Asymptotic safety of gravity and the Higgs boson mass

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Abstract

If gravity is asymptotically safe, the ultimate theory may happen to be just the Standard Model (minimally supplemented by few light particles, to accommodate neutrino masses and oscillations, dark matter and baryon asymmetry of the Universe) plus gravity. If this is indeed the case, the mass of the Higgs boson can be predicted ($m_H = m_{\rm min} \simeq 126$ GeV, with only a few GeV uncertainty) or constrained to be in the interval $m_{\rm min} < m_H < m_{\rm max} \simeq 174$ GeV.

1 Introduction

The most minimalistic approach to quantum gravity is associated with asymptotic safety [1]. Though General Relativity is non-renormalizable by perturbative methods, it may exist as a field theory non-perturbatively, exhibiting a non-trivial ultraviolet fixed point (FP) (for a review see [2]). Within this setting a very economical description of all interactions in Nature may be possible. One can assume that there is no new physics associated with any intermediate energy scale (such as Grand Unified scale or low energy supersymmetry) between the Fermi scale and the Planck scale $M_P = 2.44 \times 10^{18}$ GeV. All confirmed observational signals in favour of physics beyond the Standard Model (SM) such as neutrino masses and oscillations, dark matter and dark energy, baryon asymmetry of the Universe and inflation can be associated with new physics *below* the electroweak scale, for reviews see [3, 4] and references therein. The minimal model – νMSM, contains, in addition to the SM particles, 3 relatively light singlet Majorana fermions and the dilaton. These fermions could be responsible for neutrino masses, dark matter and baryon asymmetry of the Universe. The dilaton may lead to dynamical dark energy [5, 6] and realizes spontaneously broken scale invariance which either emerges from the cosmological approach to a fixed point [5, 7] or is an exact quantum symmetry [8, 9]. Inflation can take place either due to the SM Higgs [10] or due to the asymptotically safe character of gravity [11]. Yet another part of new physics, related, for example, to the strong CP problem or to the flavor problem, may be associated with the Planck scale.

There is, however, an obstacle against this point of view, which is related to Landau-pole problem for a number of the SM (or the νMSM) couplings. Namely, the U(1) gauge coupling $g' \equiv g_1$, the Higgs self-interaction λ, and Yukawa couplings (most notably, that of the top-quark, y_t) are not asymptotically free. This fact makes it impossible to formulate the *fundamental* SM, leaving it the role of the *effective* field theory, valid only below some energy scale.

In this talk, based on the paper we written together with Christof Wetterich [12], I will discuss a scenario which can overcome this difficulty. Our proposal leads to a prediction of the Higgs mass, which can be tested at the LHC. The paper is organised as follows. Section 2 provides a short overview of asymptotic safety, in Section 3 we discuss how the asymptotically

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safe SM may emerge due to its combination with asymptotically safe gravity and present the Higgs mass predictions, in Section 4 we conclude.

2 Asymptotic safety

A search for "good" quantum field theory can proceed along the following lines:

- Take some specified set of quantum fields and write the most general Lagrangian respecting chosen symmetries, including operators of arbitrary dimension.
- Compute all scattering amplitudes in all orders of perturbation theory.
- Require that the theory is unitary, Lorentz - invariant, and causal. This will lead to an infinite number of conditions for infinite number of coupling constants defining the theory.
- Solve these consistency equations. Hopefully, the theory will be characterised by a finite number of essential parameters - coupling constants, making the predictions possible.

Of course, it is very difficult, if not impossible, to realise this program. Some approach to it is based on renormalisation group (RG) [1]. Let us introduce dimensionless coupling constants g_i for all terms in the action:

$$g_i = \mu^{D_i} G_i \ , \tag{1}$$

where D_i is the canonical dimension of coupling constant G_i, and μ is an arbitrary parameter with dimension of mass. The RG equations are derived from requirement that physical amplitudes are μ-independent. This leads to the running of couplings $g = \{g_i\}$ as

$$\mu \frac{\partial g_i}{\partial \mu} = \beta_i(g) \ , \tag{2}$$

and fixes the β-functions.

The renormalizable asymptotically free theories correspond to Gaussian ultra-violet (UV) fixed points: essential couplings $g_i(\mu) \to 0$ at $\mu \to \infty$. The number of these couplings is finite - only operators with dimension ≤ 4 are allowed. The well known examples of asymptotically free theories include Quantum Chromodynamics, certain Grand Unified Theories, and renormalizable theories in 2 and 3 dimensional space-time.

The asymptotically safe theories are associated with non-Gaussian UV fixed points $g^* \neq 0$: $\beta_i(g^*) = 0$. Though they are non-renormalisable, they are predictive, if the dimensionality of the critical surface in the space of coupling constants (which points are attracted to g^* when $\mu \to \infty$) is finite. The known examples include the scalar field theory in 3d at the Wilson-Fischer fixed point (critical surface is 2-dimensional), non-linear σ model [13], and gravity in $2+\epsilon$ dimensions [14, 15, 1, 16].

To determine whether some theory is asymptotically safe is very complicated since the standard perturbative expansion fail. The common methods include ϵ - expansion [17], lattice simulations [18, 19], and functional renormalisation group [20, 21]. An original conjecture by Weinberg that gravity may happen to be asymptotically safe was based on ϵ expansion. The extensive studies of functional RG for gravity initiated by Reuter [22] provided yet further evidence in favour of it. A number of recent references include [23, 24]. In what follows we will assume that gravity is indeed asymptotically safe.

3 Asymptotically safe SM and Higgs boson mass

The stand alone Standard Model is neither asymptotically free nor asymptotically safe. It suffers from Landau-pole behaviour of the U(1) gauge constant, the Yukawa terms, and the Higgs self-coupling. However, it is not excluded that a combination of the SM with asymptotically safe gravity may change the situation and lead to a consistent theory. Let us discuss how this may happen.

We will concentrate on the evolution of the SM gauge coupling constants g_1, $g \equiv g_2$ and g_3, corresponding to the U(1), SU(2) and SU(3) groups respectively, and on Higgs and top Yukawa couplings, λ and y_t. As for the gauge couplings, we will fix their values at small energies to the experimental ones, but will leave λ and y_t undetermined for the time being.

The renormalisation group equations for the matter self-interactions receive contribution from gravity sector [1, 25, 27]. Generically, the RG equations for these couplings with gravity corrections incorporated, have the form

$$\frac{dh}{dt} = \beta_h^{\mathrm{SM}} + \beta_h^{\mathrm{grav}} \ , \tag{3}$$

where $t = \log \mu$, h is any of the couplings g_i, λ or y_t, β_h^{SM} is the SM contribution, and β_h^{grav} are the gravity corrections. In one-loop approximation

$$\beta_1^{\mathrm{SM}} = \frac{1}{16\pi^2}\frac{41}{6}g_1^3 \ , \quad \beta_2^{\mathrm{SM}} = -\frac{1}{16\pi^2}\frac{19}{6}g_2^3 \ , \quad \beta_3^{\mathrm{SM}} = -\frac{1}{16\pi^2}7g_3^3 \ , \tag{4}$$

$$\beta_y^{\mathrm{SM}} = \frac{1}{16\pi^2}\left[\frac{9}{2}y_t^3 - 8g_3^2 y_t - \frac{9}{4}g_2^2 y_t - \frac{17}{12}g_1^2 y_t\right] \ , \tag{5}$$

$$\beta_\lambda^{\mathrm{SM}} = \frac{1}{16\pi^2}\left[24\lambda^2 + 12\lambda y_t^2 - 9\lambda(g_2^2 + \frac{1}{3}g_1^2) - 6y_t^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right] \ . \tag{6}$$

The structure of gravity corrections can be deduced from dimensional analysis:

$$\beta_h^{\mathrm{grav}} = \frac{a_h}{8\pi}\frac{\mu^2}{M_P^2(\mu)}h \ , \tag{7}$$

where a_1, a_2, a_3, a_y and a_λ are some constants (anomalous dimensions) corresponding to g_1, g_2, g_3, y_t and λ and $M_P^2(\mu)$ is the running Planck mass. From the studies of the functional renormalization group one infers a characteristic scale dependence of the gravitational constant or Planck mass,

$$M_P^2(\mu) = M_P^2 + 2\xi_0\mu^2 \ , \tag{8}$$

where ξ_0 is a pure number, the exact value of which is not essential for our considerations. From investigations of simple truncations of pure gravity one finds $\xi_0 \approx 0.024$ from a numerical solution of functional RG equations [22, 25, 26]. Thus for large momentum transfer $q^2 \gg M_P^2$ the effective gravitational constant $G_N(q^2)$ scales as $\frac{1}{16\pi\xi_0 q^2}$, ensuring the regular behaviour of high energy scattering amplitudes. The explicit computations of different anomalous dimensions has been carried out in [25]-[32]. Note, however, that there is no agreement between different authors on the magnitude and even signs of the coefficients a_i. Moreover, the definitions of the matter couplings used in different papers are not the same. The coefficients a_i, found in different works, are dependent on the gauge used and on the form of truncation of the functional RG equations. We will assume that some gauge-invariant definition of these couplings will eventually be possible. Most probably, it could be based on gauge-invariant high energy scattering amplitudes, as was suggested in [1]. We stress that this definition of couplings does not coincide with that based on a minimal subtraction scheme, cf. [33].

The running of different couplings in the SM can be divided to two regimes. Up to the scales $\mu^2 \sim M_P^2$ the gravitational corrections to beta functions of the SM are suppressed by the factor μ^2/M_P^2 and are therefore small. The couplings run logarithmically up to $\mu^2 \sim M_P^2$. For $\mu^2 \gtrsim M_P^2$ the corrections coming from gravity become important. If the gravitational part of the β-functions dominates and $\mu^2 \gtrsim \frac{M_P^2}{2\xi_0}$, the running is a power low,

$$h \propto \mu^{\frac{a_h}{16\pi\xi_0}} \ . \tag{9}$$

Clearly, the signs of anomalous dimensions a_h play a crucial role for validity of the SM at any energy scale.

First, let us look at the gauge sector. Assume for simplicity that $a_1 = a_2 = a_3 = a$, which is true for one-loop computations, performed up to now, due to the universality of the gravitational interactions. Then all gauge constants are asymptotically free if $a < a^{\rm crit}$, $a^{\rm crit} \simeq -0.013$[1]. If this is the case, the Landau-pole problem for the U(1) coupling is solved by the gravity contribution to the RG running. And, indeed, the computations of [28, 27] give a *negative* sign for a, with $|a| \sim 1$. In what follows we will assume that

$$a < a^{\rm crit}. \tag{10}$$

In this case the gauge coupling constants cannot be predicted. If $a = a^{\rm crit}$, the value of the U(1) coupling is predictable (provided $a^{\rm crit}$ is reliably computed).

Consider now the top Yukawa coupling. Taking, for example, $a = -1$, and the (central) experimental value of the top quark mass, $m_t = 171.3$ GeV [34], one finds that at $a_y < a_y^{\rm crit}$, where $a_y^{\rm crit} \simeq -0.005$, the behaviour of y_t is asymptotically free, at $a_y = a_y^{\rm crit}$ it corresponds to a non-Gaussian fixed point with $y_t^* \simeq 0.38$, and if $a_y > a_y^{\rm crit}$ one gets the Landau-pole behaviour. The critical value of a_y is only weakly sensitive to a. For example, for $a = -0.02$ one gets $a_y^{\rm crit} \simeq -0.002$ and $y_t^* \simeq 0.25$. For smaller values of the top quark mass the corresponding values of $a_y^{\rm crit}$ are even closer to zero, while larger m_t move $a_y^{\rm crit}$ further from zero.

Suppose that $a_y > a_y^{\rm crit}$. Then, in order to have a consistent theory for all energy scales, one has to put $y_t = 0$. This corresponds to the massless top quark and is therefore rejected by experiments. In other words, if this happens to be the case, one should give up the assumption on the absence of new physics between the Fermi and Planck scales, to modify the pattern of y_t RG running. Therefore, the hypothesis of the fundamental character of the SM or νMSM can only be true if $a_y \le a_y^{\rm crit}$. Unfortunately, we were not able to extract the reliable value and the sign of a_y from existing literature. For example, in [35] it was shown that gravity contributions make Yukawa coupling asymptotically free in quantum R^2 gravity with matter. Ref. [32] studied the gravitational running of Yukawa couplings f in functional RG approach for the Einstein-Hilbert type of truncation and found different signs for a_y in different gauges. Also, in this paper the wave function renormalisation for the fermions and scalars was not included and the sensitivity to the truncation type was not investigated. So, in what follows we will simply assume that $a_y < a_y^{\rm crit}$. As in the case of the U(1) coupling, the special case $a_y = a_y^{\rm crit}$ would lead to a prediction of m_t.

Let us turn now to the behaviour of the scalar self-coupling λ. The gravitational corrections can only promote the SM to the rank of fundamental theory if the running of λ does not lead to any pathologies up to the Planck scale. In other words, the Landau pole must be absent for $k \lesssim M_P$ [36, 37, 38, 39], and λ must be positive for all momenta up to M_P [40, 41, 42], ensuring the stability of the electroweak vacuum. There is a large parameter space available on the plane m_H, m_t, where both conditions are satisfied. Close to the experimental value of the top mass, it is described by

$$m_{min} < m_H < m_{max}\ . \tag{11}$$

Here

$$m_{min} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ \mathrm{GeV}\ , \tag{12}$$

and

$$m_{\max} = [173.5 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.118}{0.002} \times 0.1]\ \mathrm{GeV}\ , \tag{13}$$

where α_s is the strong coupling at the Z-mass, with theoretical uncertainty in m_{min} equal to ± 2.2 GeV. These numbers are taken from the recent two-loop analysis of [43] (see also [44, 45]

[1]The value $a^{\rm crit}$ corresponds to the fixed point $g_1^* \simeq 0.5$ in the U(1) one-loop coupling running, if it starts from the experimental value at low energies.

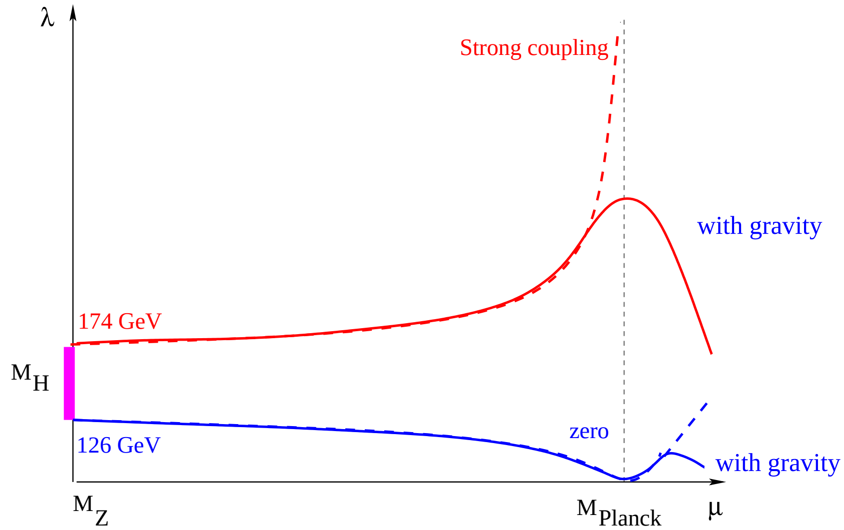

Figure 1: Evolution of the Higgs self-coupling λ in the SM and asymptotically safe gravity for the case of negative anomalous dimension a_λ.

and earlier computations in [46, 47, 48, 49]). The value of $m_{\rm max}$ corresponds to the (somewhat arbitrary) criterion $\lambda(M_P) < 6$. The admitted region contains also very small top and Higgs masses, excluded experimentally.

Suppose first that a_λ is negative and has sufficiently large magnitude,

$$a_\lambda < a_\lambda^{\rm crit} \simeq -\frac{24\xi_0\lambda(M_P)}{\pi} \ , \tag{14}$$

($a_\lambda^{\rm crit} \simeq -1$ if $\lambda(M_P) \simeq 6$). Then the Higgs coupling is asymptotically free in all the region of the parameter space, constrained by (11). The gravity contribution removes the Landau-pole behaviour at energies exceeding the Planck mass, as shown in Fig. 1. For $m_t = 171.3$ GeV (remember that the top mass cannot be predicted, if $a_y < a_y^{\rm crit}$), and neglecting uncertainties in theoretical computations and in α_s, one gets that the Higgs mass must lie in the interval $[126.3,\ 173.5]$ GeV. The upper limit on the Higgs mass goes down, if the actual value of $a_\lambda^{\rm crit}$ is smaller, than one.

The most interesting situation is realised if a_λ is positive, leading to a specific prediction of the Higgs boson and top quark masses. In fact, an evidence that this is indeed the case comes from computations of [25, 26], giving

$$a_\lambda \simeq +3.1 \ . \tag{15}$$

The contribution with the same sign and similar magnitude was found previously in [50].

Let us elucidate the structure of the solution to the RG equation for λ in this case. Due to the positive sign of a_λ, the generic solution to (3) diverges at $\mu \to \infty$, leading to inconsistent theory. However, there may exist a particular solution, leading to $\lambda \to 0$ (or, in a special case, $\lambda \to const \neq 0$) in the ultraviolet. It is easy to see the the required behaviour is only possible if the t-quark contribution, coming with negative sign to $\beta_\lambda^{\rm SM}$, dominates over the gauge contribution at $t \to \infty$, leading to the constraint

$$a \leq a_y \leq a_y^{\rm crit} \ . \tag{16}$$

If $a < a_y < a_y^{\rm crit}$, then the ultraviolet asymptotic for λ reads

$$\lambda(\mu) \approx \frac{6y_t^4(\mu)\xi_0}{\pi a_\lambda} \ . \tag{17}$$

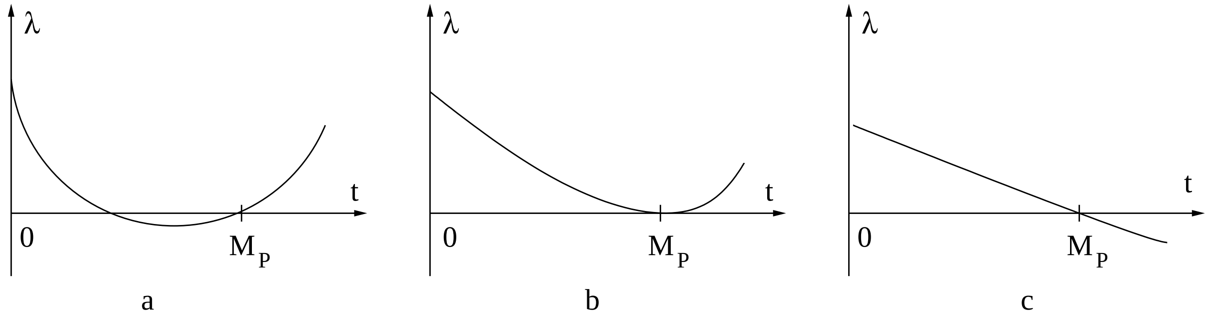

Figure 2: Evolution of the Higgs self-coupling λ in the SM with the boundary condition $\lambda(M_P) = 0$.

For $a_y = a$ one has $\lambda(\mu) \propto \mu^{\frac{a}{2\pi\xi_0}}$, whereas for $a_y = a_y^{\rm crit}$ there is a non-Gaussian fixed point for λ, which obeys the equation

$$24\lambda^{*2} + 12\lambda^* y_t^{*2} - 6y_t^{*4} + \frac{\pi a_\lambda \lambda^*}{\xi_0} = 0 \ . \tag{18}$$

For $a_\lambda \geq 0$ the amplitude of scalar self-coupling does not exceed $\lambda^* < 0.3 y_t^{*2}$.

To summarise: for any given set of a_h, satisfying (10,16), and $a_\lambda \geq 0$, there is a unique value of the low-energy scalar-self coupling, which leads to a consistent theory. This means that the Higgs boson mass can be predicted. To find m_H, one should solve the RG equations fixing the initial values (say, at Z-mass) for the gauge and Yukawa couplings and adjust λ in a way that it goes to zero at $\mu \to \infty$ or approaches the fixed point λ^*. Moreover, only RG trajectories with positive λ can be accepted.

The following consideration allows to locate the values of the Higgs boson masses, leading to consistent theory. The RG equation for λ, satisfying the asymptotic safety requirement, can be rewritten as an integral equation

$$\lambda(\mu) = -\int_\mu^\infty \frac{d\mu'}{\mu'} \left(\frac{1 + 2\xi_0 \mu^2/M_P^2}{1 + 2\xi_0 \mu'^2/M_P^2} \right)^{\frac{a_\lambda}{32\pi\xi_0}} \times \beta_\lambda^{\rm SM}(h(\mu')) \ .$$

Assuming that all couplings fall as in (9), we arrive to the boundary condition at $\mu = M_P$

$$\lambda(M_P) = -C \beta_\lambda^{\rm SM}(h(M_P)) \ , \tag{19}$$

where C is positive and is of the order of 1. Since $\beta_\lambda^{\rm SM} \ll \lambda$ at point $k = M_P$, this can be replaced by

$$\lambda(M_P) \approx 0 \ . \tag{20}$$

So, the SM running of λ must bring it close to zero at the Planck scale.

This is not all the story. To have a consistent theory, $\lambda(\mu)$ must be positive at *all* energy scales. To find the consequences from this requirement, let us consider the SM evolution of λ for $\mu < M_P$ with boundary condition (20). Three different possibilities are shown in Fig. 2. The case (a), when λ hits zero before the Planck scale, is excluded – the SM here breaks down below M_P. The case (c) is potentially dangerous: the negative value of $\beta_\lambda^{\rm SM}$ at $k = M_P$, by continuity, will push λ to negative values *above* the Planck scale. In other words, not only the scalar self-coupling must be close to zero, but *also* its SM β-function should be small at $k = M_P$:

$$\beta_\lambda^{\rm SM}(M_P) \approx 0 \ . \tag{21}$$

How accurately the equations (20,21) should be satisfied, depends on the specific values of the anomalous dimensions a_h and requires a numerical solution of the RG equations. It is important that there are two conditions instead of one, allowing to fix (or, at least constraint) the Higgs and top masses *simultaneously.*

For better accuracy, in the numerical computations we used the two-loop RG equations and one-loop pole matching of the physical parameters, see [51, 44] and also [43]. We describe below the most essential features of our findings.

The requirement of positivity of λ at all energy scales leads to strong bounds on the top mass. The lower bound is $m_t \gtrsim 170$ GeV, which practically does not depend on anomalous dimensions a_h. Basically, if $m_t < 170$ GeV, one gets the behaviour shown in Fig. 2(a), leading to unstable vacuum. Larger m_t correspond to the pattern shown in Fig. 2(c). If the magnitudes of a and a_y are sufficiently large, the constants g_i and y_t go to zero quickly at $k > M_P$, leading to a small value of $\beta_\lambda^{\rm SM}$ above the Planck scale, and thus to the healthy behaviour of λ. If the magnitudes of a and a_y are smaller, the absolute value of $\beta_\lambda^{\rm SM}$ right above the Planck scale increases, knocking λ to the negative region. Thus, the upper limit on the mass of the top, derived from the positivity considerations, depends substantially on a and a_y. For example, for $a = a_y = -1$, and $a_\lambda = 3$ the admitted RG trajectories exist for a large variety of top masses: $m_t = 171.3$ GeV leads to $m_H \simeq 126$ GeV, whereas $m_t = 230$ GeV requires $m_H \simeq 227$ GeV. The choice of $a = a_y = -0.25$, $a_\lambda = 3$ leads to an upper bound $m_t \lesssim 174$ GeV, which is very close to the lower limit. The fact that the experimental value of the top mass is amazingly close to the lower limit (and to the upper limit for small enough a_y) can be considered as a support of the ideas presented in this paper.

Let us now choose the experimental value for the top quark mass and determine the Higgs boson mass. The prediction is quite insensitive to the specific values of a, a_y and a_λ and reads

$$m_H = m_{min} \ , \tag{22}$$

where m_{min} is given in (12). It is not difficult to understand why this is the case. The SM behaviour of λ, corresponding to $m_H = m_{min}$ and $m_t = 171.3$ GeV is exactly what is shown in Fig. 2 (b). Decreasing m_H moves us to Fig. 2 (a), what is excluded. Increasing m_H makes $\lambda(M_P)$ positive, and drives it to infinity above the Planck scale for $a_\lambda > 0$, which is excluded as well. The latter behaviour can only be modified if the top Yukawa coupling has a non-Gaussian fixed point, $a_y = a_y^{\rm crit}$, which leads to the existence of the non-trivial fixed point in λ. Taking, as an example, $a = -1$, $a_y \simeq -0.005$, one gets, that $\lambda^* < 0.043$, what shifts up the prediction of the Higgs mass by not more than 8 GeV. Taking smaller a decreases this shift. This situation, however, requires some fine tuning and therefore looks improbable.

Our prediction (22) (or (11), if a_λ is in fact negative) can be verified at the LHC. Given the fact that the accuracy in the Higgs mass measurements at the LHC can reach 200 MeV, the reduction of theoretical uncertainty and of experimental errors in the determination of the top quark mass and of the strong coupling constant are highly desirable. As was discussed in [43], the theoretical error can go down from 2.2 GeV to 0.4 GeV if one upgrades the one-loop pole matching at the electroweak scale and two loop running up to the Planck scale to the two-loop matching and 3-loop running. Note that 3-loop beta-functions for the SM are not known by now, and that the two-loop pole matching has never been carried out.

The prediction $m_H \approx m_{\rm min}$ does not only hold for the hypothesis that the SM plus gravity describes all the physics relevant for the running of couplings. It generalizes to many extensions of the SM and gravity, including possibly even higher dimensional theories. Of course, the precision of the prediction gets weaker if a much larger class of models is considered. Nevertheless, only two crucial ingredients are necessary for predicting $m_H \approx m_{\rm min}$: (i) Above a transition scale k_{tr} the running should drive the quartic scalar coupling rapidly to an approximate fixed point at $\lambda = 0$, only perturbed by small contributions to β_λ from Yukawa and gauge couplings. This is generically the case for a large enough anomalous dimension $a_\lambda > 0$. (ii) Around k_{tr} there should be a transition to the SM-running in the low energy regime. This transition may actually involve a certain splitting of scales as "threshold effects", for example by extending the SM to a Grand Unified theory at a scale near k_{tr}. It is sufficient that these threshold effects do not lead to a rapid increase of λ in the threshold region. This will be the case if the λ-independent contributions to β_λ only involve perturbatively small couplings in a threshold region extending over only a few orders of magnitude.

Some comments are now in order.

(i) The amazing fact that the SM scalar self-coupling is equal to zero *together* with its β-function at the Planck scale for the particular values of the top-quark and Higgs masses was first (to the best of our knowledge) noticed in [52]. These authors put forward the hypothesis of a "multiple point principle", stating that the effective potential for the Higgs field must have two minima, the one corresponding to our vacuum, whereas another one must occur at the Planck scale. Our reasoning is completely different. Though the sense of the "multiple point principle" remains unclear to us, we would like to note that the prediction of the Higgs mass from it coincides with ours (the specific numbers in [52] are different, as they were based on one-loop computation).

(ii) The values of the Higgs mass we found are consistent with a possibility of inflation due to the SM Higgs boson [10]. The Higgs-inflation requires the consistency of the SM up to the lower, than M_P energy scale $k \sim \frac{M_P}{\xi}$, where $\xi = 700 - 10^5$ is the value of the non-minimal coupling of the Higgs field to the curvature Ricci scalar [53, 43] (see also [54, 55]), the smaller ξ correspond to smaller Higgs masses.

(iii) We implicitly assumed in this paper that the Fermi scale is fixed to its experimental value. Refs. [25, 26] found that in a scalar-gravity system the anomalous dimension of the scalar mass is negative, making it the relevant (and thus unpredictable) coupling. If this is indeed the case for the SM, then the smallness of the Fermi scale in comparison with the Planck scale remains the puzzle. If, on the contrary, this anomalous dimension happens to be positive for the SM, the consistency of the theory will require to put the Fermi scale to zero in the asymptotic region. If true, this may eventually shed light on the huge difference between the electroweak and Planck scales.

4 Conclusions

In conclusion, we discussed the possibility that the SM, supplemented by the asymptotically safe gravity plays the role of a fundamental, rather than effective field theory. We found that this may be the case if the gravity contributions to the running of the Yukawa and Higgs coupling have appropriate signs. The mass of the Higgs scalar is predicted $m_H = m_{\min} \simeq 126$ GeV with a few GeV uncertainty if all the couplings of the Standard Model, with the exception of the Higgs self-interaction λ, are asymptotically free, while λ is strongly attracted to an approximate fixed point $\lambda = 0$ (in the limit of vanishing Yukawa and gauge couplings) by the flow in the high energy regime. This can be achieved by a positive gravity induced anomalous dimension for the running of λ. A similar prediction remains valid for extensions of the SM as grand unified theories, provided the split between the unification and Planck-scales remains moderate and all relevant couplings are perturbatively small in the transition region. Detecting the Higgs scalar with mass around 126 GeV at the LHC could give a strong hint for the absence of new physics influencing the running of the SM couplings between the Fermi and Planck/unification scales.

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