

$$\partial^2\Pi_{1/2} = -\frac{1}{16}D^a\bar{D}^2D_a \tag{12}$$

is a supersymmetric transversal projector. We calculate

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\lambda_0,\Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = -\frac{d\alpha_0^{-1}}{d\ln\Lambda} = \frac{\beta(\alpha_0)}{\alpha_0^2}. \tag{13}$$

The anomalous dimension is defined similarly. First we consider the two-point Green function for the matter superfield in the massless limit:

$$\Gamma^{(2)}_\phi = \frac{1}{4}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, (\phi^*)^i(-p,\theta)\,\phi_j(p,\theta)\,(ZG)_i{}^j(\alpha,\lambda,\mu/p), \tag{14}$$

where Z denotes the renormalization constant for the matter superfield. Then anomalous dimensions is defined by

$$\gamma_i{}^j\Big(\alpha_0(\alpha,\lambda,\Lambda/\mu)\Big) = -\frac{\partial}{\partial\ln\Lambda}\Big(\ln Z(\alpha,\lambda,\Lambda/\mu)\Big)_i{}^j. \tag{15}$$

3 Two-loop β-function for $N = 1$ supersymmetric Yang-Mills theory

The calculation of two-loop diagrams gives the following result for the β-function

$$\beta(\alpha) = -\frac{3\alpha^2}{2\pi}C_2 + \alpha^2 T(R)I_0 + \alpha^3 C_2^2 I_1 + \frac{\alpha^3}{r}C(R)_i{}^jC(R)_j{}^i I_2 + \alpha^3 T(R)C_2 I_3 + \alpha^2 C(R)_i{}^j\frac{\lambda^*_{jkl}\lambda^{ikl}}{4\pi r}I_4 + \ldots, \tag{16}$$

where the following notation is used:

$$\text{tr}\,(T^AT^B) \equiv T(R)\,\delta^{AB};\quad (T^A)_i{}^k(T^A)_k{}^j \equiv C(R)_i{}^j;\quad f^{ACD}f^{BCD} \equiv C_2\delta^{AB};\quad r \equiv \delta_{AA}. \tag{17}$$

Here we do not write the integral for the one-loop ghost contribution, and the integrals I_0–I_4 are presented below. With Pauli–Villars contributions

$$I_i = I_i(0) - \sum_I I_i(M_I), \qquad i = 0, 2, 3 \tag{18}$$

where

$$I_0(M) = 8\pi \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{q^2} \frac{d}{dq^2} \Big\{ \frac{1}{2} \ln\big(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2\big)$$
$$+ \frac{M^2}{2(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)} - \frac{mq^{2m}/\Lambda^{2m} q^2 (1+q^{2m}/\Lambda^{2m})}{q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2} \Big\}; \tag{19}$$

$$I_1 = 96\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{k^2} \frac{d}{dk^2} \Big\{ \frac{1}{q^2(1+q^{2n}/\Lambda^{2n})(q+k)^2(1+(q+k)^{2n}/\Lambda^{2n})} \times$$
$$\times \Big(\frac{n+1}{(1+k^{2n}/\Lambda^{2n})} - \frac{n}{(1+k^{2n}/\Lambda^{2n})^2} \Big) \Big\}; \tag{20}$$

$$I_2(M) = -64\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{q^2} \frac{d}{dq^2} \Big\{ \frac{q^2(1+(q+k)^{2m}/\Lambda^{2m})}{((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2 + M^2)} \times$$
$$\times \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})} \Big[\frac{q^2(1+q^{2m}/\Lambda^{2m})^3}{(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)^2} + \frac{mq^{2m}/\Lambda^{2m}}{q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2}$$
$$- \frac{2mq^{2m}/\Lambda^{2m} M^2}{(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)^2} \Big] \Big\}; \tag{21}$$

$$I_3(M) = 16\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \Big\{ \frac{\partial}{\partial q_\alpha} \Big[\frac{k_\alpha(1+q^{2m}/\Lambda^{2m})}{(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)} \times$$
$$\times \frac{1}{(k+q)^2(1+(q+k)^{2n}/\Lambda^{2n})} \Big(- \frac{(1+k^{2m}/\Lambda^{2m})^3}{(k^2(1+k^{2m}/\Lambda^{2m})^2 + M^2)^2} - \frac{mk^{2m-2}/\Lambda^{2m}}{k^2(1+k^{2m}/\Lambda^{2m})^2 + M^2}$$
$$+ \frac{2mk^{2m-2}/\Lambda^{2m} M^2}{(k^2(1+k^{2m}/\Lambda^{2m})^2 + M^2)^2} \Big) \Big] -$$
$$- \frac{1}{k^2} \frac{d}{dk^2} \Big[\frac{2(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}{(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)\,((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2 + M^2)}$$
$$\times \Big(\frac{1}{(1+k^{2n}/\Lambda^{2n})} + \frac{nk^{2n}/\Lambda^{2n}}{(1+k^{2n}/\Lambda^{2n})^2} \Big) \Big] \Big\}; \tag{22}$$

$$I_4 = 64\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{q^2} \frac{d}{dq^2} \Big[\frac{1}{k^2(1+k^{2m}/\Lambda^{2m})(q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})} \times$$
$$\times \Big(\frac{1}{(1+q^{2m}/\Lambda^{2m})} + \frac{mq^{2m}/\Lambda^{2m}}{(1+q^{2m}/\Lambda^{2m})^2} \Big) \Big]. \tag{23}$$

It is easy to see that all these integrals are integrals of total derivatives, due to the identity

$$\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} \frac{d}{dq^2} f(q^2) = \frac{1}{16\pi^2} \Big(f(q^2=\infty) - f(q^2=0) \Big). \tag{24}$$

(This expression is reduced to a total derivative in the four-dimensional spherical coordinates.) Calculating the integrals we obtain the result for a two-loop β-function:

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\Big(3C_2 - T(R)\Big) + \frac{\alpha^3}{(2\pi)^2}\Big(-3C_2^2 + T(R)C_2 + \frac{2}{r} C(R)_i{}^j C(R)_j{}^i\Big) - \frac{\alpha^2 C(R)_i{}^j \lambda^*_{jkl}\lambda^{ikl}}{8\pi^3 r} + \ldots. \tag{25}$$

Comparing this with the one-loop anomalous dimension

$$\gamma_i{}^j(\alpha) = -\frac{\alpha C(R)_i{}^j}{\pi} + \frac{\lambda^*_{ikl}\lambda^{jkl}}{4\pi^2} + \ldots, \qquad (26)$$

we find the agreement with the exact NSVZ β-function [31]

$$\beta(\alpha) = -\frac{\alpha^2\Big[3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha)/r\Big)\Big]}{2\pi(1 - C_2\alpha/2\pi)}. \qquad (27)$$

in the considered approximation. Result (25) also agrees with the DRED calculations up to notation.[4]

4 Three-loop calculation for SQED

From the results, describe above, it is possible to suggest that a β-function in supersymmetric theories is given by integrals of total derivatives. In order to confirm this conjecture we also present the result for the three-loop β-function in $N = 1$ SQED, regularized by higher derivatives. This theory is described by the action

$$S = \frac{1}{4e^2}\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab} f(\partial^2/\Lambda^2) W_b + \frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi\Big), \qquad (28)$$

where $f(\partial^2/\Lambda^2)$ is a regulator, for example $f = 1 + \partial^{2n}/\Lambda^{2n}$. In this case

$$\Gamma^{(2)} = \int \frac{d^4p}{(2\pi)^4}\, d^4\theta\,\Big(-\frac{1}{16\pi} V(-p)\,\partial^2\Pi_{1/2}V(p)\, d^{-1}(\alpha,\mu/p) + \\ + \frac{1}{4}(\phi^*)^i(-p,\theta)\,\phi_j(p,\theta)\,(ZG)_i{}^j(\alpha,\mu/p)\Big). \qquad (29)$$

The calculation of a three-loop β-function was made in Ref. [19]. The result of this paper in a more beautiful form was written in Ref. [23]. Here we will not write explicit expressions for the renormgroup functions, which in this case are also given by integrals of total derivatives. The main result has the following form:

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = -\frac{d}{d\ln\Lambda}\alpha_0^{-1}(\alpha,\mu/\Lambda) = \\ = \frac{1}{\pi}\Big(1 - \frac{d}{d\ln\Lambda}\ln G(\alpha_0,\Lambda/q)\Big|_{q=0}\Big) = \frac{1}{\pi} + \frac{1}{\pi}\frac{d}{d\ln\Lambda}\Big(\ln ZG(\alpha,\mu/q) - \ln Z(\alpha,\Lambda/\mu)\Big)\Big|_{q=0} = \\ = \frac{1}{\pi}\Big(1 - \gamma\Big(\alpha_0(\alpha,\Lambda/\mu)\Big)\Big). \qquad (30)$$

It should be noted that the left hand side and the right hand side here are well defined integrals. In the three-loop approximation the integrals, obtained with the higher covariant derivative regularization, can not be calculated analytically. However, it is possible to find relation (30) between them. It is easy to see that the origin of this relation is the factorization of integrands into total derivatives.

[4]For example, in order to obtain notation of Ref. [2] it is necessary to make the following substitutions: $\lambda_{ijk} \to 2Y_{ijk}$, $\alpha \to g^2/4\pi$, $\gamma(\alpha) \to 2\gamma(g)$, $\beta(\alpha) \to g\beta(g)/2\pi$.

5 Factorization into total derivatives in $N=1$ SUSY QED for some classes of diagrams

Now let us try to partially explain the factorization of integrands into total derivatives. For this purpose we consider $N=1$ SQED, described by action (28), and calculate the two-point Green function of the gauge superfield. Due to the Ward identity this function is transversal and, therefore, we will obtain has the following structure:

$$\int d^4\theta V\partial^2\Pi_{1/2}V \times \frac{d}{d\ln\Lambda}(\text{Momentum integral})\Big|_{p=0}. \qquad (31)$$

Therefore, in order to find a β-function it is possible to make the substitution

$$V \to \bar\theta^a\bar\theta_a\theta^b\theta_b \equiv \theta^4, \qquad (32)$$

so that

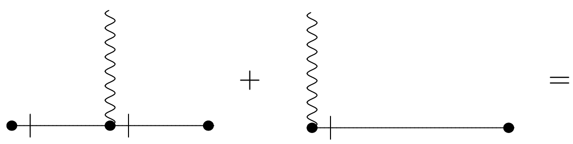
$$\int d^4\theta V\partial^2\Pi_{1/2}V \to -4. \qquad (33)$$

This is possible, because we make calculations in the limit $p\to 0$, where p is an external momentum. (In this limit the gauge superfield V does not depend on the coordinates x^μ.)

We will try to reduce a sum of Feynman diagrams for the considered theory to integrals of total derivatives. In the coordinate representation such an integral can be written as

$$\mathrm{Tr}\Big([x^\mu, \text{Something}]\Big) = 0. \qquad (34)$$

In order to extract such commutators first we consider diagrams, containing the vertex to which only one external line (and no internal lines) is attached. We can sum such a diagram with the diagram, in which the external line is shifted to the nearest vertex [32]. This corresponds to a sum of the subdiagrams, presented below. Making the substitution $V\to\theta^4$ we obtain



$$= -\theta^a\theta_a\bar\theta^b\frac{\bar D_b D^2}{4\partial^2} + \theta^a\theta_a\frac{D^2}{4\partial^2} + i\bar\theta^b(\gamma^\mu)_b{}^a\theta_a\frac{\bar D^2 D^2\partial_\mu}{\partial^4} - i\theta^a(\gamma^\mu)_a{}^b\frac{\bar D_b D^2\partial_\mu}{4\partial^4} + \frac{\bar D^2 D^2}{16\partial^4} \qquad (35)$$

Only the first and the third terms give nontrivial contributions to the two-point function of the gauge superfield, because they contain $\bar\theta$. Really, finally it is necessary to obtain

$$\int d^4\theta\, \theta^a\theta_a\bar\theta^b\bar\theta_b,$$

while calculating the θ-part of the graph can not increase powers of θ or $\bar\theta$. Therefore we should have $\bar\theta^a\bar\theta_a$ from the beginning.

Now let us proceed to calculation of Feynman diagrams. In the one-loop approximations all calculations have been already done, so that we can ignore this case. (In the one-loop approximation contributions of the Pauli–Villars field are very essential, but here we do not consider them. For investigation of the Pauli–Villars contribution it is necessary to consider the massive case. In principle, this is made similarly, but all expressions become more complicated.)

First we consider a case, in which external V-lines are attached to different loops of matter superfields. Let us denote a chain of propagators, connecting vertexes with quantum gauge field, by $*$.[5] Then each loop will be proportional to

$$\mathrm{Tr}\Big(- i\bar\theta^c(\gamma^\nu)_c{}^d\theta_d\frac{\bar D^2 D^2\partial_\nu}{8\partial^4} * -\theta^c\theta_c\bar\theta^d\frac{\bar D_d D^2}{4\partial^2} *\Big) = \mathrm{Tr}\Big(- \theta^c\theta_c\bar\theta^d * \frac{\bar D_d D^2}{4\partial^2} *$$
$$-\bar\theta^d\theta^c * \frac{\bar D^2 D_c}{4\partial^2} * \frac{\bar D_d D^2}{4\partial^2} * -i\bar\theta^c(\gamma^\nu)_c{}^d\theta_d * \frac{\bar D^2 D^2\partial_\nu}{8\partial^4} * +\theta^2, \bar\theta^1, \theta^1, \theta^0 \text{ terms}\Big). \qquad (36)$$

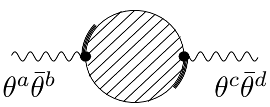
After some simple algebra the first three terms can be written as

$$\mathrm{Tr}\Big(- 2\theta^c\theta_c\bar\theta^d[\bar\theta_d, *] - i\bar\theta^c(\gamma^\nu)_c{}^d\theta_d[y^*_\nu, *] + \ldots\Big) = 0 + \ldots$$

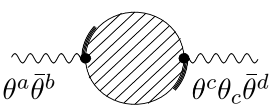
where $y^*_\nu = x_\nu - i\bar\theta^a(\gamma_\nu)_a{}^b\theta_b/2 + i\theta^a(\gamma_\nu)_a{}^b\bar\theta_b/2$, and dots denote θ^2, $\bar\theta^1$, θ^1, θ^0 terms. Calculating the two-point function we should multiply two such expressions. Then the terms, denoted by dots, vanish due to the integration over $d^4\theta$. Therefore, the sum of all such diagrams is given by an integral of a total derivative and is equal to 0.

Next, it is necessary to consider a case, in which external **V**-lines are attached to a single loop of the matter superfields.

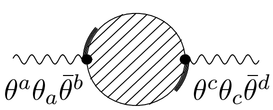
We shift θ-s to an arbitrary point of the loop, commuting them with matter propagators. This gives



$$\sim -\frac{1}{128}\mathrm{Tr}\Big(\theta^4\frac{\bar D^2 D^2\partial^\mu}{\partial^4} * \frac{\bar D^2 D^2\partial_\mu}{\partial^4} *\Big)$$

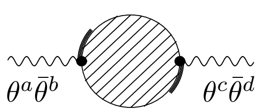


$$\sim i(\gamma^\mu)_d{}^c\mathrm{Tr}\Big(\theta^4\Big(\frac{\bar D^2 D_c\partial_\mu}{16\partial^4} * \frac{\bar D^d D^2}{\partial^2} * + \frac{\bar D^2 D^2\partial_\mu}{16\partial^4} * \frac{\bar D^2 D_c}{16\partial^2} * \frac{\bar D^d D^2}{\partial^2} *\Big)\Big)$$

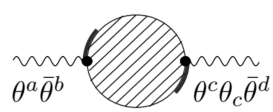


$$\sim \mathrm{Tr}\Big(\theta^4\Big(- \frac{\bar D_d D^2}{4\partial^2} * \frac{\bar D^2}{8\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * - \frac{\bar D_d}{2\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * + \frac{\bar D_d D^c}{2\partial^2} *$$
$$\times\frac{\bar D^2 D_c}{8\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * + \frac{\bar D_d D^2}{4\partial^2} * \frac{\bar D^2 D^c}{8\partial^2} * \frac{\bar D^2 D_c}{8\partial^2} * \frac{\bar D^d D^2}{4\partial^2} *\Big)\Big)$$

In order to write the sum of these diagrams as an integral of a total derivative, we will start with the calculation of the following sum of diagrams:



$$+\ \frac{1}{2}$$



Using the identity

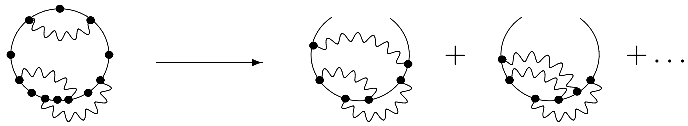
$$[x^\mu, \frac{\partial_\mu}{\partial^4}] = [-i\frac{\partial}{\partial p_\mu}, -\frac{ip_\mu}{p^4}] = -2\pi^2\delta^4(p) \qquad (37)$$

after some algebra we obtain

$$\sim \mathrm{Tr}\Big(\theta^4\Big(- \frac{\pi^2}{8} * \bar D^2 D^2\delta^4(\partial_\alpha) - \Big[y^*_\mu, \frac{\bar D^2 D^2\partial^\mu}{16\partial^4} *\Big]\Big)\Big) = -\mathrm{Tr}\Big(\frac{\pi^2}{8}\theta^4 * \bar D^2 D^2\delta^4(\partial_\alpha)\Big). \qquad (38)$$

[5]It is possible to give a rigorous definition of $*$. This will be done in [33]. Below for simplicity we omit some details, needed in the rigorous approach.

δ-function allows to perform integration over the momentum of the considered matter loop. This corresponds to cutting the diagram, that gives diagrams for the two-point Green function of the matter superfield [21]. For example,



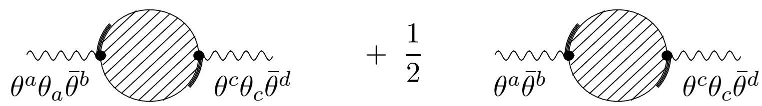
For diagrams, which can not be made disconnected by two cuts of the matter loop, this gives the corresponding contribution to the anomalous dimension.

In a method, based on substituting solution of the Ward identity into the Schwinger–Dyson equation, a sum of the considered diagrams is included into the effective diagram, which can be expressed in terms of the two-point function of the matter superfield. This effective diagram gives

$$\beta(\alpha) \leftarrow -\frac{\alpha^2}{\pi}\gamma(\alpha). \tag{39}$$

(A part of this contribution, corresponding to higher terms in the expansion of $\ln Z$, comes from diagrams, considered below.)

Now let us calculate the remaining diagrams (which can not be expressed in terms of the two-point function by substituting solution of the Ward identity into the Schwinger–Dyson equation, if a diagram can not be made disconnected by two cuts of the matter line)

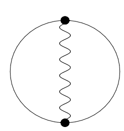


The considered sum is given by

$$\mbox{Tr}\theta^4\Big(-\frac{i}{32}(\gamma^\mu)_d{}^c\Big(\frac{\bar D^2 D^2\partial_\mu}{\partial^4} * \frac{\bar D^2 D_c}{16\partial^2} * \frac{\bar D^d D^2}{\partial^2} * + \frac{\bar D^2 D_c\partial_\mu}{\partial^4} * \frac{\bar D^d D^2}{\partial^2} * \Big) - \frac{\bar D_d}{2\partial^2} * \frac{\bar D^d D^2}{4\partial^2} *$$
$$- \frac{\bar D_d D^2}{4\partial^2} * \frac{\bar D^2}{4\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * + \frac{\bar D_d D^c}{2\partial^2} * \frac{\bar D^2 D_c}{8\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * + \frac{\bar D_d D^2}{4\partial^2} * \frac{\bar D^2 D^c}{8\partial^2} * \frac{\bar D^2 D_c}{8\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * \Big). \tag{40}$$

This expression was not yet completely factorized into a total derivative. However, for planar diagrams with a single loop of the matter superfield the factorization can be proven in all orders:

Let us assume that a diagram contain a line, which cut it:



Then there are the following possibilities:

1. There is

$$*\frac{\bar D^a D^2}{8\partial^2}* = [*,\bar\theta^a] = 0 \quad \mbox{or} \quad * \frac{D^2 D^a}{8\partial^2}* = [*,\theta^a] = 0 \tag{41}$$

on the left (right) side. The equality to 0 arises, because θ can be carried along the wavy line due to the identity $\theta_1\delta^4(\theta_1-\theta_2) = \theta_2\delta^4(\theta_1-\theta_2)$.

2. There is $*\frac{\bar D^a D^2}{16\partial^2} * \frac{\bar D_a D^2}{16\partial^2}*$ on the left (right) side. Then on the other side we obtain

$$\sim \{\theta^b[\theta_b, *]\} = 0. \tag{42}$$

3. There is $*\dfrac{\bar{D}^a D^2}{16\partial^2} * \dfrac{\bar{D}^2 D_b}{16\partial^2}*$ on the left (right) side. Then on the other side we obtain

$$\sim (\gamma^\mu)_a{}^b [y^*_\mu, *]\theta^4.$$

Carrying y^*_μ along the wavy line evidently gives a derivative of the photon propagator with respect to the loop momentum.

4. All modified propagators are on the same side of the diagram.

The considered (planar, with a single matter loop) diagrams can be cut into parts with no more than two matter propagators. Therefore, an expression for a diagram is split into the parts, proportional to

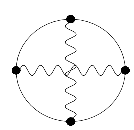
$$\mathrm{Tr}\Big(-\frac{i}{32}(\gamma^\mu)_d{}^c \frac{\bar{D}^2 D_c \partial_\mu}{\partial^4}\cdot\frac{\bar{D}^d D^2}{\partial^2} - \frac{\bar{D}_d}{2\partial^2}\cdot\frac{\bar{D}^d D^2}{4\partial^2}\Big)\theta^4 \sim (\gamma^\mu)_c{}^d \mathrm{Tr}\Big(\Big[y^*_\mu, \frac{\bar{D}^2 D_d}{\partial^2}\cdot\frac{\bar{D}^c D^2}{\partial^2}\Big]\theta^4\Big)$$
$$+\text{terms, proportional to the } \delta\text{-function.} \tag{43}$$

In this expression we also take into account possible terms, coming from item 3. These terms allows to commute y^*_μ with the wavy lines. (Terms with δ-functions arise from the product

$$\frac{\bar{D}^2 D_d}{\partial^2}\cdot\frac{\bar{D}^c D^2}{\partial^2} \tag{44}$$

if at least two momenta in the matter loop coincide, that occurs for diagrams, which can be made disconnected by two cuts of the matter loop. These contributions are certainly taken into account in Eq. (39).)

Thus, planar diagrams with a single loop of matter superfields are given by integrals of total derivatives and are equal to 0, if there are no coinciding momenta in the matter loop. Using this method it is also possible to prove factorization for some non-planar diagrams, for example, for



In particular this explains the result of the three-loop calculation in $N=1$ SQED.

6 Conclusion

In this paper we investigate the conjecture that in supersymmetric theories, regularized by higher covariant derivatives, all integrals, defining the β-fucntion, are integrals of total derivatives. This fact allows to calculate at least one of the integrals analytically, so that the factorization of integrands into total derivatives is in the close connection with the NSVZ exact β-function. We have verified this explicitly for the most general $N=1$ supersymmetric Yang–Mills theory in the two-loop approximation and for the $N=1$ SQED in the three-loop approximation.

For $N=1$ SQED, regularized by higher derivatives, it is also possible to prove factorization of integrals into total derivative for some classes of diagrams, for example, for planar diagrams with a single matter loop. (Here we present the proof formally, without PV contributions.) The sketch of a prove for a general case, generalizing the arguments, presented here, is given in [34]. The complete rigorous proof (for $N = 1$ SQED), which, in particular, includes calculation of Pauli–Villars fields contributions) will be described in a future paper [33].

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