

Internal structure of Maxwell-Gauss-Bonnet black hole

K. A. Rannu^{1*}, S. O. Alexeyev^{a†}, A. Barrau^{b,c‡}

^a *Sternberg Astronomical Institute*

Lomonosov Moscow State University, Universitetsy Prospect, 13, Moscow 119991, RUSSIA

^b *Laboratoire de Physique Subatomique et de Cosmologie*

UJF-INPG-CNRS, 53, avenue des Martyrs, 38026 Grenoble cedex, France

^c *Institut de Hautes Etudes Scientifiques*

35, route de Chartres, 91440, Bures-sur-Yvette, France

Abstract

The influence of the Maxwell field on a static, asymptotically flat and spherically-symmetric Gauss-Bonnet black hole is considered. Numerical computations suggest that if the charge increases beyond a critical value, the inner determinant singularity is replaced by an inner singular horizon.

1 Introduction

The internal structure of black holes described by the action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[m_{pl}^2 (-R + 2\partial_\mu \phi \partial^\mu \phi) - e^{-2\phi} F_{\mu\nu} F^{\mu\nu} + \lambda e^{-2\phi} S_{GB} \right], \quad (1)$$

where m_{pl} is the Planck mass, ϕ is the dilaton field, R is the scalar curvature, $S_{GB} = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2$ is the Gauss-Bonnet term, $F_{\mu\nu}F^{\mu\nu}$ is the Maxwell field and λ is the string coupling constant. The influence of the magnetic charge of the black hole on the behavior of the metric functions was considered and it was shown that there exists a “critical value” of the charge beyond which the influence of the Maxwell term becomes more important than the Gauss-Bonnet one. The inner determinant singularity at $r = r_s$ is then replaced by a smooth local minimum. The focus was made on the behavior of the curvature invariant $R_{ijkl}R^{ijkl}$ near this critical point and in the vicinity of the main singularity at $r = r_x$.

2 Field equations, curvature invariant and calculation aspects

Considering a static, asymptotically flat and spherically symmetric black hole solution, we focus on the following metric:

$$ds^2 = \Delta dt^2 - \frac{\sigma^2}{\Delta} dr^2 - f^2 (d\theta^2 + \sin^2\theta d\varphi^2), \quad (2)$$

where Δ , σ and f are functions that depend on the radial coordinate r only. To simplify the problem, only the magnetic charge was taken into account. Therefore, for the Maxwell tensor

***e-mail:** rannu@xray.sai.msu.ru

†**e-mail:** alexeyev@sai.msu.ru

‡**e-mail:** barrau@in2p3.fr

$F_{\mu\nu}$, one can use the ansatz $F = q \sin \theta d\theta \wedge d\varphi$. The corresponding field equations in the GHS gauge ($\sigma(r) = 1$) are as follows:

$$m_{Pl}^2 [ff'' + f^2(\phi')^2] + 4e^{-2\phi} \lambda [\phi'' - 2(\phi')^2] \Delta (f')^2 - 1 + 4e^{-2\phi} \lambda \phi' 2\Delta f' f'' = 0, \quad (3)$$

$$m_{Pl}^2 [1 + \Delta f^2(\phi')^2 - \Delta' f f' - \Delta (f')^2] + 4e^{-2\phi} \lambda \Delta' \phi' [1 - 3\Delta (f')^2] - e^{-2\phi} q^2 f^{-2} = 0, \quad (4)$$

$$m_{Pl}^2 [\Delta'' f + 2\Delta' f' + 2\Delta f'' + 2\Delta f(\phi')^2] + 4e^{-2\phi} \lambda [\phi'' - 2(\phi')^2] 2\Delta \Delta' f' + \\ + 4e^{-2\phi} \lambda \phi' 2[(\Delta')^2 f' + \Delta \Delta'' f' + \Delta \Delta' f''] - 2e^{-2\phi} q^2 f^{-3} = 0, \quad (5)$$

$$- 2m_{Pl}^2 [\Delta' f^2 \phi' + 2\Delta f f' \phi' + \Delta f^2 \phi''] + 4e^{-2\phi} \lambda [(\Delta')^2 (f')^2 + \Delta \Delta'' (f')^2 + 2\Delta \Delta' f' f'' - \Delta''] - \\ - 2e^{-2\phi} q^2 f^{-2} = 0. \quad (6)$$

The behavior of the metric functions and of the dilatonic field near the horizon are described by a simple Taylor expansion:

$$\begin{aligned} \Delta &= d_1 x + d_2 x^2 + O(x^2), \\ f &= f_0 + f_1 x + f_2 x^2 + O(x^2), \\ e^{-2\phi} &= e^{-2\phi_0} + \phi_1 x + \phi_2 x^2 + O(x^2), \end{aligned} \quad (7)$$

where ($x = r - r_h, \ll 1$).

Without the Gauss-Bonnet term, the Gibbons-Maeda-Garfinkle-Horowitz-Strominger solution (GM-GHS) should be recovered as the basic solution of the Einstein equations with the dilaton and Maxwell terms. This solution is given by:

$$\begin{aligned} ds^2 &= \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 \\ &\quad - r \left(r - \frac{q^2 \exp(2\phi_0)}{M}\right) d\Omega, \\ \exp(-2\phi) &= \exp(-2\phi_0) - \frac{q^2}{Mr}, \end{aligned} \quad (8)$$

where M stands for the black hole mass. In the limit $\lambda \rightarrow 0$, the solution of equations (3)–(6) at infinity should coincide with Eq. (8).

This establishes the boundary conditions near the horizon and at the infinity respectively. The computation process was divided into two parts. First, the GM-GHS solution Eq. (8) was taken as the initial condition at infinity. Solutions for the metric functions and the dilaton outside the event horizon were found. Then, the results near the horizon were taken as new initial conditions.

In order to determine the two metric functions and the dilatonic field, three equations are required. Among the four equations (3)–(6), only equations (3), (5) and (6), which contain the second derivatives of the metric functions and the dilaton, are used. In contrast, Eq. (4), which contains the first derivative only, is considered as a constraint to check the solution.

To solve the system (3), (5), (6) the equations are rewritten using $E = e^{-2\phi}$ instead of the dilaton itself. Furthermore, the case $\lambda = 1$ is considered. In the chosen metric gauge the squared Riemann tensor is given by:

$$\begin{aligned} R_{ijkl} R^{ijkl} &= \Delta'^2 + 4\Delta'^2 \frac{f'^2}{f^2} + 8\Delta'^2 \frac{f''^2}{f^2} + 8\Delta \Delta' \frac{f' f''}{f^2} \\ &\quad + \frac{4}{f^4} - 8\Delta \frac{f'^2}{f^4} + 4\Delta^2 \frac{f'^4}{f^4}. \end{aligned} \quad (9)$$

Its behavior of the squared Riemann tensor was the main question of this work because it is needed to determine if the singularity is the coordinate one or the real scalar one. Thus it describes the inner structure of the black hole.

3 Results

Metric function Δ has a particular point that causes the inner singularity r_s when the charge is small. However if the charge is larger than some critical value q_{cr} the inner singularity disappears and function Δ displays the local minimum in a point not far from the function's f zero, as it can be seen on Fig. 1. This is the difference between the given solution and the GM-GHS.

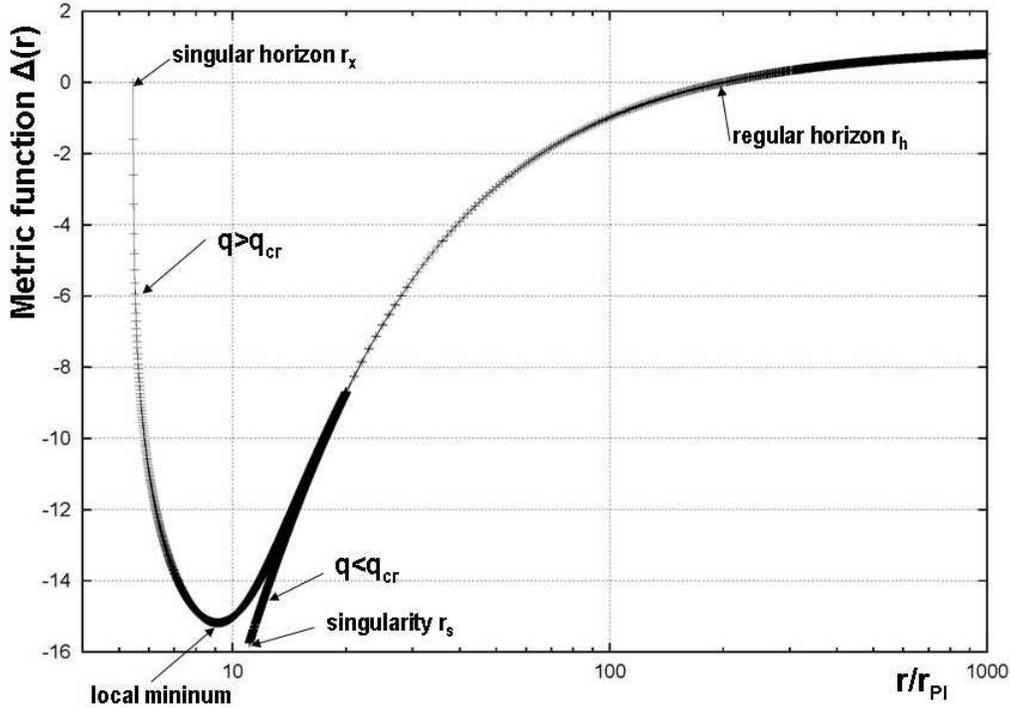


Figure 1: Metric function Δ as a function of the radial coordinate r for $q = 21.50 < q_{cr}$ (left curve) and $q = 24.81 > q_{cr}$ (right curve) when $r_h = 200.0$ Planck units.

Functions' f (Fig. 2) and $e^{-2\phi}$ (Fig. 3) behavior is analogous to the GM-GHS case. If $q < q_{cr}$ they decrease monotonously till $r = r_s$. Metric function f can be approximately described as $f = r$, but it reaches it's zero before r does even if $q < q_{cr}$. Function $e^{-2\phi}$ also approaches it's zero near the function's f zero. This leads to a fact that near the function's f zero influence of the Maxwell and Gauss-Bonnet terms become negligible and ordinary Einstein gravitation is realised.

It was confirmed that the behavior of the curvature invariant $R_{ijkl}R^{ijkl}$ under the black hole's event horizon is about zero almost everywhere and near the r_s $R_{ijkl}R^{ijkl} \rightarrow \infty$. When black hole charge q reaches it's critical value and metric function's Δ local minimum replaces singularity in r_s . In this case the value of curvature invariant $R_{ijkl}R^{ijkl}$ does not increase (Fig. 4). So it is obvious that metric function's Δ local minimum is not singular.

Function f plays the role of the radial coordinate in our solution. When r_s vanishes, the new point r_x in which f reaches it's zero, appears. In this case curvature invariant increases near the r_x . So we can consider r_x to be the singular horizon. When $q < q_{cr}$ this horizon belongs to a second branch of the system's (3)–(6) solution. This branch is nonphysical.

Near the singular horizon r_x curvature invariant increases much more rapidly than near the

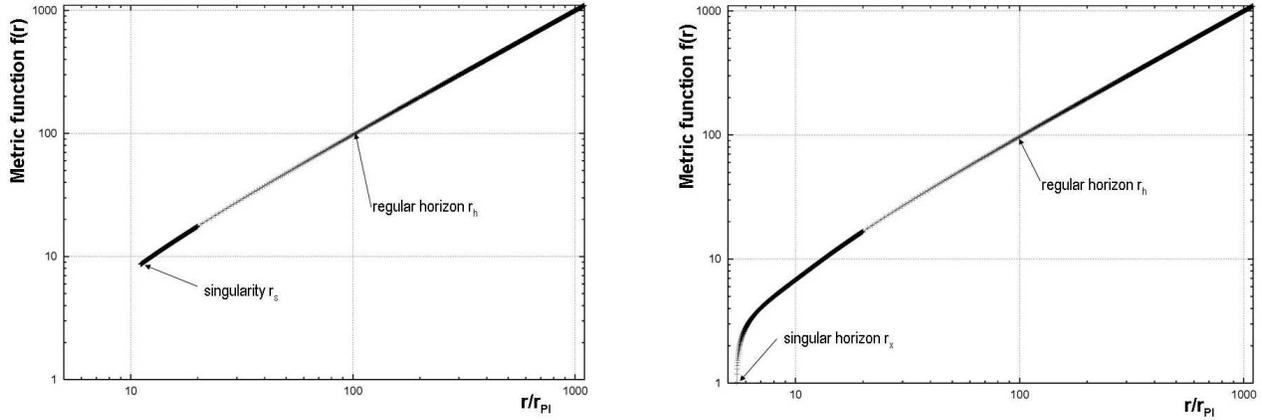


Figure 2: Metric function f as a function of the radial coordinate r for $q = 21.50 < q_{cr}$ (left plot) and $q = 24.81 > q_{cr}$ (right plot) when $r_h = 200.0$ Planck units.

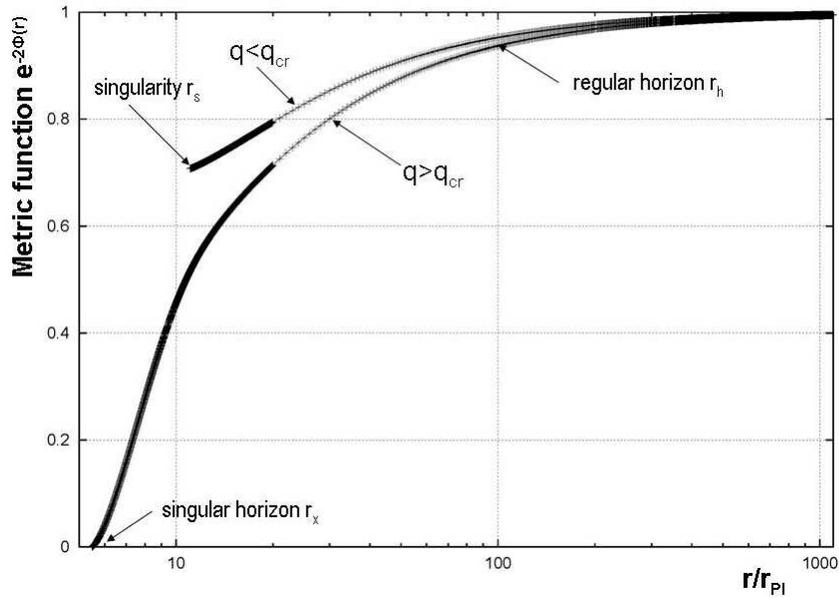


Figure 3: Dilatonic exponent $E = e^{-2\phi}$ as a function of the radial coordinate r for $q = 21.50 < q_{cr}$ (left plot) and $q = 24.81 > q_{cr}$ (right plot) when $r_h = 200.0$ Planck units.

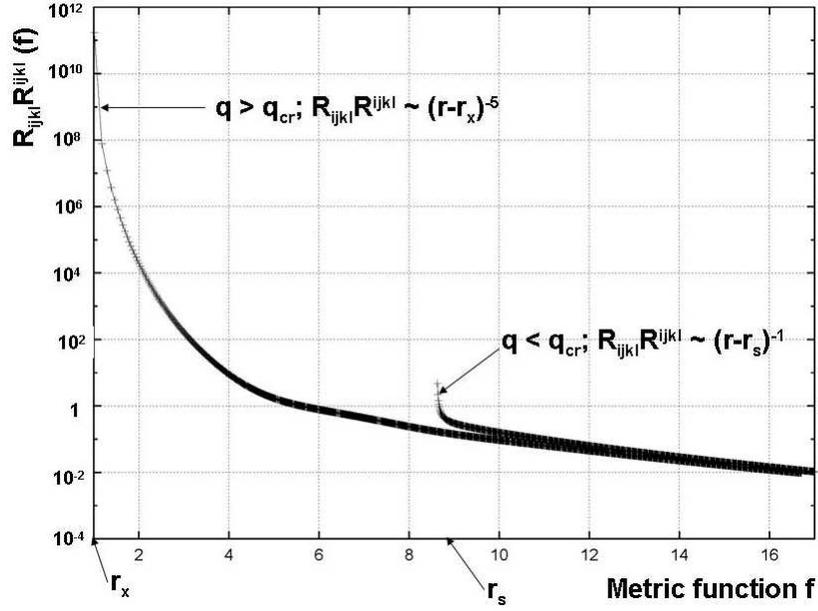


Figure 4: Curvature invariant $R_{ijkl}R^{ijkl}$ as a function of the metric function f for $q = 21.50 < q_{cr}$ (left curve) and $q = 24.81 > q_{cr}$ (right curve), with $r_h = 200.0$ Planck units.

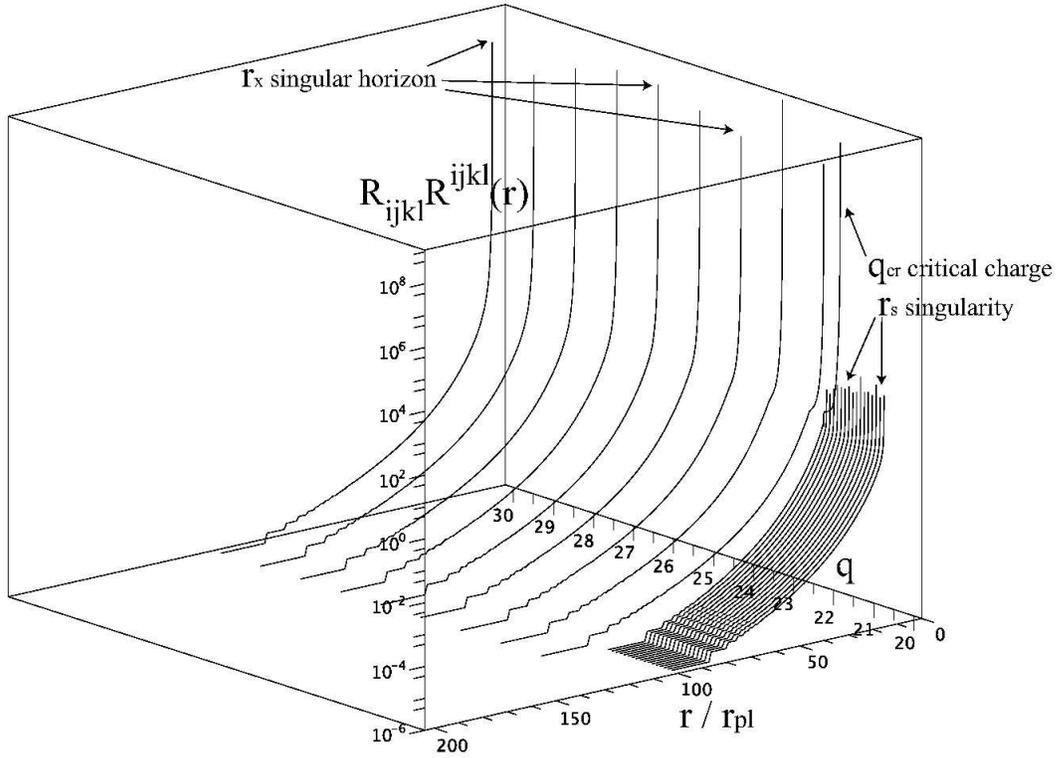


Figure 5: Three dimensional dependence of curvature invariant $R_{ijkl}R^{ijkl}$ on charge q and radial coordinate r in case $r_h = 200$ (black hole mass is $M = 100$).

singularity r_s .

$$\begin{aligned}
f(r \rightarrow r_s) &= f_s + f_{s2}(\sqrt{r - r_s})^2 + f_{s3}(\sqrt{r - r_s})^3 + \dots \\
f(r \rightarrow r_x) &= f_x + f_{x1}\sqrt{r - r_x} + f_{x2}(\sqrt{r - r_x})^2 + \dots \\
\text{for } f \rightarrow f_s & \quad R_{ijkl}R^{ijkl} \sim \text{const}_1 \times (f - f_s)^{-1} \\
\text{for } f \rightarrow f_x & \quad R_{ijkl}R^{ijkl} \sim \text{const}_2 \times (f - f_x)^{-5}
\end{aligned}$$

So we can conclude that if singularity r_s is replaced by the function's Δ local minimum the singularity in r_x is much stronger than the one in r_s .

4 Conclusions

When the black hole charge becomes larger than the critical value the singularity r_s is replaced by a local minimum of the function $\Delta(r)$ and the solution exists till the singular horizon r_x .

Function $f(r)$ is the radius of S^2 , so it plays the role of the radial coordinate. If $q < q_{cr}$ it decreases monotonously till $r = r_s$ like in GHS. When r_s disappears the function $f(r)$ reaches its zero in the new point r_x .

Curvature invariant increases much more rapidly (as $(r - r_x)^{-5}$) near the singular horizon r_x than near the singularity r_s (as $(r - r_s)^{-1}$), so the singularity in r_x is much stronger than the one in r_s .

New kind of singularity inside black hole was found. Unfortunately Maxwell-Gauss-Bonnet black hole cannot help wormholes' or multiverse theories because this singularity is very strong.

References

- [1] Alexeyev S O, Barrau A and Rannu K A 2009 *Phys. Rev. D* **79** 067503
- [2] Alexeyev S O and Pomazanov M V 1997 *Phys. Rev. D* **55** 2110
- [3] Alexeyev S O and Sazhin M V 1998 *Gen. Relativ. Grav.* **30** 1187
- [4] Alexeyev S O, Sazhin M V and Pomazanov M V 2001 *J. Mod. Phys. D* **10** 225
- [5] Gibbons G W and Maeda K 1988 *Nucl. Phys. B* **298** 741
- [6] Garfincle D, Horowitz G and Strominger A 1991 *Phys. Rev. D* **43** 3140
- [7] Garfincle D, Horowitz G and Strominger A 1992 *Phys. Rev. D* **45** 3888