The study of ambiguity in nonabelian gauge theories

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Abstract

The problem of ambiguity in quantization of nonabelian gauge theories is reviewed. A method of unambigous quantization of nonabelian gauge theories is proposed.

1 Introduction

I this talk I am going to discuss the problem of ambiguity in quantization of gauge theories. The standard formulation of the Yang-Mills theory does not allow a unique gauge fixing. It was shown by V.N.Gribov [1] that the Coulomb gauge condition $\partial_i A_i = 0$ does not choose a unique representative in the class of gauge equivalent configurations, as the condition

$$\partial_i A_i^{\Omega} = 0 \tag{1}$$

considered as the equation for the elements of the gauge group Ω at the surface $\partial_i A^i = 0$ for sufficiently large A has nontrivial solutions fast decreasing at the spatial infinity. This result was generalised by I.Singer [2] to arbitrary covariant gauge conditions.

In the framework of perturbation theory, that is for sufficiently small A the equaton (1) has only trivial solutions. Hence the Gribov umbiguity in this case is absent. However beyond the perturbation theory this ambiguity exists, which makes problematic the standard way of canonical quantization of nonabelian gauge theories.

One may try to avoid the ambiguity by using so called algebraic gauge conditions, for example choosing the Hamiltonian gauge $A_0 = 0$. However such gauge violates explicitly the Lorentz invariance complicating the analysis enormously. It introduces also some other problems which will not be discussed here.

According to the common lore in the quantization of nonabelian gauge theories one faces the dilemma: differential gauge conditions like the Lorentz gauge $\partial_\mu A_\mu = 0$ are plagued by the Gribov ambiguity, and algebraic gauge conditions result in the absence of a manifest Lorentz invariance and serious problems in renormalizabilty. A possible way out is given by a new formulation of nonabelian gauge theories introducing a bigger number of auxilliary fields. Example of such reformulation is given by the Higgs model, described by the Lagrangian

$$L = L_{YM} + (D_\mu \varphi)^*(D_\mu \varphi) - \lambda^2(\varphi^*\varphi - \mu^2)^2 \tag{2}$$

where L_{YM} is the usual Yang-Mills Lagrangian. After the shift $\varphi = \varphi' + \hat{\mu}$, $\hat{\mu} = \{0,\ \mu\}$ φ'_a, $a = 1,2,3$ becomes a gauge field:$\varphi'_a \to \varphi'_a + \mu\eta^a(x) + \ldots$. Unitary gauge $\varphi'_a = 0$ is algebraic, but Lorentz invariant. At the same time the Higgs scalar is described not by one field, but by four component $SU(2)$-spinor, three unphysical components of which may be gauged away.The unitary gauge is however nonrenormalizable.

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A similar idea may be used for an ambiguity free renormalizable formulation of the Yang-Mills theory. Recently I proposed an explicitly Lorentz invariant formulation of the quantum Yang-Mills theory in which the effective Lagrangian of ghost fields is gauge invariant [3]. In the present talk I will show that in this approach the Yang-Mills theory allows a quantization procedure which is free of the Gribov ambiguity and hence may serve as a starting point for nonperturbative constructions [4]. In perturbation theory the model is renormalizable, although renormalization includes a nonmultiplicative redefinition of the fields [5].

2 Unambiguos quantization of the Yang-Mills field.

We consider the model described by the classical Lagrangian

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\varphi)^*(D_\mu\varphi) - (D_\mu\chi)^*(D_\mu\chi) + i[(D_\mu b)^*(D_\mu e) - (D_\mu e)^*(D_\mu b)] \quad (3)$$

To save the place we shall consider the model with the $SU(2)$ gauge group. Generalization to other groups makes no problem. Here $F^a_{\mu\nu}$ is the standard curvature tensor for the Yang-Mills field. The scalar fields φ, χ, b, e form the complex $SU(2)$ doublets parametrized by the Hermitean components as follows:

$$c = \left(\frac{ic_1 + c_2}{\sqrt{2}}, \frac{c_0 - ic_3}{\sqrt{2}}\right) \quad (4)$$

where c denotes any of doublets. The fields φ and χ are commuting, and the fields e and b are anticommuting. In the eq.(3) D_μ denotes the usual covariant derivative, hence the Lagrangian (3) is gauge invariant. Note that due to the negative sign of the χ field Lagrangian, this field possesses negative energy.

Let us make the following shifts in the Lagrangian (3):

$$\varphi \to \varphi + g^{-1}\hat{m}; \quad \chi \to \chi - g^{-1}\hat{m}; \quad \hat{m} = (0, m) \quad (5)$$

where m is a constant parameter. Due to the negative sign of the Lagrangian of the field χ the terms quadratic in m arising due to the shifts of the fields φ and χ mutualy compensate and the Lagrangian acquires a form

$$\begin{aligned} L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} &+ (D_\mu\varphi)^*(D_\mu\varphi) - (D_\mu\chi)^*(D_\mu\chi) \\ +g^{-1}[(D_\mu\varphi)^* + (D_\mu\chi)^*](D_\mu\hat{m}) &+ g^{-1}(D_\mu\hat{m})^*[D_\mu\varphi + D_\mu\chi] \\ &+ i[(D_\mu b)^*(D_\mu e) - (D_\mu e)^*(D_\mu b)] \end{aligned} \quad (6)$$

As before this Lagrangian describes massless vector particles.

The Lagrangian (6) is obviously invariant with respect to the "'shifted"' gauge transformations. In particular the transformation of the field $\varphi^a_- = \frac{\varphi-\chi}{\sqrt{2}}$ is

$$\delta\varphi^a_- = m\eta^a + \frac{g}{2}\varepsilon^{abc}\varphi^b_-\eta^c + \frac{g}{2}\varphi^0_-\eta^a \quad (7)$$

The Lagrangian (6) except for gauge invariance possesses also the invariance with respect to the supersymmetry transformations

$$\begin{aligned} \delta\varphi(x) &= i\epsilon b(x) \\ \delta\chi(x) &= -i\epsilon b(x) \\ \delta e(x) &= \epsilon[\varphi(x) + \chi(x)] \\ \delta b(x) &= 0 \end{aligned} \quad (8)$$

where ϵ is a constant anticommuting parameter.

In the future we shall see that invariance with respect to the supersymmetry transformations provides unitarity of the theory in the space which includes only physical exitations of the fields. An explicit form of interaction is not essential. Only the symmetry properties are important. In principle any counterterms which preserve gauge invariance and supersymmetry are allowed.

The field φ_-^a is shifted under the gauge transformation by an arbitrary function $m\eta^a$. It allows to impose Lorentz invariant algebraic gauge condition $\varphi_-^a = 0$.

However imposing the Lorentz invariant gauge condition $\varphi_-^a = 0$ does not solve the problem of ambiguity completely. The field φ_-^a satisfying the condition $\varphi_-^a = 0$ is transformed by the gauge transformation to $\varphi_-'^a = (m + \frac{g}{2}\varphi_-^0)\eta^a$. For some x the factor $(m + \frac{g}{2}\varphi_-^0(x))$ may vanish, leading to nonuniqueness of the gauge fixing. To avoid the problem of ambiguity completely we redefine the fields as follows

$$\begin{aligned} \varphi_-^0 = \frac{2m}{g}(\exp\{\frac{gh}{2m}\} - 1); \quad \varphi_-^a = \tilde{M}\tilde{\varphi}_-^a \\ \varphi_+^a = \tilde{M}^{-1}\tilde{\varphi}_+^a; \quad \varphi_+^0 = \tilde{M}^{-1}\tilde{\varphi}_+^0 \\ e = \tilde{M}^{-1}\tilde{e}; \quad b = \tilde{M}\tilde{b} \end{aligned} \tag{9}$$

where

$$\tilde{M} = 1 + \frac{g}{2m}\varphi_-^0 = \exp\{\frac{gh}{2m}\} \tag{10}$$

The new Lagrangian has the form

$$\begin{aligned} \tilde{L} = -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + \partial_\mu h \partial_\mu \tilde{\varphi}_+^0 - \frac{g}{2m}\partial_\mu h \partial_\mu h \tilde{\varphi}_+^0 \\ + m\tilde{\varphi}_+^a \partial_\mu A_\mu^a - [((D_\mu \tilde{b})^* + \frac{g}{2m}\tilde{b}^* \partial_\mu h)(D_\mu \tilde{e} - \frac{g}{2m}\tilde{e}\partial_\mu h) + h.c.] \\ + \frac{mg}{2}A_\mu^2 \tilde{\varphi}_+^0 + g\partial_\mu h A_\mu^a \tilde{\varphi}_+^a \ldots \end{aligned} \tag{11}$$

Here $\ldots$ denote the terms $\sim \tilde{\varphi}_-^a$. By construction this Lagrangian is invariant with respect to the gauge transformations written in terms of the new variables. In particular $\delta\tilde{\varphi}_-^a = \eta^a$, and the ambiguity is absent.

Obviously the lagrangian is also invariant with respect to the supersymmetry transformations written in terms of the transformed variables. However imposing the gauge condition $\tilde{\varphi}_-^a = 0$ we break the invariance of the effective action with respect to the supersymmetry transformation (8). The transition from one gauge to the other one may be achieved by a gauge transformation, and in the gauge $\partial_i A_i = 0$ the effective action is invariant with respect to the supertransformation (8). Therefore in the gauge $\tilde{\varphi}_-^a = 0$ it also must be invariant with respect to some supertransformation. The corresponding gauge function is a solution of the equation.

$$\int d^4x \lambda^a(x) \partial_i (A^\Omega)_i^a(x) = \int d^4x \lambda^a(x) \tilde{\varphi}_-^a(x) \tag{12}$$

The solution of this equation may be found explicitly.

The effective Lagrangian (11) is invariant with respect to the supertransformations mentioned above and global $SU(2)$ transformations, which do not change the fields $\tilde{\varphi}_\pm^0$ and $\tilde{\varphi}_-^a$.

The spectrum of the theory looks as follows: Ghost exitations:

$\varphi_\pm, b, e$, longitudinal and temporal components of A_μ^a.

Physical excitations: three dimensionally transversal components of the Yang-Mills field.

The supersymmetry of the effective action generates a conserved nilpotent charge Q. Physical states are separated by the condition:

$$Q|\psi>_{ph} = 0 \tag{13}$$

The states separated by this condition describe only three dimensionally transversal components of the Yang-Mills field. The ghost excitations decouple.

3 Renormalization

The field h enters interaction only with derivative $\partial_\mu h$. Hence the divergency index of a diagram with n external $h(\varphi^0_-)$ lines decreases by n.

The index of divergency of an arbitrary diagram is

$$n = 4 - 2L_{\varphi^0_+} - 2L_{\varphi^0_+} - L_A - L_e - L_b - L_h \tag{14}$$

The theory is manifestly renormalizable.

In terms of the old (nontransformed) variables the theory is not manifestly renormalizable. Transition to the new variables simultaneously eliminates the residual ambiguity and makes the theory manifestly renormalizable.

Renormalization preserves all the symmetries of the theory. The possible counterterms may be classified on the basis of ST-Identities, associated with the symmetry, which combines the gauge invariance of the effective action and supersymmetry.

$$S(\Gamma) = \int d^4x \sum_\Phi \{\frac{\delta\Gamma}{\delta\Phi^*(x)}\frac{\delta\Gamma}{\delta\Phi(x)}\} = 0 \tag{15}$$

Φ are the fields:$A_\mu, \varphi^\alpha_+, e^\alpha, b^\alpha$;

Φ^* are the antifields introducing the variations of the fields Φ, e.g.$\int dx\{-\frac{2i}{m}A^{*a}_\mu(D_\mu b)^a\}$

The most general solution of S-T identities compatible with the power counting and the residual $SU(2)$ symmetry is:In the classical action one should renormalize the parameters $g' = Z_g g; \quad m' = Z_m m$, and redefine the fields

$$\tilde{e}' = Z_1\tilde{e}; \quad \tilde{b}' = Z_m\tilde{b}; \quad A^{a'}_\mu = Z_2A^a_\mu; \quad h' = Z_mZ_3h$$
$$\tilde{\varphi}^{a'}_+ = Z_4\tilde{\varphi}^a_+ + Z_5\partial_\mu A^a_\mu + \frac{Z_6}{m}\partial_\mu hA^a_\mu + Z_7(\tilde{e}^0\tilde{b}^a - \tilde{e}^a\tilde{b}^0 - \varepsilon^{abc}\tilde{e}^b\tilde{b}^c)$$
$$\tilde{\varphi}^{0'}_+ = Z_8\tilde{\varphi}^0_+ + Z_9\frac{\partial^2}{m}h + Z_{10}\frac{1}{m^2}\partial_\mu h\partial_\mu h + Z_{11}A^2 + Z_{12}(\tilde{e}^0\tilde{b}^0 + \tilde{e}^a\tilde{b}^a) \tag{16}$$

To satisfy the ST-identity (15)one has to redefine also the antifields. Corresponding equations also can be solved.

The renormalized action differs of the unrenormalized one only by the renormalization of the parameters which enter the unrenormalized effective action and redefinition (16) of the fields.

$$\Gamma_R(g', m'\Phi') = \Gamma_{cl}(Z_g g, Z_m m, \Phi(\Phi')) \tag{17}$$

All the ultraviolet divergencies can be recursively removed by a suitable choice of the parameters Z_g, Z_m, Z_j.

4 Gauge independence of observables

All the redefinitions of the fields were local and did not include the field $\tilde{\varphi}^a_-$. Therefore assuming the invariant
regularization like dimensional one and making the inverse redefinitions we arrive to the path integral representation for the scattering matrix

$$S = \int \exp\{i\int[L_{gi} + \lambda^a\varphi^a_-\}\det(M_{ab})d\mu \tag{18}$$

where L_{gi} is the gauge invariant classical Lagrangian with renormalized parameters.

$$(\det(M_{ab}))^{-1}_{\varphi^a_-=0} = \int \delta((\varphi^\Omega)^a_-)d\Omega \tag{19}$$

Multiplying the integral (18) by "1", $1 = \Delta_L \int \delta(\partial_\mu A_\mu^\Omega) d\Omega$ and changing the variables $\Phi^\Omega = \Phi'$,we arrive to the expression for the scattering matrix in the Lorentz gauge

$$S = \int \exp\{i \int [L_{gi}(x) + \lambda^a(x) \partial_\mu A_\mu(x) + \partial_\mu \bar{c}^a [D_\mu c]^a dx\} d\mu \tag{20}$$

This expression can be easily transformed to the standard expression for the Yang-Mills scattering matrix.

5 Conclusion.

In this talk I showed that a renormalizable manifestly Lorentz invariant formulation of the Yang-Mills theory which allows a canonical quantization without Gribov ambiguity is possible. In perturbation theory the scattering matrix (infrared regularized) and the gauge invariant correlators of the Yang-Mills fields coincide with the standard ones. Analogous formalism may be developped for the Higgs-Kibble model, which describes the massive Yang-Mills field and the massive Higgs scalar. In this case infrared singularities are absent and the scattering matrix is well defined. As in the case of the massless Yang-Mills field, this model allows a modified formulation, in which one can impose the ambiguity free gauge preserving a manifest Lorentz invariance and renormalizability. It would be interesting to carry out semi-analytic and numerical calculations in this formalism beyond the perturbation theory and compare the results with the existing calculations.

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