Null Energy Condition Violation and Classical Stability in the Bianchi I Metric

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Abstract

The stability of isotropic cosmological solutions in the Bianchi I model with the Null Energy Condition violation is considered. We prove that the stability of isotropic solutions in the Bianchi I metric for a positive Hubble parameter follows from their stability in the Friedmann–Robertson–Walker metric. We obtained the sufficient conditions of stability of the solutions tending to a fixed point in cases of one- and two-field cosmological models. These results are applied to models inspired by string field theory, which violate the null energy condition. Examples of stable isotropic solutions are presented.

1 Introduction

Field theories which violate the null energy condition (NEC) are of interest for the solution of the cosmological singularity problem [1, 2, 3] and for models of dark energy with the equation of state parameter $w < -1$ (see [4]–[14] and references therein). Generally speaking, models that violate the NEC have ghosts, and therefore are unstable and physically unacceptable.

However, the possibility of the existence of dark energy with $w < -1$ on the one hand[1] and the cosmological singularity problem on the other hand encourage the investigation of models which violate the NEC. It is almost clear that such a possibility can be realized within an effective theory, while the fundamental theory should be stable and admit quantization. From this point of view the NEC violation might be a property of a model that approximates the fundamental theory and describes some particular features of the fundamental theory. With the lack of quantum gravity, we can just trust string theory or deal with an effective theory admitting the UV completion.

There have been several attempts to realize these scenarios [18, 19, 20]. The ghost condensation model [18, 21, 22, 23] proposed to describe a wide class of cosmological perturbations has a ghost in the perturbative vacuum and has no ghost in the ghost condensation phase within an effective theory. The new ekpyrotic scenario [20, 24, 25, 26] is a development of the ekpyrotic [27] and the cyclic scenarios [2, 28], and it attempts to solve the singularity problem, among others, by involving violation of the NEC. Nonlocal cosmological models [19, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38] inspired by the string field theory (SFT) [39, 40, 41] admit a regime with $w < -1$.

The NEC violating models can admit classically stable solutions in the Friedmann–Robertson–Walker (FRW) cosmology. In particular, there are classically stable solutions for self-interacting ghost models with minimal coupling to gravity. Moreover, there exists an attractor behavior (for details about attractor solutions for inhomogeneous cosmological models, see [45]) in a class of the phantom cosmological models [46, 47, 48]. One can study the stability of the FRW metric,

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[1] This possibility is not excluded experimentally [15], see [16, 17] for reviews of dynamical dark energy models.

specifying a form of fluctuations. It is interesting to know whether these solutions are stable under the deformation of the FRW metric to an anisotropic one, for example, to the Bianchi I metric. In comparison with general fluctuations we can get an explicit form of solutions in the Bianchi I metric, which can probably clarify some nontrivial issues of theories with NEC violation.

Stability of isotropic solutions in the Bianchi models [49, 50, 51] (see also [52]) has been considered in inflationary models (see [53, 54] and references therein for details of anisotropic slow-roll inflation). Assuming that the energy conditions are satisfied, it has been proved that all initially expanding Bianchi models except type IX approach the de Sitter space-time [55] (see also [56, 57, 58, 59]). The Wald theorem [55] shows that for space-time of Bianchi types I–VIII with a positive cosmological constant and matter satisfying the dominant and strong energy conditions, solutions which exist globally in the future have certain asymptotic properties at $t\to\infty$. It is interesting to consider a similar question in the case of phantom cosmology and string inspired models [19, 30, 34, 36, 61, 62].

In our works we investigated classical stability of isotropic solutions in the Bianchi I metric in the presence of phantom scalar fields. In my report I'll tell you about the results of these investigations. The report is based on the works [63, 64].

2 The Bianchi I cosmological model with scalar and phantom scalar fields and the CDM

Let us start with a cosmological model with N scalar fields ϕ_1, ϕ_2, …, ϕ_N in the Bianchi I metric

$$ds^2 = -\,dt^2 + a_1^2(t)dx_1^2 + a_2^2(t)dx_2^2 + a_3^2(t)dx_3^2. \tag{1}$$

The action is

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \sum_{k=1}^{N}\frac{C_k}{2}g^{\mu\nu}\partial_\mu\phi_k\partial_\nu\phi_k - V(\phi_1,\dots,\phi_N) - \Lambda\right), \tag{2}$$

where the potential V is a twice continuously differentiable function, G_N is the Newtonian gravitational constant, Λ is a cosmological constant, and C_k are nonzero real numbers. The sign of C_k defines whether field ϕ_k is the phantom field ($C_k<0$) or the ordinary scalar field ($C_k>0$).

The Einstein equations have the following form:

$$H_1H_2 + H_1H_3 + H_2H_3 = 8\pi G_N\varrho, \tag{3}$$

$$\dot H_2 + H_2^2 + \dot H_3 + H_3^2 + H_2H_3 = -\,8\pi G_N p, \tag{4}$$

$$\dot H_1 + H_1^2 + \dot H_2 + H_2^2 + H_1H_2 = -\,8\pi G_N p, \tag{5}$$

$$\dot H_1 + H_1^2 + \dot H_3 + H_3^2 + H_1H_3 = -\,8\pi G_N p, \tag{6}$$

where

$$\varrho = \sum_{k=1}^{N}\frac{C_k}{2}\dot\phi_k^2 + V(\phi_1,\dots,\phi_N) + \Lambda + \rho_m, \tag{7}$$

$$p = \sum_{k=1}^{N}\frac{C_k}{2}\dot\phi_k^2 - V(\phi_1,\dots,\phi_N) - \Lambda, \tag{8}$$

$$H_1 = \frac{\dot a_1}{a_1},\qquad H_2 = \frac{\dot a_2}{a_2},\qquad H_3 = \frac{\dot a_3}{a_3} \tag{9}$$

and a dot denotes a time derivative.

Note that we couple, in a minimal way, pressureless matter (the CDM) with the energy density ρ_m to our model. The equation for the CDM energy density is as follows:

$$\dot{\rho}_m = -(H_1 + H_2 + H_3)\rho_m. \tag{10}$$

Introducing $\psi_k = \dot{\phi}_k$ we obtain from action (2) the following equations:

$$\begin{aligned} \dot{\phi}_k &= \psi_k, \\ \dot{\psi}_k &= -(H_1 + H_2 + H_3)\psi_k - \frac{1}{C_k} V'_{\phi_k}, \end{aligned} \tag{11}$$

where $V'_{\phi_k} \equiv \frac{\partial V}{\partial \phi_k}$, $k = 1, 2, \ldots, N$.

It is convenient to express the initial variables a_i in terms of new variables a and β_i (we use notations from [73]), subject to the following constraint:

$$\beta_1 + \beta_2 + \beta_3 = 0. \tag{12}$$

One has the following relations

$$a_i(t) = a(t)e^{\beta_i(t)}, \quad \text{hence,} \quad a(t) = (a_1(t)a_2(t)a_3(t))^{1/3}, \tag{13}$$

$$H_i \equiv H + \dot{\beta}_i, \qquad \text{and} \qquad H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{14}$$

where $H \equiv \dot{a}/a$. To obtain (14) we have used the following consequence of (12):

$$\dot{\beta}_1 + \dot{\beta}_2 + \dot{\beta}_3 = 0. \tag{15}$$

Note that β_i are not components of a vector and, therefore, are not subjected to the Einstein summation rule. In the case of the FRW metric all β_i are equal to zero and H is the Hubble parameter. Following [50, 73] (see also [52]) we introduce the shear

$$\sigma^2 \equiv \dot{\beta}_1^2 + \dot{\beta}_2^2 + \dot{\beta}_3^2. \tag{16}$$

It is useful to write equations (3)–(6), (10) and (11) in terms of new variables.

Using relation (15) we can write equation (3) as follows

$$3H^2 - \frac{1}{2}\sigma^2 = 8\pi G_N \varrho. \tag{17}$$

Summing equations (4)–(6) one can obtain

$$2\dot{H} + 3H^2 + \frac{1}{2}\sigma^2 = -8\pi G_N p. \tag{18}$$

Therefore

$$\dot{H} + 3H^2 = 4\pi G_N(\varrho - p). \tag{19}$$

Note that equations (10) and (11) in new variables,

$$\dot{\phi}_k = \psi_k, \qquad \dot{\psi}_k = -3H\psi_k - \frac{1}{C_k} V'_{\phi_k}, \tag{20}$$

$$\dot{\rho}_m = -3H\rho_m, \tag{21}$$

as well as equation (19), look like the corresponding equations in the FRW metric.

For evolution of the new variables we obtain the following equations

$$\ddot{\beta}_i = -3H\dot{\beta}_i, \tag{22}$$

$$\frac{d}{dt}\left(\sigma^2\right) = -6H\sigma^2. \tag{23}$$

3 A few known facts about stability

Let us remember a few facts about the stability [71, 72, 75] of solutions for a general system of the first order autonomic equations

$$\dot{y}_k = F_k(y), \qquad k = 1, 2, \ldots, N. \tag{24}$$

By definition a solution (a trajectory) $y_0(t)$ is attractive (stable) if

$$\|\tilde{y}(t) - y_0(t)\| \to 0 \quad \text{at} \quad t \to \infty \tag{25}$$

for all solutions $\tilde{y}(t)$ that start close enough to $y_0(t)$.

If all solutions of the dynamical system that start out near a fixed (equilibrium) point y_f,

$$F_k(y_f) = 0, \qquad k = 1, 2, \ldots, N \tag{26}$$

stay near y_f forever, then y_f is *a Lyapunov stable point*. If all solutions that start out near the equilibrium point y_f converge to y_f, then the fixed point y_f is *an asymptotically stable* one. Asymptotic stability of fixed point means that solutions that start close enough to the equilibrium not only remain close enough but also eventually converge to the equilibrium. A solution $y_0(t)$ of (24), which tends to the fixed point y_f, is attractive if and only if the point y_f is asymptotically stable.

The Lyapunov theorem [71, 72] states that to prove the stability of fixed point y_f of non-linear system (24) it is sufficient to prove the stability of this fixed point for the corresponding linearized system

$$\dot{y} = Ay, \qquad A_{ik} = \frac{\partial F_i(y)}{\partial y_k}|_{y=y_f}. \tag{27}$$

The stability of the linear system means that real parts of all solutions of the characteristic equation

$$\det\left(\frac{\partial F}{\partial y} - \lambda I\right)|_{y=y_f} = 0$$

are negative.

4 Stability of isolated fixed points and kink-type solutions in one-field models with the CDM

Let us consider the gravitational model with one scalar field ϕ and an arbitrary potential $V(\phi)$, described by action (2) at $N = 1$. Equations (19) and (42) for one-field models are as follows

$$\begin{aligned} \dot{H} &= -3H^2 + 8\pi G_N(V(\phi) + \Lambda), \\ \dot{\phi} &= \psi, \\ \dot{\psi} &= -3H\psi - \frac{1}{C}V'_{\phi}. \end{aligned} \tag{28}$$

This system of three first order equations is valid in the Bianchi I metric as well as in the FRW one. Different initial values of σ^2 in (39) specify these different cases.

Let us define

$$I = \frac{3}{8\pi G_N}H^2 - \frac{C}{2}\psi^2 - V(\phi) - \Lambda. \tag{29}$$

From system (28) it is follows that the function I should be a solution of the following equation:

$$\dot{I} = -6HI. \tag{30}$$

If the case $H(t) \equiv 0$ is excluded, then I is an integral of motion of (28) if and only if $I = 0$. From (39) we see that

$$I = \frac{1}{16\pi G_N}\sigma^2, \tag{31}$$

so, I is an integral of motion only at $\sigma^2 = 0$, i.e. in the FRW metric. From (30) and (31) it follows that equation (46) is a consequence of (28).

We are interested in the stability of kink and lump solutions, namely, we consider such solutions in which the Hubble parameter tends to a finite value at $t \to +\infty$. In this case $\phi(t)$ tends to a finite value as well. Thus, there exists a fixed point $y_f \equiv (H_f, \phi_f, \psi_f)$, which corresponds to $t = +\infty$. We consider the stability of isotropic solutions only, so $\sigma_f^2 = 0$ and $\dot{\beta}_{i_f} = 0$. It is easy to see that

$$\psi_f = 0, \qquad V'_\phi(\phi_f) = 0, \qquad H_f^2 = \frac{8}{3}\pi G_N\left(\Lambda + V(\phi_f)\right). \tag{32}$$

Having analyzed the solutions of the linearized system we came to the conclusion that the stability conditions for the fixed point and for the solution that tends to it are

$$\frac{V''_\varphi\left(\phi_f\right)}{C} > 0 \quad \text{and} \quad H_f > 0. \tag{33}$$

As we can see the NEC is not necessary here, these conditions must be satisfied.

If we introduce the CDM into our model then the result will be the same.

5 Connections between the first order corrections to isotropic solutions in the FRW and Bianchi I metrics

In the previous section we studied one-field models and the first corrections near a fixed point. In this section we consider the first corrections of an arbitrary isotropic solution.

Let's consider an N-field cosmological model, which is described by action (2) and the Einstein equations (4)–(11). In this section we do not assume that the isotropic solution tends to a fixed point. We do not prove the stability of solutions, we only analyse the first corrections in the FRW and Bianchi I metrics.

To study the stability of this solution, we present solutions whose initial conditions are close to the isotropic one, in the following form:

$$\begin{aligned}
H_i(t) &= H_0(t) + \varepsilon h_i(t) + \mathcal{O}(\varepsilon^2), && (34)\\
\phi_k(t) &= \phi_{0k}(t) + \varepsilon\varphi_k(t) + \mathcal{O}(\varepsilon^2), && (35)\\
\psi_k(t) &= \psi_{0k}(t) + \varepsilon\chi_k(t) + \mathcal{O}(\varepsilon^2), && (36)\\
\rho_m(t) &= \rho_{m0}(t) + \varepsilon\tilde{\rho}_m(t) + \mathcal{O}(\varepsilon^2), && (37)
\end{aligned}$$

where $i = 1, 2, 3$ and $k = 1, \ldots, N$.

Having analyzed the solutions of the system of equations for the corrections we obtained that the following result is valid.

Theorem 1

Let $H_0(t)$ be a smooth function bounded at all finite values of time and $\int\limits_0^\infty H_0(\tau)d\tau$ be bounded from below, in other words, this integral is equal to either a finite number or plus infinity. Functions $h_1(t)$, $h_2(t)$, $h_3(t)$, $\tilde{\rho}_m(t)$, and $\varphi_k(t)$, which are solutions of the linearized system of equations, are bounded if and only if isotropic solutions, namely, solutions, which satisfy the condition $h_1(t) = h_2(t) = h_3(t)$, are bounded.

Note that Theorem 1 connects the stability properties of the FRW and Bianchi I metrics not only for solutions which tend to a fixed point, but also for solutions which tend to infinity at $t \to \infty$.

6 Two-field cosmological models

Let's consider two-field cosmological models. Two-field models with the crossing of the cosmological constant barrier $w_{DE} = -1$ are known as quintom models and include one phantom scalar field and one ordinary scalar field. Quintom models are being actively studied at present time [61, 48]. We'll consider not not only quintom models, but also models with arbitrary nonzero constants before the kinetic terms.

The action for these models looks as follows

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \left(\frac{C_1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{C_2}{2}g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi - V(\phi,\xi)\right)\right), \qquad (38)$$

where the potential $V(\phi,\xi)$ is a twice continuously differentiable function, which can include the cosmological constant Λ, G_N is the Newtonian gravitational constant ($8\pi G_N = 1/M_P^2$, where M_P is the Planck mass,), ϕ and ξ are either scalar or phantom scalar fields in dependence on signs of constants C_1 and C_2.

The Einstein equations have the following form:

$$3H^2 - \frac{1}{2}\sigma^2 = 8\pi G_N\varrho. \qquad (39)$$

$$2\dot{H} + 3H^2 + \frac{1}{2}\sigma^2 = -8\pi G_N p. \qquad (40)$$

$$\dot{\phi} = \psi, \qquad \dot{\psi} = -3H\psi - \frac{1}{C_1}\frac{\partial V}{\partial\phi}, \qquad (41)$$

$$\dot{\xi} = \zeta, \qquad \dot{\zeta} = -3H\zeta - \frac{1}{C_2}\frac{\partial V}{\partial\xi}, \qquad (42)$$

where

$$\varrho = \frac{C_1}{2}\dot{\phi}^2 + \frac{C_2}{2}\dot{\xi}^2 + V(\phi,\xi), \qquad (43)$$

$$p = \frac{C_1}{2}\dot{\phi}^2 + \frac{C_2}{2}\dot{\xi}^2 - V(\phi,\xi). \qquad (44)$$

For β_i and σ^2 we obtain the following equations

$$\ddot{\beta}_i = -3H\dot{\beta}_i, \qquad (45)$$

$$\frac{d}{dt}\left(\sigma^2\right) = -6H\sigma^2. \qquad (46)$$

We assume that the fields ϕ and ξ tend to finite limits at $t \to +\infty$.

Summing equations (39) and (40) we obtain the following system

$$\begin{aligned}
&\dot{H} = -3H^2 + 8\pi G_N V(\phi,\xi),\\
&\dot{\phi} = \psi,\\
&\dot{\psi} = -3H\psi - \frac{1}{C_1}\frac{\partial V}{\partial\phi},\\
&\dot{\xi} = \zeta,\\
&\dot{\zeta} = -3H\zeta - \frac{1}{C_2}\frac{\partial V}{\partial\xi}.
\end{aligned} \qquad (47)$$

System (47) has a fixed point $y_f = (H_f, \phi_f, \psi_f, \xi_f, \zeta_f)$ if and only if

$$\begin{aligned} \psi_f &= 0, && (48) \\ \zeta_f &= 0, && (49) \\ V'_\phi &= 0, && (50) \\ V'_\xi &= 0, && (51) \\ H_f^2 &= \frac{8\pi G_N}{3} V(\phi_f, \xi_f), && (52) \end{aligned}$$

where $V'_\phi \equiv \frac{\partial V}{\partial \phi}(\phi_f, \xi_f)$ and $V'_\xi \equiv \frac{\partial V}{\partial \xi}(\phi_f, \xi_f)$.

All fixed points y_f correspond to $\psi_f = 0$ and $\zeta_f = 0$. We denote the fixed point $y_f = (H_f, \phi_f, \psi_f, 0, 0)$ as $y_f = (H_f, \phi_f, \psi_f)$. To analyse the stability of y_f we study the stability of this fixed point for the corresponding linearized system of equations and use the Lyapunov theorem.

Having analyzed the solutions of the linearized system we came to the conclusion that the stability conditions for the fixed point and the solution that tends to it are

$$H_f > 0, \qquad \frac{V''_{\xi\xi}}{C_2} + \frac{V''_{\phi\phi}}{C_1} > 0, \qquad \frac{V''_{\xi\xi} V''_{\phi\phi}}{C_1 C_2} > \frac{V''^{\,2}_{\phi\xi}}{C_1 C_2}. \tag{53}$$

We obtained that the NEC is not necessary here again.

If we introduce CDM the into our model then the result will be the same.

7 Examples of isotropic stable solutions in the SFT inspired models

As the examples of application of the results obtained by us let's consider some String Field Theory inspired cosmological models with stable exact solutions.

In the examples we use a dimensionless parameter $m_p^2 \sim M_p^2 = 1/(8\pi G_N)$. The coefficient of proportionality arises when we construct effective cosmological models from the original SFT action (see [60, 30, 36] for details). For convenience, we write the Einstein equations for the SFT inspired cosmological models in the following form:

$$\begin{aligned} \dot{H} &= -\frac{3}{2} H^2 - \frac{1}{2m_p^2}\left(\frac{C\psi^2}{2} - V(\phi) - \Lambda\right), \\ \dot{\phi} &= \psi, && (54) \\ \dot{\psi} &= -3H\psi - \frac{1}{C} V'_\phi(\phi). \end{aligned}$$

We also have

$$3m_p^2 H^2 - \frac{C}{2}\phi^2 - V(\phi) = \Lambda. \tag{55}$$

7.1 Model with a kink solution and the sixth degree potential

An exact solution to the Friedmann equations with a string inspired phantom scalar matter field has been constructed in [60] (see also [46]). The notable features of the model are a phantom sign of the kinetic term ($C = -1$) and a special polynomial form of the effective tachyon potential:

$$V(\phi) = \frac{1}{2}\left(1 - \phi^2\right)^2 + \frac{1}{12m_p^2}\,\phi^2\left(3 - \phi^2\right)^2. \tag{56}$$

Note that this potential has been used in the string gas cosmology [8].

System (54) has the following exact kink-type solution [60]:

$$\phi_0(t) = \tanh(t), \qquad H_0(t) = \frac{1}{2m_p^2}\tanh(t)\left(1-\frac{1}{3}\tanh(t)^2\right). \tag{57}$$

Let us analyse the stability of this solution. At $t \to \infty$ solution (57) tends to a fixed point,

$$H_f = \frac{1}{3m_p^2}, \qquad \phi_f = 1. \tag{58}$$

It is easy to see that

$$V'_\phi(1) = 0, \qquad V''_{\phi\phi}(1) = 2\left(2-\frac{1}{m_p^2}\right). \tag{59}$$

Using (33), we obtain that solution (57) is attractive in the Bianchi I metric at $m_p^2 < 1/2$. Note that this solution is stable with respect to small fluctuations of the initial value of the CDM energy density as well.

In [60] we have showed that the first corrections $\varphi(t)$ and $h(t)$ satisfy the following system:

$$\begin{aligned}
\dot{h} &= \frac{1}{m_p^2}\Big(1-\tanh(t)^2\Big)\dot{\varphi},\\
\dot{\varphi} &= \frac{\left(3-4m_p^2+4(m_p^2-1)\tanh(t)^2+\tanh(t)^4\right)\tanh(t)}{2m_p^2\left(1-\tanh(t)^2\right)}\varphi - \\
&- \frac{\left(3-\tanh(t)^2\right)\tanh(t)}{1-\tanh(t)^2}h,
\end{aligned} \tag{60}$$

and have the following explicit form:

$$\begin{aligned}
\varphi(t) &= 2m_p^2C_1\left(1-\tanh(t)^2\right) + \\
&+ 2m_p^2C_2\frac{2J(t)+(\cosh(2t)-1)(\cosh(t))^{2-\frac{1}{m_p^2}}e^{\left(\frac{1}{2m_p^2(\cosh(2t)+1)}\right)}}{\cosh(2t)+1},\\
h(t) &= C_1\left(1-\tanh(t)^2\right)^2 - \frac{4m_p^2C_2J(t)}{(\cosh(2t)+1)^2},
\end{aligned} \tag{61}$$

where C_1 and C_2 are arbitrary constants,

$$J(t) = \int_0^t \sinh(\tau)\,(\cosh(\tau))^{1-1/m_p^2}\left(2\left(2m_p^2-1\right)\cosh(\tau)^2-1\right)e^{\frac{1}{4m_p^2\cosh(\tau)^2}}\,d\tau.$$

It is easy to see that if $m_p^2 > 1/2$ then at $C_2 \neq 0$ the function $\varphi(t)$ tends to infinity as $t \to \infty$ and, therefore, solution (57) is not stable. At $m_p^2 = 1/2$ we obtain from (61) that

$$\begin{aligned}
h(t) &= \left(\tanh(t)^2-1\right)^2(C_1-C_2J_2),\\
\varphi(t) &= -\left(\tanh(t)^2-1\right)(C_1-C_2J_2) - \frac{1}{2}C_2e^{-\tanh(t)^2/2},
\end{aligned}$$

where $J_2 = \int_0^t e^{-\tanh(\tau)^2/2}\tanh(\tau)d\tau$. Thus, $\varphi(t)$ and $h(t)$ are bounded functions at $m_p^2 = 1/2$.

The functions h_i have the form

$$h_i(t) = h(t) + \tilde{C}_i\, e^{-\frac{\tanh^2(t)}{4m_p^2}}\left(1-\tanh^2(t)\right)^{1/\left(2m_p^2\right)}, \tag{62}$$

where $\tilde{C}_i$ are real constants, $i = 1,2,3$, which satisfy the following relation:

$$\tilde{C}_1 + \tilde{C}_2 + \tilde{C}_3 = 0. \tag{63}$$

We conclude that exact solutions obtained in [60] are stable in the Bianchi I metric at $m_p^2 < 1/2$ and unstable at $m_p^2 > 1/2$. The case of $m_p^2 = 1/2$ needs a more detailed analysis. The first corrections are bounded.

7.2 Model with a lump solution

In the previous subsection kink solutions were considered. In this subsection we consider the stability of a lump solution in the model [30] which is motivated by a description of D-brane decay within the string field theory framework. We take the one-field cosmological model with the potential

$$V(\phi) = 2(1-\phi)\phi^2 - \frac{4(\phi-1)^3(2+3\phi)^2}{75m_p^2} \tag{64}$$

and $C = -1$. The Friedmann equations (54) have the following exact solution [30]:

$$\phi_0 = \operatorname{sech}^2(t), \tag{65}$$

$$H_0 = \frac{2(3+2\cosh(t)^2)\tanh^3(t)}{15m_p^2\cosh(t)^2}. \tag{66}$$

At $t \to \infty$ solution (7.2) tends to a fixed point:

$$H_f = \frac{4}{15m_p^2}, \qquad \phi_f = 0. \tag{67}$$

It is easy to see that

$$V'_\phi(0) = 0, \qquad V''_{\phi\phi}(0) = 4\left(1 - \frac{2}{5m_p^2}\right). \tag{68}$$

Using (33), we obtain that solution (7.2) is attractive in the Bianchi I metric at $m_p^2 < 2/5$. In [30] the authors consider a model without the CDM, at the same time, the results of Section 2 show that solution (7.2) is stable with respect to small fluctuations of the initial value of the CDM energy density as well.

Let us perturb the Friedmann equations in the standard way,

$$H = H_0(t) + \epsilon h(t), \quad \phi = \phi_0(t) + \epsilon\varphi(t). \tag{69}$$

To first order in ϵ we have the following system of equations:

$$\begin{aligned}
&\dot h + \frac{2}{m_p^2}\operatorname{sech}^2(t)\tanh^2(t)\dot\varphi = 0,\\
&\frac{1}{m_p^2}\left(\frac{4}{5}(4+\cosh(2t))\operatorname{sech}^2(t)\tanh^3(t)h + (6\operatorname{sech}^4(t) - 4\operatorname{sech}^2(t))\varphi\right) -\\
&- \frac{4(2+3\operatorname{sech}^2(t))^2}{25m_p^2}(\tanh^4(t) - 2\tanh^6(t))\varphi + 2\operatorname{sech}^2(t)\tanh(t)\dot\varphi = 0.
\end{aligned} \tag{70}$$

System (70) has the following solutions:

$$\varphi = \frac{1}{2\sinh(t)\cosh^3(t)}\left(5C_2m_p^2\cosh(t)^{\left(\frac{-4+30m_p^2}{5m_p^2}\right)}e^{\left(\frac{2\cosh^2(t)-3}{10m_p^2\cosh^4(t)}\right)} - 2C_1\cosh^2(t) -\right.$$

$$- 2C_2\int\frac{1}{\sinh^3(t)}(-15m_p^2\cosh^4(t) + 10m_p^2\cosh^6(t) + 8\cosh^2(t) - 6 - 4\cosh^6(t) + 2\cosh^4(t))\times$$

$$\left.\times\cosh(t)^{\left(\frac{-4+5m_p^2}{5m_p^2}\right)}e^{\left(\frac{2\cosh^2(t)-3}{10m_p^2\cosh^4(t)}\right)}(\cosh^2(t) - m_p^2)dt + 2C_1\right),$$

$$h = \frac{16(\cosh(2t)-1)}{\cosh(6t) + 6\cosh(4t) + 15\cosh(2t) + 10}\left(C_1 + C_2\int\frac{\cosh(t)^{\left(\frac{-4+5m_p^2}{5m_p^2}\right)}e^{\left(\frac{2\cosh^2(t)-3}{10m_p^2\cosh^4(t)}\right)}}{\sinh^3(t)}\times\right.$$

$$\times[-15m_p^2\cosh^4(t)+10m_p^2\cosh^6(t)+8\cosh^2(t)-6-4\cosh^6(t)+2\cosh^4(t)]dt\Big).$$

Using (66) we get

$$h_i = h + \tilde{C}_i \cosh(t)^{-\frac{4}{5m_p^2}} e^{\frac{\sinh(t)^2\left(3+\cosh(t)^2\right)}{10m_p^2\cosh(t)^4}}, \tag{71}$$

where $\tilde{C}_i$ are arbitrary real constants which satisfy (63).

It is easy to verify that $h(t)$ and $h_i(t)$ are bounded functions for any values of the parameters. Taking into account that

$$\lim_{t->\infty} \exp\left(\frac{2\cosh^2(t)-3}{10m_p^2\cosh^4(t)}\right) = 1, \tag{72}$$

we obtain that φ is bounded at $m_p^2 \leqslant 2/5$ and unbounded at $m_p^2 > 2/5$. The stability in the case of $m_p^2 = 2/5$ cannot be analysed without using high order corrections.

8 Conclusion

We have analysed the stability of isotropic solutions for the models with NEC violation in the Bianchi I metric.

In our papers for the one-field and two-field models with the CDM we used the Lyapunov theorem and found sufficient conditions for stability of kink-type and lump-type solutions both in the FRW metric and in the Bianchi I metric. The obtained results allow us to prove that the exact solutions, found in string inspired phantom models [60, 30], are stable.

We found the explicit form of the connection between $h_1(t)$, $h_2(t)$, and $h_3(t)$, which define metric perturbations in the Bianchi I metric, and h_0, which defines perturbations in the FRW metric. We have proved that fluctuations for the fields and the CDM energy density in both metric are the same. In particular, for $H_0 \geqslant 0$ the boundedness of h_0 is a sufficient and necessary condition for the boundedness of $h_1(t)$, $h_2(t)$, and $h_3(t)$.

Our study of the stability of isotropic solutions for the models with NEC violation in the Bianchi I metric shows that the NEC is not a necessary condition for classical stability of isotropic solutions. In these papers we have shown that the models [30, 36, 60] have stable isotropic solutions and that large anisotropy does not appear in these models.

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