UV Behavior of Scalar Propagators

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Abstract

Problems in the available treatments of the ultraviolet behavior of scalar fields are pointed out. A new approach is advertised. Results of its application to ϕ^4-model are presented. In particular, it is explained that the running scalar mass in units of the normalization point has a minimum at a certain location of the normalization point. An energy scale at which the new qualitative features in the behavior of scalar propagators may appear is pointed out.

Since the late seventies it is widely accepted that scalar fields are in some sense unnatural. The unnaturalness of scalar fields shows itself in the fact that quantum corrections to the inverse scalar propagator computed with momentum subtractions involve powers of momentum squared, while for the rest of the fields these corrections involve only logarithms of the momentum squared. (The appearance of the powers in the corrections is related to the presence of the quadratic divergences in the quantum corrections to scalar mass [1].) Therefore, it is not obvious how to apply conventional renormalization group treatment to scalar fields. This fact was pointed out in [2]. Later on it was motivating technicolor models and supersymmetry [3], [4].

Remarkably, this general observation is not showing up on the level of practical computations employing minimal subtractions. Since the paper [5] it is well known that scalar fields can be treated within dimensional regularization and minimal subtractions. As for the fields with nonzero spin, the renormgroup summation of leading logarithms softens the ultraviolet behavior of the free scalar propagator:

$$D_{MS}(Q^2) \approx \frac{1}{(Q^2)^{1-\gamma_\phi}\mu^{2\gamma_\phi}}, \tag{1}$$

where MS stands for minimal subtractions, γ_ϕ is the anomalous dimension of the scalar field, and μ is the dimensional regularization mass unit. It may seem that dimensional regularization and minimal subtractions allow us to ignore quadratic divergences and power-like corrections to the scalar self energy appearing in other regularization and renormalization schemes. But this is not exactly the case.

It was pointed out in [6] that the quadratic divergences show up within dimensional regularization as poles in the self energy of the scalar fields at nonphysical dimensions $4-2/l$, where l is the number of loops:

$$\Sigma_{reg}(Q^2) = \sum_{l=1}^{\infty} \frac{2\mu^2(-1)^{l+1}Z_l}{l!l^2(D-4+2/l)} + \ldots, \tag{2}$$

where Σ_{reg} is the self energy (which is the sum of loop corrections to the inverse propagator) regularized with dimensional regularization, dots denote terms without quadratic divergences, Z_l are dimensionless residues depending on couplings and masses (in units of μ) of the model.

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Notice that these residues do not depend on the momentum squared Q^2. (The residues Z_l have been computed up to four loops within the standard model [7]; Condition $Z_1 = 0$ has the form $M_Z^2 + 2M_W^2 + M_H^2 = 4M_t^2$ and is known as "Veltman condition" [6].)Minimal subtractions do not subtract these poles because they are finite at $D = 4$ and finite number of loops.

At the same time the terms from (2) can be summed up as follows:

$$\Sigma_{reg}(Q^2) = \mu^2 Z(D)\Gamma(\frac{2}{D-4}) + \ldots, \tag{3}$$

where the function of dimension $Z(D)$ is related to the above residues in the poles: $Z(4 - 2/l) = Z_l$. We see that the poles concentrating toward the physical dimension form an essential singularity at $D = 4$.

Should we do anything with the singularity of Eq. (3)? Can we trust the result of Eq. (1) despite the fact that it is obtained within a scheme ignoring this singularity? We do not have clear answers to these questions.

Recently, an alternative approach to constructing finite perturbation theory in quantum field theories has been suggested [1]. Differential equations for connected Green functions with small number of external legs appear naturally within the new approach. These equations describe the dependence of the Green functions on the momenta. For the fields with nonzero spins these equations are similar to conventional renormalization group equations. A qualitative difference appears in the evolution equation for scalar propagators. Namely, these equations turn out to be equations of the second order in the derivatives (they involve second derivatives over momentum squared). The presence of the second derivatives in the evolution equation for the scalar propagator is related to the presence of quadratic divergences. Solving these new evolution equations sums up powers and logarithms of momentum squared appearing in the perturbative corrections to the scalar propagator. The ultraviolet asymptotics of the solution disagrees with the minimal subtraction result (1).

In this contribution, we advertise the new approach to perturbative quantum field theory of [1].

We start with discussing two plots where the results yielded by the new approach are compared to the results obtained with the minimal subtractions. The plots are drawn for ϕ^4-model, and show properties of the renormalized scalar propagators with perturbative corrections summed by means of two versions of the evolution equations—the minimal subtraction renormgroup evolution and the new second order evolution equation. The dependence on the self coupling enters both evolution equations via the anomalous dimension, γ_ϕ. On both plots, for the sake of the illustration, we take a large value $\gamma_\phi = 0.3$. (In the leading approximation of ϕ^4, $\gamma_\phi = g^2/(12(16\pi^2)^2)$, and this value of γ_ϕ is too large for perturbative considerations. We had chosen this unrealistic value to sharpen our illustrative comparison of the two approaches.)

The first plot (Fig. 1) shows the ratio of the propagator to the free propagator as a function of the euclidean momentum squared in units of the pole mass squared. Physically, it is a "K-factor" for an amplitude dominated by an exchange of a scalar particle that self interacts with the ϕ^4 interaction.

For the minimal subtractions, the normalization of the field and the dimensional regularization mass unit were tuned to make the inverse propagator equal to $2M^2$, and the derivative of the inverse propagator in Q^2 equal to unity at the point $Q^2 = M^2$. (This means that M is the pole mass of the scalar particle.) The same initial conditions were imposed on the inverse propagator when the new evolution equation was used. We see on Fig. 1 that the K-factor computed with the minimal subtractions is larger than the one computed with the new approach. So, minimal subtractions overestimate the corrections to the tree amplitude dominated by the exchange of the scalar particle.

What may be more interesting is the qualitative difference in the momentum dependence of the two curves. While the minimal subtraction K-factor slowly grows at large Q^2 as $(Q^2)^{\gamma_\phi}$, the new result grows almost linearly at large Q^2.

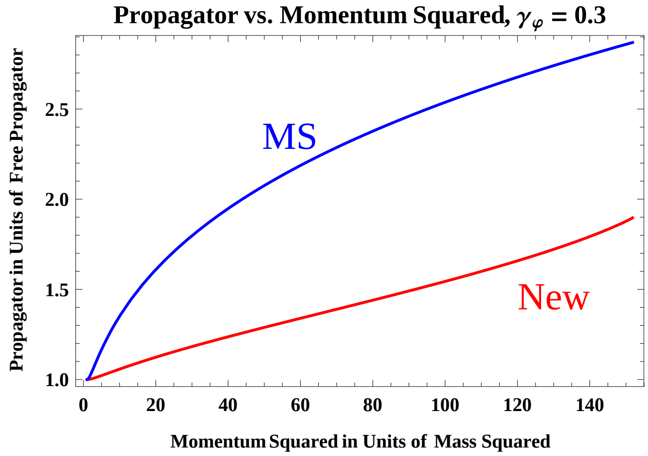

Figure 1: Dependence of scalar propagator on momentum

This qualitative difference between the results of the two approaches is more prominent on the second plot, Fig. 2. Here we plotted the running mass squared in units of the euclidean normalization point for three cases: the free theory, the minimal subtractions, and the new approach. We have three different ultraviolet asymptotics for the running mass. For the free theory, the mass in units of the normalization point goes to zero; for minimal subtractions it goes to the constant $\gamma_\phi/(1-\gamma_\phi)$; and for the new approach it goes to infinity.

Let us comment on what is the running mass plotted on Fig. 2. Consider the inverse propagator $R(Q^2)$ near some value Q_0^2 of the momentum squared. Expand it in power series around Q_0^2 up to the term linear in $(Q^2 - Q_0^2)$. Rescale the inverse propagator by the derivative $R'(Q_0^2)$ to achieve the canonical normalization of the Q^2-term. (The overall normalization of the inverse propagator is defined by the normalization of the filed and is in our disposal.) The inverse propagator becomes $Q^2 + M^2(Q_0^2)$, where $M^2(Q_0^2) \equiv R(Q_0^2)/R'(Q_0^2) - Q_0^2$ is the running mass depending on the normalization point Q_0^2. On Fig. 2, we plotted $M^2(Q^2)/Q^2$ as a function of Q^2/M^2, where M^2 is the pole mass of the scalar particle. (For the free theory, $R(Q^2) = Q^2 + M^2$, and $M^2(Q^2)/Q^2 = M^2/Q^2$, because the running mass coincides with the pole mass in this case.)

The running mass is an observable. We conclude that the new approach may lead to observable new predictions for the processes involving scalar particles.

Let us now characterize the new approach in more details. In most general terms, it is a particular realization of the program discussed, for example, in [8]. It was pointed out in this paper that the complications related to regularizing a quantum field theory are after all only of a technical nature. The infinites disappear (for renormalizable theories) if one expresses the predictions of a theory in terms of a finite set of observables. If so, it should be possible to avoid infinites altogether. It was demonstrated in [8] that it is indeed possible, at least, within perturbation theory.

Such "renormalization without infinities" has obvious advantages (there is no need in any regularization, etc.). Its disadvantage is also obvious: it is technically inconvenient (at least, in the version of [8]). The elegant methods involving dimensional regularization have been developed to a high degree of sophistication, and real perturbative computations are performed exclusively with dimensional regularization and minimal subtractions.

It is not clear at the moment if the approach of [1] that we advertise here overcomes the technical inconvenience of the previous attempts of renormalizing without infinities. But we stress that the new approach has yielded the new results about propagators of scalar fields, as discussed above. These results disagree qualitatively with the results yielded by the minimal subtractions. And, because the minimal subtractions ignore the singularity of Eq. (3), they

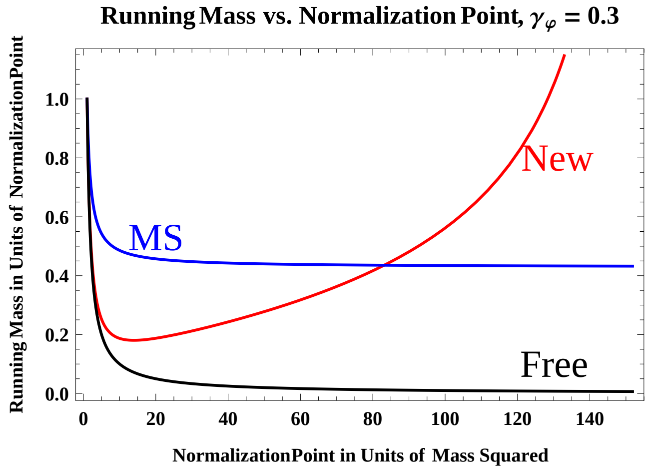

Figure 2: Dependence of running mass on normalization point

cannot be used to disprove the new results.

What singles the new approach out from the previous attempts of renormalizing without infinities is the particular choice of the finite set of observables parameterizing the theory. They are quantities contained in the connected amplitudes of the theory, while in the previous attempts the parameters of the theory were related to the one-particle irreducible diagrams.

Specifically, let $W(J)$ be the generating functional of connected Green functions of the theory, and let $\bar{W}(J)$ be $W(J)$ with the part quadratic in the sources omitted (it is assumed that the fields are counted from their vacuum values, which implies that the expansion of $\bar{W}$ in powers of the sources starts from cubic terms). So, $W(J) = JDJ/2 + \bar{W}(J)$, where D is the propagator. The key object for the new approach is the generating functional of connected amplitudes defined as follows: $V(\phi) \equiv \bar{W}(R\phi)$, where R is the inverse propagator, $R \equiv D^{-1}$. The parameters used in the new approach to parameterize the theory are contained in the inverse propagator, R, and in the vertex functional $V(\phi)$.

These parameters are extracted from R and $V(\phi)$ with an operation P_μ. This operation acts on any functional of the fields and yields a local functional with couplings of nonnegative dimensions. The projector P_μ plays in the new approach the role played by the bare action $I_B(\phi)$ in the conventional approach. Namely, P_μ defines the model in the new approach. The connection between the new approach and the conventional one is established by the condition $P_\mu I_B(\phi) = I_B(\phi)$. There is a considerable arbitrariness in the choice of P_μ. This arbitrariness is parameterized with the normalization point μ. (It is a point in a multidimensional space).

The finite parameters that are used by the new approach to parameterize the theory are the couplings involved in the so called normalized action defined as follows: $I_\mu(\phi) \equiv P_\mu(-\phi R\phi/2 + V(\phi))$. Like bare action, the normalized action I_μ is a local functional whose couplings have nonnegative dimensions. So, it is similar to the bare action of the theory with an important distinction: the normalized action is a finite functional.

The purpose of the theory is to determine the propagator (or, equivalently, the inverse propagator R) and the vertex functional $V(\phi)$. It turns out that it is possible to derive a new equation for these objects that does not involve the bare action. The new equation reads

$$(1 - P_\mu)(-\phi\frac{R}{2}\phi + T_c\, e^V) = 0, \tag{4}$$

where T_c is an operation similar to T-product. (The subscript "c" stands for "connected"; it recalls that T_c suppresses the disconnected diagrams. Definition of T_c involves the propagator D; for more details see [1].) Because this equation does not involve the bare action, we call it *the inaction equation*, and the new approach based on (4), *the inaction approach.*

It turns out that if the normalized action I_μ is given, the inaction equation (4) suffices to determine both R and V as a power series in powers of $V_\mu \equiv P_\mu V(\phi)$. This perturbation theory expands the finite amplitudes in powers of the finite couplings involved in V_μ. No infinites appear in the process.

The inaction approach to perturbative quantum field theory is based on the inaction equation (4) and the requirement that the inverse propagator R and the vertex functional $V(\phi)$ do not depend on the normalization point. This requirement is expressed as follows:

$$\frac{\partial P_\mu}{\partial \mu}(-\phi\frac{R}{2}\phi + T_c\, e^V) = 0. \tag{5}$$

To derive this equation from the inaction equation (4), take the derivative of the inaction equation in μ, and keep in mind that R and $V(\phi)$ do not depend on the normalization point.

The equation (5) determines the dependence of the normalized action $I_\mu(\phi)$ on the normalization point μ. This dependence, if determined by (5), guarantees that the inverse propagator R and vertex functional $V(\phi)$ determined with the inaction equation (4) by a given $I_\mu(\phi)$ do not depend on μ. In this respect, the equation (5) is similar to the renormalization group equations of the conventional approach.

At a particular choice of P_μ, the renormalization group equation (5) determines the dependence on the momentum squared of the inverse propagator $R(Q^2)$. If we apply this consideration to ϕ^4-theory, we obtain the following equation:

$$R'' = -\frac{8\gamma_\phi}{(R')^3 Q^2}\int_0^\infty J_3(x)[mK_1(mx)]^3 x\, dx + \ldots, \tag{6}$$

where $\gamma_\phi \equiv g^2/(12(16\pi^2)^2)$, $m^2 \equiv R/(R'Q^2) - 1$, the primes denote the derivatives in Q^2, the dots denote higher order corrections, J_3 is the Bessel function of the third order, and K_1 is the modified Bessel function of the first order. (Here g is the coupling defined as the four-particle connected amplitude taken at some external momenta; for details see [1].)

To study the solution of (6), let us rewrite the second order differential equation (6) as a pair of coupled first order equations. As discussed in [1], convenient variables are $m^2(Q^2) \equiv \big(R/(Q^2R') - 1\big)$ and $n(Q^2) \equiv (R')^4$. Also it is convenient to use as evolution parameter a log of Q^2, $t = \log(Q^2/M^2)$. It is easy to check that (6) is equivalent to the following pair of the first order equations:

$$\frac{d}{dt}\, m^2 = -m^2 + \frac{\gamma_\phi}{n}\,(1+m^2)\,\Phi(m), \tag{7}$$

$$\frac{d}{dt}\, n = -4\gamma_\phi\,\Phi(m), \tag{8}$$

where

$$\Phi(m) \equiv 8\int_0^\infty J_3(x)\,[mK_1(mx)]^3\, x\, dx \tag{9}$$

This evolution system should be given the initial conditions. We take that $m^2(0) = 1$ and $n(0) = 1$, which implies that M in the definition of the evolution parameter is the physical (pole) mass.

Next step is to study properties of function $\Phi(m)$ involved in (7) and (8). For our purposes, we use the approximation

$$\Phi(m) \approx \frac{0.3609}{6\, m^2 + 0.3609}. \tag{10}$$

(See [1] for details.)

Now we are ready to study the dynamical system (7), (8).

First, we notice that nothing interesting happens at momenta smaller than M. The terms in the right-hand-sides of (7), (8) involving γ_ϕ can be safely neglected. This is the case because

m^2 is growing at decreasing negative t and the derivative of n is decreasing in absolute value, while n itself is slightly growing with momentum decreasing. Numerical experiments confirm this observation. At low momenta inverse propagator $R(Q^2)$ is very close to the free inverse propagator $Q^2 + M^2$.

The situation at momentum growing larger than M is much more interesting. The key point is that there are two competing terms in the right-hand-side of (7). The first term is negative and decreasing, the second term is positive and increasing. They will inevitably cancel each other at a certain value of Q^2. At this Q^2 the derivative of mass squared in t vanishes and mass stays approximately constant. This happens around the point $M^2/Q^2 = \gamma_\phi$. Here the minimal value of the running mass is reached, $m^2_{min} \approx \gamma_\phi$. At larger momentum the derivative of mass becomes positive. The running mass in units of normalization point starts to grow in contradiction with the dimensional analysis! The normalization factor n is slowly decreasing around this point: $n = 1 - 4\gamma_\phi \log(Q^2/M^2)$. This decrease causes extra growth in m^2. At this stage, $m^2 \approx \gamma_\phi/n$. We conclude that at this stage the running mass can be expressed in terms of the anomalous dimension of the field:

$$M^2(Q^2) \approx \frac{\gamma_\phi \, Q^2}{1 - 4\gamma_\phi \log(Q^2/M^2)}, \tag{11}$$

which describes the growth of the running mass squared in units of the normalization momentum squared (see Fig. 2). The range of validity of this equation is $M^2/\gamma_\phi < Q^2 \ll M^2 \exp\left(1/(4\gamma_\phi)\right)$. When Q^2 grows beyond this range, perturbation theory becomes unreliable.

To summarize, applying the inaction approach to ϕ^4 theory we discovered a new phenomenon: For scalar field the running mass in units of the normalization point is not a monotonous function of the normalization point. It has a minimum at a certain value of the normalization point.

May this new phenomenon be of importance for the LHC era? Within ϕ^4 the discussed phenomenon is a two-loop effect. The momentum needed to reach the minimum of the running mass is $\left(1/\sqrt{\gamma_\phi}\right)$ times larger than the physical mass of the scalar particle. If we take that the self-coupling of the scalar field is close to unit, we obtain that we need to reach the momentum which is $16\pi^2\sqrt{12} \approx 174$ times larger than the physical mass, which is not very promising. Within the standard model, there is a contribution to the anomalous dimension of the scalar field already in the one-loop approximation. Optimistically, the momentum needed to observe the unnaturalness of the scalar field may be only $4\pi \approx 12.6$ times larger than the scalar mass. We conclude that an application of the considered formalism to the standard model is a necessary objective.

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