Interpreting multiple dualities conjectured from superconformal index identities

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60th October Anniversary Prospect, 7a, 117312 Moscow, Russia

Abstract

We consider field theory side of new multiple Seiberg dualities conjectured within superconformal index matching approach. We study the case of $SU(2)$ supersymmetric QCD and find that the numerous conjectured duals are different faces of handful of master theories. These different faces are inequivalent to each other in a very peculiar sense. We confirm that all index identities correspond to theories flowing to one and the same theory in the infrared, thus supporting the conjecture of index matching for Seiberg dual theories.

1 Introduction and summary

New method of exploring Seiberg dualities has been suggested recently. In Refs. [1, 2], the generalisation of the Witten index for superconformal field theories was introduced, and it was conjectured that the indices of theories related by Seiberg duality should coincide [3]. The coincidence was checked for several known dual theories [4, 5]. The index is a character of a relevant representation of certain subgroup of superconformal group and counts ground states invariant under the action of a particular supercharge. Thus, the conjecture seems natural to hold for models flowing to the same infrared conformal fixed point.

Superconformal indices for gauge theories are given in terms of elliptic hypergeometric integrals, and duality relations correspond to their highly non-trivial transformation properties [6, 7]. Thus, the coincidence of indices is new independent argument in favour of duality conjecture. Comprehensive list of dualities and corresponding relations for elliptic hypergeometric integrals, as well as introduction to this recently emergent branch of special function theory and its relation to Seiberg dualities can be found in Ref. [8].

On the other hand, known transformation properties of elliptic hypergeometric integrals lead to conjectures of new dualities between supersymmetric gauge theories [5, 8]. Although this method provides only field content of conjectured duals, it must be possible to construct complete field theories including their superpotentials. A remarkable feature of the superconformal index approach is that it suggests a multiplicity of duals to a single "electric" theory. By making use of this approach, Spiridonov and Vartanov [5] (SV in what follows) have recently conjectured 71 dual descriptions for supersymmetric QCD with $N_c = 2$ colours and $N_f = 4$ quark flavours whereas only three were known before that. For the SQCD with $N_f = 3$ flavours, they have suggested 35 dual gauge theories. In the latter case, the low energy description in terms of non-gauge theory was found in Ref. [9], but no non-trivial dual gauge theory was known.

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We studied the phenomenon of multiple duals. We consider duals conjectured by SV for $N_c = 2$, $N_f = 4, 3$ SQCD. This theory has three "master" magnetic descriptions with different field contents and superpotentials [11, 12, 13]. Although the original electric theory has global symmetry group $SU(8)$, two of its duals have lower symmetries. The latter symmetries get enhanced in the infrared, where the full $SU(8)$ is restored [10]. Hence, part of $SU(8)$ global symmetry is accidental in magnetic descriptions.

We find that the relationship between theories behind the superconformal index identities can be called duality with reservations. There are two types of extra dualities. The first one is inherent in electric theories whose duals have enhanced accidental symmetries in the infrared (for the discussion of enhanced symmetries in the Seiberg duality context see, e.g., Ref. [10]). Per se, these electric theories have only a few "master" duals. Dualities proliferate once some flavour current is coupled to external gauge field. Multiple dual theories then have the same field content, but differ by the structure of currents coupled to the external field.

Dualities of the second type are obtained from known ones by introducing a relevant operator into superpotential. This leads to new infrared behaviour of both electric and magnetic theories obtained by integrating out some heavy fields or after symmetry breaking. As the theories flow to the infrared, they can pass through intermediate stages; some of these intermediate theories are captured by the superconformal index approach. These theories are proper duals in the sense that they have the same infrared description, but usually they are not considered as new non-trivial independent dualities.

On the positive side, the superconformal index identities are in remarkable correspondence with field theory dualities understood in the above extended way. Field theories related by these identities do flow to the same infrared theory, and all index identities have their field theory counterparts. Thus, our study can be considered as a check of the conjecture that superconformal indices of Seiberg dual theories do match and contain important group-theoretical information on the structure of a theory.

The $N_c = 2$, $N_f = 4$ SQCD theory has three "master" magnetic descriptions with different field contents and superpotentials [11, 12, 13]. Although the original electric theory has global symmetry group $SU(8)$, two of its duals have lower symmetries. The latter symmetries get enhanced in the infrared, where the full $SU(8)$ is restored [10]. Hence, part of $SU(8)$ global symmetry is accidental in magnetic descriptions.

Since part of $SU(8)$ is accidental in the two magnetic theories, there is an ambiguity in identifying the operators of the electric theory and their magnetic counterparts. The ambiguity becomes physical when the currents corresponding to Cartan generators of $SU(8)$ are coupled to external gauge fields. Numerous magnetic theories in external gauge fields obtained in this way are inequivalent, and become identical in the infrared only. This construction is in one-to-one correspondence with SV counting based on superconformal index identities, hence giving the interpretation of multiple SV dualities in $N_f = 4$ theory.

2 $N_c = 2$, $N_f = 4$ SQCD and its duals

Let us recall known properties of supersymmetric QCD with $SU(2)$ gauge group and $N_f = 4$ flavours of quarks, Q^i in $\mathbf{2}$ representation and $\tilde{Q}_{\tilde{j}}$ in $\overline{\mathbf{2}}$ representation ($i, \tilde{j} = 1, 2, 3, 4$). Its feature is that fundamental and antifundamental representations of the gauge group are equivalent, so "left" quarks Q and "right" quarks $\tilde{Q}$ are combined into one multiplet of $SU(8)$ flavour group.

The theory is believed to have non-trivial infrared fixed point [11]. It has at least three Seiberg duals, i.e., theories which flow to the same fixed point in the infrared. All of them have $SU(2)$ gauge group and at least $SU(4)_L \times SU(4)_R \times U(1)_B$ global symmetry. They differ by field content, field representations and superpotentials. The pattern of the dual theories can be understood by considering their moduli spaces.

The moduli space is parametrized by all possible gauge invariants (modulo classical relations

Field content	Quark gauge invariants	Superpotential	Moduli
Q^i $(\mathbf{4},\mathbf{1})^{+1}$ $\tilde{Q}_{\tilde{j}}$ $(\mathbf{1},\mathbf{4})^{-1}$	$B^{ij} \equiv Q^i \cdot Q^j$ $(\mathbf{6},\mathbf{1})^{+2}$ $\tilde{B}_{\tilde{i}\tilde{j}} \equiv \tilde{Q}_{\tilde{i}} \cdot \tilde{Q}_{\tilde{j}}$ $(\mathbf{1},\mathbf{6})^{-2}$ $M^i_{\tilde{j}} \equiv Q^i \cdot \tilde{Q}_{\tilde{j}}$ $(\mathbf{4},\mathbf{4})^0$		$\begin{pmatrix} B^{ij} & M^i_{\tilde{j}} \\ -M^j_{\tilde{i}} & \tilde{B}_{\tilde{i}\tilde{j}} \end{pmatrix}$
q^i $(\mathbf{4},\mathbf{1})^{-1}$ $\tilde{q}_{\tilde{j}}$ $(\mathbf{1},\mathbf{4})^{+1}$ B^{ij} $(\mathbf{6},\mathbf{1})^{+2}$ $\tilde{B}_{\tilde{i}\tilde{j}}$ $(\mathbf{1},\mathbf{6})^{-2}$	$C^{ij} \equiv q^i \cdot q^j$ $(\mathbf{6},\mathbf{1})^{-2}$ $\tilde{C}_{\tilde{i}\tilde{j}} \equiv \tilde{q}_{\tilde{i}} \cdot \tilde{q}_{\tilde{j}}$ $(\mathbf{1},\mathbf{6})^{+2}$ $N^i_{\tilde{j}} \equiv q^i \cdot \tilde{q}_{\tilde{j}}$ $(\mathbf{4},\mathbf{4})^0$	$\frac{1}{4\mu}\epsilon_{ijkl}B^{ij}\, q^k \cdot q^l +$ $\frac{1}{4\mu}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\tilde{B}_{\tilde{i}\tilde{j}}\, \tilde{q}_{\tilde{k}} \cdot \tilde{q}_{\tilde{l}}$	$\begin{pmatrix} B^{ij} & N^i_{\tilde{j}} \\ -N^j_{\tilde{i}} & \tilde{B}_{\tilde{i}\tilde{j}} \end{pmatrix}$
q_i $(\bar{\mathbf{4}},\mathbf{1})^{+1}$ $\tilde{q}^{\tilde{j}}$ $(\mathbf{1},\bar{\mathbf{4}})^{-1}$ $M^i_{\tilde{j}}$ $(\mathbf{4},\mathbf{4})^0$	$C_{ij} \equiv q_i \cdot q_j$ $(\bar{\mathbf{6}},\mathbf{1})^{+2}$ $\tilde{C}^{\tilde{i}\tilde{j}} \equiv \tilde{q}^{\tilde{i}} \cdot \tilde{q}^{\tilde{j}}$ $(\mathbf{1},\bar{\mathbf{6}})^{-2}$ $N_i^{\tilde{j}} \equiv q_i \cdot \tilde{q}^{\tilde{j}}$ $(\bar{\mathbf{4}},\bar{\mathbf{4}})^0$	$\frac{1}{\mu}M^i_{\tilde{j}}\, q_i \cdot \tilde{q}^{\tilde{l}}$	$\begin{pmatrix} \epsilon^{ijkl}C_{kl} & M^i_{\tilde{j}} \\ -M^j_{\tilde{i}} & \epsilon_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\tilde{C}^{\tilde{k}\tilde{l}} \end{pmatrix}$
q_i $(\bar{\mathbf{4}},\mathbf{1})^{-1}$ $\tilde{q}^{\tilde{j}}$ $(\mathbf{1},\bar{\mathbf{4}})^{+1}$ B^{ij} $(\mathbf{6},\mathbf{1})^{+2}$ $\tilde{B}_{\tilde{i}\tilde{j}}$ $(\mathbf{1},\mathbf{6})^{-2}$ $M^i_{\tilde{j}}$ $(\mathbf{4},\mathbf{4})^0$	$C_{ij} \equiv q_i \cdot q_j$ $(\bar{\mathbf{6}},\mathbf{1})^{-2}$ $\tilde{C}^{\tilde{i}\tilde{j}} \equiv \tilde{q}^{\tilde{i}} \cdot \tilde{q}^{\tilde{j}}$ $(\mathbf{1},\bar{\mathbf{6}})^{+2}$ $N_i^{\tilde{j}} \equiv q_i \cdot \tilde{q}^{\tilde{j}}$ $(\bar{\mathbf{4}},\bar{\mathbf{4}})^0$	$\frac{1}{\mu}M^i_{\tilde{j}}\, q_i \cdot \tilde{q}^{\tilde{l}} +$ $\frac{1}{2\mu}B^{ij}\, q_i \cdot q_j +$ $\frac{1}{2\mu}\tilde{B}_{\tilde{i}\tilde{j}}\, \tilde{q}^{\tilde{i}} \cdot \tilde{q}^{\tilde{j}}$	$\begin{pmatrix} B^{ij} & M^i_{\tilde{j}} \\ -M^j_{\tilde{i}} & \tilde{B}_{\tilde{i}\tilde{j}} \end{pmatrix}$

Table 1: $N_c = 2$, $N_f = 4$ supersymmetric QCD and its duals.

implied by their definitions) giving extremum to the superpotential. In original electric theory, there is no superpotential and moduli space is spanned by expectation values of gauge invariants constructed from quarks. These are mesons $M^i_{\tilde{j}} = Q^i \cdot \tilde{Q}_{\tilde{j}}$, baryons $B^{ij} = Q^i \cdot Q^j$ and antibaryons $\tilde{B}_{\tilde{i}\tilde{j}} = \tilde{Q}_{\tilde{i}} \cdot \tilde{Q}_{\tilde{j}}$. Because of enhanced flavour symmetry, they form together antisymmetric tensor representation of $SU(8)$. Thus, all other descriptions of this infrared fixed point must have the same moduli space. In case of smaller flavour group, additional symmetry should accidentally emerge in the infrared.

Electric SQCD and all three duals are described in Table 1, with the representations of the common global symmetry group $SU(4)_L \times SU(4)_R \times U(1)_B$. The original theory and the third (last) dual possess $SU(8)$ flavour symmetry. For the first two duals, it is impossible to arrange elementary fields in the $SU(8)$ multiplets, and flavour $SU(8)$ emerges accidentally in the infrared.

The tree duals are constructed by introducing to SQCD some of the moduli of electric theory as elementary fields. Then one has to get rid of similar composite moduli in order to restore the proper moduli space structure. To this end, a superpotential is introduced. Representations of magnetic quarks q, $\tilde{q}$ are then completely fixed by demanding the coincidence of moduli spaces. In order to match elementary scalars to electric composites of canonical dimension 2, a new energy scale μ is introduced.

The first dual description in our table was considered by Csáki et al. [12] and contains baryons B^{ij} and antibaryons $\tilde{B}_{\tilde{i}\tilde{j}}$ of electric QCD as elementary fields. The presence of these fields explicitly breaks $SU(8)$ symmetry down to $SU(4)_L \times SU(4)_R \times U(1)_B$. There is the superpotential, which gives the mass to composite magnetic baryons $C^{ij} \equiv q^i \cdot q^j$ and antibaryons $\tilde{C}_{\tilde{i}\tilde{j}} \equiv \tilde{q}_{\tilde{i}} \cdot \tilde{q}_{\tilde{j}}$. This superpotential is crucial to match moduli spaces. The moduli space of this dual theory is parametrized by $SU(8)$ antisymmetric tensor containing elementary baryons and composite mesons[1] $N^i_{\tilde{j}} \equiv q^i \cdot \tilde{q}_{\tilde{j}}$. With this identification of moduli spaces, one relates electric

[1]In electric theory, there are classical relations between mesons and baryons. In $SU(8)$ language, they state that the rank of the antisymmetric matrix parameterizing the moduli space does not exceed 2. In magnetic theory, the rank of the moduli matrix is constrained partly by the superpotential and partly by the fact that

gauge invariants to magnetic ones surviving in the infrared. Thus, one identifies baryons B^{ij} and antibaryons $\tilde{B}_{\tilde{i}\tilde{j}}$ present in both theories, and mesons $M^{i}_{\tilde{j}}$ of electric theory with magnetic ones $N^{i}_{\tilde{j}}$. Composite baryons C^{ij}, $\tilde{C}_{\tilde{i}\tilde{j}}$ are absent in the infrared because of the superpotential; they do not have electric counterparts.

The generalisation of this duality to higher rank gauge groups has been found recently by using superconformal index identities ($SU-SP$ series of [8]).

The second dual was considered in the original paper by Seiberg [11] as part of the series of dual theories with $SU(N_c)$ gauge groups. Electric mesons $M^{i}_{\tilde{j}}$ are introduced as elementary fields and, together with composite baryons $C_{ij} \equiv q_i \cdot q_j$ and antibaryons $\tilde{C}^{\tilde{i}\tilde{j}} \equiv \tilde{q}^{\tilde{i}} \cdot \tilde{q}^{\tilde{j}}$, they parametrize the moduli space. Superpotential takes care of now redundant composite mesons $N^{\tilde{j}}_{i} \equiv q_i \cdot \tilde{q}^{\tilde{j}}$. One identifies mesons $M^{i}_{\tilde{j}}$ of electric and dual theories, and electric baryons B^{ij}, $\tilde{B}_{\tilde{i}\tilde{j}}$ with the Hodge-duals of magnetic baryons $\epsilon^{ijkl}C_{kl}$ and $\epsilon_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\tilde{C}^{\tilde{k}\tilde{l}}$. Similarly to the first dual, the existence of elementary mesons explicitly breaks $SU(8)$ symmetry down to $SU(4)_L \times SU(4)_R \times U(1)_B$.

The third dual theory was proposed by Intriligator and Pouliot in [13], where they generalise the original Seiberg series of dual theories to $SP(N_c)$ gauge groups. The full set of electric colour singlets is added as fundamental fields, and all composite singlets are made massive by the superpotential. Thus, the moduli space is trivially the same as electric one, and all electric composites are identified with corresponding elementary fields in the dual theory. As the full set of SQCD gauge invariants fits in $SU(8)$ multiplet, this dual has $SU(8)$ global symmetry and does not exhibit accidental symmetry in the infrared.

3 Duality in the presence of the accidental symmetry. Matching with the index approach

The first and second duals of the previous Section possess only $SU(4)_L \times SU(4)_R \times U(1)_B$ global symmetry, which gets promoted in the infrared to the full $SU(8)$ of the electric theory. This is not in contradiction with the concept of Seiberg duality which relates different theories with the same *infrared* properties. The dual theories may have not only different gauge groups and field contents, but also different global symmetries.

It is worth mentioning that the 't Hooft anomaly matching conditions [14] are somewhat peculiar in this situation. As the flavour group of the magnetic theory is smaller than that of its infrared descendant, anomaly matching conditions in the magnetic theory apply to smaller set of global currents. In other words, not all anomaly relations of the electric theory have their magnetic counterparts, simply because the magnetic theory has smaller set of global currents.

Once the global groups of electric theory and its magnetic dual are different, there is an ambiguity in identifying the operators of the two theories. Operators related by global symmetry in electric theory may no longer have this property in magnetic theory; this ambiguity becomes irrelevant in the infrared only. Hence, a small deformation of the electric theory may have several duals emanating from one and the same undeformed magnetic master theory. As an example, as proposed in Ref. [10], one can modify electric and magnetic theories by introducing small terms into their superpotentials. These terms are related by duality correspondence, and it is expected that duality remains valid for modified theories. Above the infrared, this correspondence is ambiguous, and unique superpotential term in theory with larger symmetry corresponds to a family of inequivalent terms in dual theory.

Consider small mass term for a pair of quarks in electric theory of the previous Section. For definiteness, take it as $m\, Q^4 \cdot \tilde{Q}_{\tilde{4}}$. Its magnetic counterpart in the first dual theory, obtained according to the above default identification, is $m\, q^4 \cdot \tilde{q}_{\tilde{4}}$. However, the $SU(8)$ rotation of the mass term, innocent in electric theory, gives rise to terms $m\, B^{34}$, $m\, \tilde{B}_{\tilde{3}\tilde{4}}$ in the superpotential

baryon vev of rank greater than 2 leads to runaway vacuum.

of the magnetic theory, as well as their linear combinations with $m\, q^4 \cdot \tilde{q}_{\tilde{4}}$. These cannot be rotated back to $m\, q^4 \cdot \tilde{q}_{\tilde{4}}$ by $SU(4) \times SU(4) \times U(1)$ of the "magnetic theory". In the infrared, $q^4 \cdot \tilde{q}_{\tilde{4}}$ is replaced by magnetic meson $N^4_{\tilde{4}}$, and all mass terms in the dual theory are related by the accidental $SU(8)$.

Likewise, the modifications of the second dual theory in addition to default magnetic counterpart $m\, M^4_{\tilde{4}}$ contain also superpotential terms $m\, q_1 \cdot q_2$, $m\, \tilde{q}^{\tilde{1}} \cdot \tilde{q}^{\tilde{2}}$ and their linear combinations with $m\, M^4_{\tilde{4}}$. These cannot be related to each other by $SU(4) \times SU(4) \times U(1)$ transformations.

We see that once introduced, the mass term leads to a family of inequivalent dual theories emanating from one master theory. In this sense, $N_c = 2$, $N_f = 4$ SQCD *with small mass term* for quark flavour has two continuous families of dual descriptions, namely, the first and second magnetic duals with inequivalent families of terms in the superpotentials.

Our point is that it is this kind of multiplicity that has been found by SV using superconformal index matching. For $N_c = 2$, $N_f = 4$ SQCD, the matching approach suggests that there are 72 theories dual to each other. These include the original electric theory, the third dual from our table and two sets of 35 dual theories corresponding to the first and second magnetic duals. Duals in each set have identical field content and Lagrangian, namely, those of the first or second magnetic theories. The only difference between them is the way the anomalies match with electric theory, i.e., the way $SU(4)_L \times SU(4)_R \times U(1)_B$ is embedded into $SU(8)$. This part of multiplicity is precisely due to the ambiguity in relating operators in dual theories.

As they stand, all 35 duals are equivalent to their master theory. To make this multiplicity physical, one deforms the electric theory. Instead of using $SU(8)$ breaking terms in superpotential as proposed in [10], we introduce external gauge field that weakly couples to global current of the Cartan subgroup of $SU(8)$. Then the family of dualities found by SV in the superconformal index context are dualities between the theories *in the external field* coupled to inequivalent global currents.

One should note that that $SU(4)_L \times SU(4)_R \times U(1)_B$ of the first and second magnetic theories is embedded in electric $SU(8)$ in non-trivial way. Remarkably, the mapping between generators of magnetic and electric flavour groups are given exactly by the transformations acting on arguments of superconformal indices that lead to the duality identities found by SV . The arguments of index are precisely the global group elements, parametrized by eigenvalues of their matrix representation. It turns out that these transformations together with permutations of quarks generate the Weyl group $W(E_7)$ of the exceptional root system E_7, which defines transformational properties of hyperelliptic integrals that give indices of dual theories.

By coupling different Cartan generators of the flavour group to the external fields one obtains the two complete sets of 35 dual theories suggested by SV. They correspond to $\frac{1}{2}C_8^4$ splittings of eight eigenvalues into two groups acting on left and right magnetic quarks, respectively, modulo interchanging the notions of left and right. Remarkably, there is one-to-one correspondence between this procedure and the index approach.

We see that multiplicity of dual descriptions suggested by superconformal index matching reveals some new aspects of duality. We have found that most of 72 dual theories suggested by superconformal index matching can be considered independent only in quite unconventional sense, namely, by coupling different global currents of the same theory to external field. Without this additional construction, there are only four theories dual to each other. It is worth noting, though, that several aspects of multiple duality, like the rôle of $W(E_7)$ transformations that somehow relate all four duals, remain unclear.

4 Acknowledgments

The author is indebted to S. Demidov, D. Gorbunov, V. Spiridonov, V. Vartanov and V. Rubakov for useful discussions. The work has been supported in part by the Russian Foundation for Basic Research grant 08-02-00473a, by the grants of the President of the Russian Federation

MK-4317.2009.2, NS-1616.2008.2 (government contract 02.740.11.0244), by FAE program (government contract P520), by the 'MassTeV' ERC advanced grant 226371, and by the 'Dynasty' foundation.

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