A simple way to take into account back reaction on pair creation

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Abstract

In this talk we propose a simple and systematic way of accounting for the back reaction on the background field due to the pair creation in four-dimensional scalr QED. In the case of QED with instantly switched on constant electric field background we obtain a remarkably simple answer.

1 Introduction

First of all I would like to thank the oranisers for this conference for the opportunity to give a talk at such representive meeting. Today I will try to propose you a simple way to take into account backreaction on pair creation.

Before one begin to study some problem, one should ask himself, why it is important to find a solution of this problem (or at least to make a step towards the solution). Fortunately, there are a lot of reasons for study the problem of back reaction on classical background. First of all it is importance of this problem for high intensity field dynamics, for Black Hole physics, for modern cosmology. Also there is usually an expectation of manifestation of new physics, especially in the gravity context of the problem. And the last but not least reason, there is no simple and systematic way of consideration of this porblem wich finally lead to the analytical answer.

2 Strategy

If there are enoight important reasons for you to try to solve this problem, then the next step you shoud do is to formulate this prblem in a solvable way. And this is exactly what we are going to do. First of all, you should take the simplest but informative example of a system, in our case it is a scalar QED on some electromagnetic background. But, for to obtain a solvable model we should also make a several assumptions about the initial moment of time $t = 0$.

We suppose that at this initial moment of time a constant everywhere in space electric field was created, so we consider the problem in a whole 3-dimensional space, and by doing this we neglect all the boundary effects.

Also we suppose that there are no electrically charged particles present on top of the field at the initial moment of time. Technically it means that we impose the following boundary conditions on our scalar field

$$\phi(x)|_{t=0} = 0 \tag{1}$$

But conceptually it means that we consider our problem in $t > 0$ half of $\mathcal{R}^{3,1}$. And this is crucial assumption for our approach.

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And finally we suppose that exactly at this initial moment of time one turn on the interaction between electromagnetic and scalar fields and the pair creation begins.

In the context of such setup we address the following question: What is the deacay rate of the electric field?

After we asked the major question, it is a good time to discuss the way how we will try to find an answer. The common strategy for back reaction type problems is to calculate the electromagnetic current, which is created by pairs, and to take into account the field due to this current. More concrete, one has to consider Heisenberg evolution of the operator of the electromagnetic current up to some moment of time t. Then one has to use it like a source in the RHS of the Maxwell equations. And after that the only thing which is remain to do is to solve this equation

$$\partial^\mu F_{\mu\nu}(x,t) = \left\langle in \middle| j_\nu(x,t) \middle| in \right\rangle \tag{2}$$

If we want to solve this equation we should simplify and then calculate it's RHS. This can be done by using some basic properties of scalar QED and all of the special features of our model.

3 Simplifications

If one recall the definition of the current for charged scalars

$$\begin{aligned} j_\nu(x) &= \phi^*(x)\left(\mathrm{ie}\overleftrightarrow{\partial}_\nu + 2e^2 A_\nu\right)\phi(x) = \\ &= \left\{\left[\mathrm{ie}\left(\frac{\partial}{\partial y^\nu} - \frac{\partial}{\partial z^\nu}\right) + 2e^2 A_\nu\right] \phi^*(y_\mu)\phi(z_\mu)\right\}_{y=z=x} \end{aligned} \tag{3}$$

then one could rewrite all through the in-in propagator of the theory

$$\left\langle in \middle| j_\nu(x,t) \middle| in \right\rangle = \left\{\left[\mathrm{ie}\left(\partial_{y^\nu} - \partial_{z^\nu}\right) + 2e^2 A_\nu\right] G^{hs}_{in-in}(y,z)\right\}_{y=z=x} \tag{4}$$

where

$$G^{hs}_{in-in}(y,z) = \left\langle in \middle| \phi^*(y)\phi(z) \middle| in \right\rangle \tag{5}$$

is the in-in propagator in the background electric field on the half of $R^{3,1}$ ($t \in [0,+\infty)$)

3.1 relationships of propagators

At the present moment our central equation reduced to the following form

$$\partial^\mu F_{\mu\nu}(x) = \left\{\left[\mathrm{ie}\left(\partial_{y^\nu} - \partial_{z^\nu}\right) + 2e^2 A_\nu\right] G^{hs}_{in-in}(y,z)\right\}_{y=z=x} \tag{6}$$

For further simplifications let's express this *in–in propagator for the half of* $R^{3,1}$ through the *in–out propagator for the full* $R^{3,1}$. This can be done in two basic steps:

- From the consideration of Bogolubov transformation one can obtain, that for the full $R^{3,1}$ space

$$G_{in-in}(z,y) = 2\mathrm{Re}\left[G_{out-in}(z,y)\right] \tag{7}$$

- Using *mirror sources* one can connect propagators for full space and for half space

$$G^{hs}_{out-in}(z,y) = G_{out-in}(z,y) - G_{out-in}(z,\overline{y}) \tag{8}$$

where $\overline{y} = (-y_0, \vec{y})$ is the position of the mirror source

Summarize all above transformations and using Euclidean reformulation one arrived to the following already comfortable for calculations version of our central equation

$$\begin{aligned}\partial_\mu F_{\mu\nu} &= -2\left[\mathrm{i}e\left(\frac{\partial}{\partial y^\nu}-\frac{\partial}{\partial z^\nu}\right)+2e^2A_\nu\right]\times \\ &\times\ \mathrm{Re}\left[\left\langle z\middle|\frac{1}{-D_\mu^2+m^2}\middle|y\right\rangle-\left\langle z\middle|\frac{1}{-D_\mu^2+m^2}\middle|\bar{y}\right\rangle\right]\Bigg|_{y=z=x}\end{aligned} \tag{9}$$

here is some refference information about the way how we will calculate propagators in given external field.

$$\begin{aligned}&G_{out-in}(z,y)=\left\langle z\middle|\frac{1}{-D_\mu^2+m^2}\middle|y\right\rangle= \\ &\left\langle z\middle|\int_0^\infty dTe^{-(-D_\mu^2+m^2)T}\middle|y\right\rangle=\int_0^\infty dTe^{-m^2T}\left\langle z\middle|e^{D_\mu^2T}\middle|y\right\rangle= \\ &=\int_0^\infty dTe^{-m^2T}\int_{x(0)=y;x(T)=z}\mathcal{D}x(\tau)\ e^{-\int_0^T(\frac{1}{4}\dot{x}^2+\mathrm{i}eA_\mu\dot{x}_\mu)d\tau}.\end{aligned} \tag{10}$$

4 Calculation

We will be even more specific in this section and consider all for the case of uniformally and almost constant electric field. We talk about *almost* constant electric field, because for back reaction type problems you need to hold timedependence of the field somewhere. Let's hold this timedependence in the LHS of our equation and put there this ansatz for electromagnetic field

$$F_{\mu\nu}=-\mathrm{i}E(x_0)\delta_{3[\mu}\delta_{\nu]0} \tag{11}$$

which correspond to the timepependent electric field. But in the RHS, i.e. into the propagators, let's put the ansatz $A_\mu = Ex_0\delta_{\mu 3}$ for constant field. And after simple we arrived to the following equation

$$\frac{dE}{dt}=-\mathrm{Re}\left(\mathrm{i}\frac{e^3E^2t}{8\pi^2}\int_0^\infty dTe^{-m^2T}\frac{1}{T\sin(eET)}\right). \tag{12}$$

And using the fact that the real part of the RHS is equal to the imaginary part of this integral

$$\mathrm{Im}\left(\int_0^\infty dTe^{-m^2T}\frac{1}{T\sin(eET)}\right)=-\ln\left(1+e^{-\frac{m^2\pi}{eE}}\right), \tag{13}$$

which is just sum of the residues. We finally arrived to the one loop answer for the decay rate of the background electric field in our approximation

$$\boxed{\frac{dE}{dt}=-\frac{e^3E^2t}{4\pi^2}\ln\left(1+e^{-\frac{m^2\pi}{eE}}\right)} \tag{14}$$

5 Semiclassical consideration

One could restore the leading approximation of our one loop result on the *general physical grounds*:
Energy of the electric field per unit volume $E^2/8\pi$ is spent on the work on the creation of pairs, which is proportional to $e\,E\,z = e\,E\,t$, here z is the separation distance between the members of the pair reached during the observation time t and also to the $w(E) \propto e^2\,E^2\,e^{-\frac{m^2\pi}{eE}}$ approximate Schwinger's pair creation probability rate per unit time and unit volume. Hence, one could write

$$\frac{d}{dt}E^2 \propto -2e\,E\,t\,w(E), \tag{15}$$

and obtain from here the leading approximation of our result

$$\frac{dE}{dt} \propto -2e^3 \, E^2 \, t \, e^{-\frac{m^2 \pi}{eE}} \tag{16}$$

6 Discussion

We can analize our result by considering two different limits. On can see that *weak* electric field $(eE << m^2)$ changes slowly in time, because in this limit $\ln\left(1 + e^{-\frac{m^2\pi}{eE}}\right) \to 0$ and therefore $dE(t)/dt \to 0$. But, In the *strong* field limit $(eE >> m^2)$ there will be a fast decay $E(t) \propto 1/t^2$. Of course, it's just a hint, because in the case of the overcritical field we can not apply our approximation