# Induced Semiclassical Processes with Defects and Electrons

M.B. Voloshin

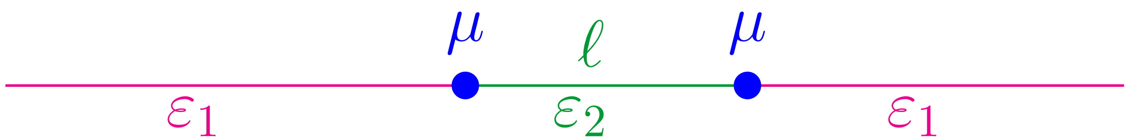
*ITEP, Moscow*

*FTPI, University of Minnesota*

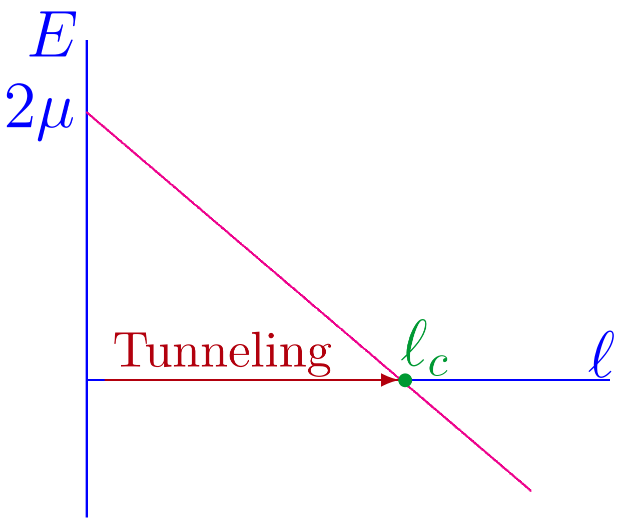
# Outline

- Metastable strings, walls, and vacua
- Tunneling exponent
- Pre-exponent for strings and walls
- Thermal effects at low temperature
- Destruction of strings and walls in collisions of Goldstone bosons. Extracting particle processes from the thermal effect.
- Schwinger $e^+e^-$ pair creation
- Thermal effects and photon-induced pair creation
- Photon-induced process from WKB. Low energy.
- Photon-induced process from WKB. High energy.

• Metastable String:



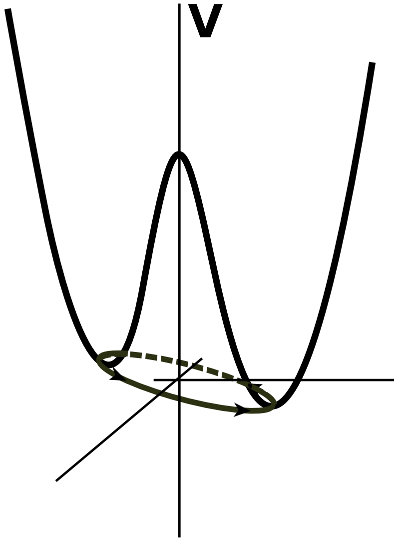

$$E = 2\mu - (\varepsilon_1 - \varepsilon_2)\,\ell$$




Critical gap: $\ell_c = \frac{2\mu}{\varepsilon_1 - \varepsilon_2}$

• Metastable Wall:

(e.g. axion wall)




Hole in the wall (bubble) of radius $r$. Energy:

$$E = 2\pi\mu r - \pi\varepsilon r^2$$

$\varepsilon$ - surface tension of the wall, $\mu$ - linear tension of the boundary.

Critical size: $R = 2\mu/\varepsilon$.

• Decay of metastable strings and walls is similar to the false vacuum decay (also Schwinger process for the strings):
A critical 'bubble' is formed by tunneling, which then expands. Probability of decay = the rate $\gamma$ of nucleation of the critical bubbles per volume/area/length.
Tunneling exponent:

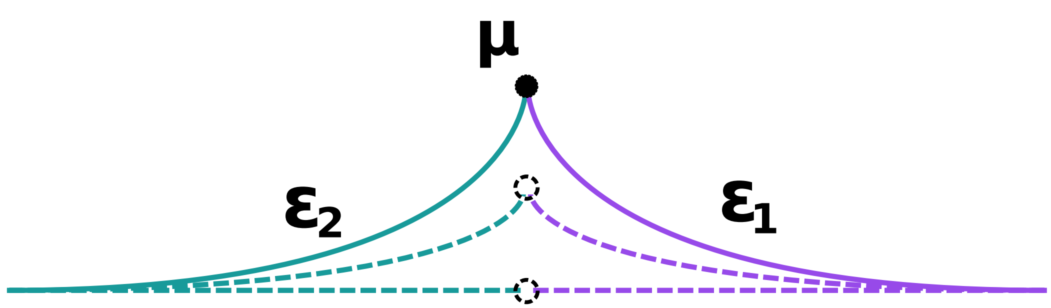
$$\text{String } (d = 2): \quad \gamma \propto \exp\left(-\frac{\pi\mu^2}{\varepsilon_1 - \varepsilon_2}\right)$$

$$\text{Wall } (d = 3): \quad \gamma \propto \exp\left(-\frac{16\pi\mu^3}{3\varepsilon^2}\right)$$

$$\text{False vacuum decay in } d = 4: \quad \gamma \propto \exp\left(-\frac{27\pi^2\mu^4}{2\varepsilon^3}\right)$$

M.B.V., I.Yu. Kobzarev and L.B. Okun, 1974

• The pre-exponent however is different
because the interface "drags" a part of string/wall.



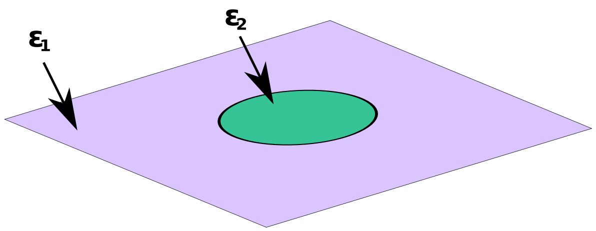
• Euclidean-space formulation (Coleman 1977)

String (the case of wall is similar): Nambu-Goto effective (Euclidean) action

$$S = \varepsilon_1 \mathcal{A}_1 + \varepsilon_2 \mathcal{A}_2 + \mu \mathcal{P}$$

$\mathcal{A}_{1,2}$ - world area of the string 1,2; $\mathcal{P}$ - length of the world line (perimeter) of the interface between 1 and 2.

Classical solutions (with the string 1 at infinity): trivial - flat string 1; bounce:




$$\text{Radius :} \qquad R = \frac{\mu}{\varepsilon_1 - \varepsilon_2}$$

(Diameter $\ell_c = 2R$)

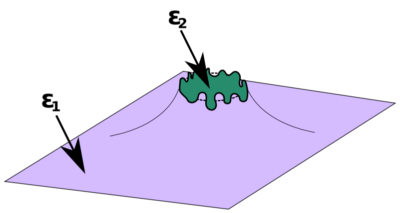
Action on the bounce (relative to the flat string) $S_B = \pi \mu^2 / (\varepsilon_1 - \varepsilon_2)$.

Decay rate $\propto \exp(-S_B)$.

- Pre-exponent

A calculation of the path integral $\mathcal{Z}$ with $e^{-S}$ over the fluctuations around the bounce is needed. Probability $\gamma_0$ per time T per length X:

$$\gamma_0 = \frac{1}{XT}\,\mathrm{Im}\frac{\mathcal{Z}_{12}}{\mathcal{Z}_1}$$




- Longitudinal — variations of the bounce boundary in the (x,t) plane - same as in false vacuum decay/Schwinger process — pre-exponential factor $(\varepsilon_1 - \varepsilon_2)/2\pi$ V. Kiselev, K. Selivanov 1984, MBV 1985
- Transverse modes — massless Goldstone bosons on the string — extra factor $F(\varepsilon_2/\varepsilon_1)$

$$F\left(\frac{\varepsilon_2}{\varepsilon_1}\right) = \sqrt{\frac{\varepsilon_1+\varepsilon_2}{2\,\varepsilon_1}}\,\Gamma\left(\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1-\varepsilon_2}+1\right)\left(\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)^{\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1-\varepsilon_2}} \times$$

$$\exp\left(\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1-\varepsilon_2}\right)\left(2\pi\,\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_1-\varepsilon_2}\right)^{-1/2}$$

$F(0) = e/\sqrt{4\pi} = 0.7668\ldots$ and $F(1) = 1$

- Total string decay rate

$$\gamma_0 = F\left(\frac{\varepsilon_2}{\varepsilon_1}\right) \frac{\varepsilon_1 - \varepsilon_2}{2\pi} \exp\left(-\frac{\pi\, \mu_R^2}{\varepsilon_1 - \varepsilon_2}\right)$$

$\mu_R$ — renormalized mass parameter $\mu$.

Renormalized by the pieces of the string dragged along with the ends.

- Similar situation in the case of wall decay: the UV terms from the NG action can all be absorbed into renormalization of $\mu$.

Not true for higher-dimensional branes.

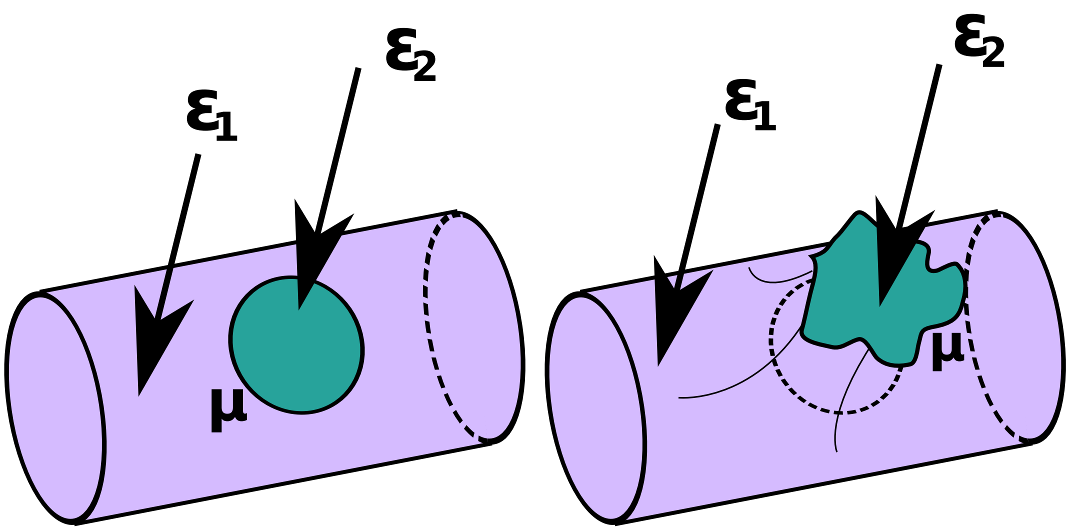
• Thermal catalysis of the decay.

Massless transverse waves (Goldstone modes) are excited at arbitrarily low temperature $T$. Expect the thermal catalysis factor $\mathcal{K}(T)$ in

$$\gamma(T) = \mathcal{K}(T)\,\gamma_0$$

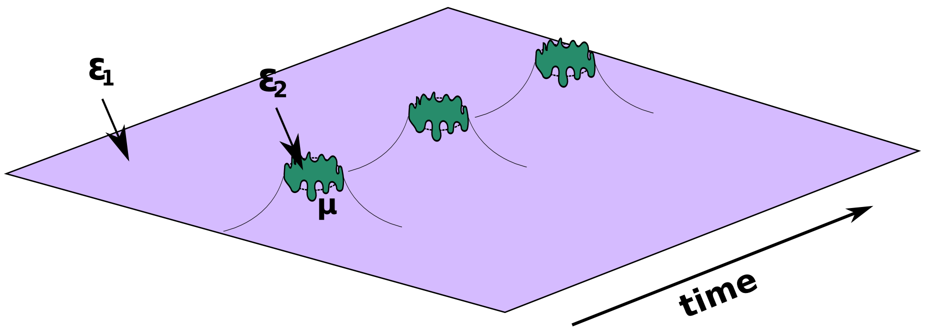
to expand in powers of $T$.

In the Euclidean thermal formulation need to consider time with the period $\beta = 1/T$.



Nothing happens to the bounce as long as $2R < \beta$. The only modification is in the path integral over the modes around the bounce.

Periodic plane:




For a single bounce the eigen modes are O(2) symmetric:

$z_k(t, x) =$Re,Im$(R^k/w^k)$ with $w = t + ix$. Can be made periodic:

$$g_k = \frac{R^k}{w^k} + \sum_{n=1}^{\infty} \left[ \frac{R^k}{(w - n\beta)^k} + \frac{R^k}{(w + n\beta)^k} \right]$$

but these are not eigen modes.

A mixing between the modes $g_k$ in the action occurs.

- General expression

$$\mathcal{K} = \mathrm{Det}\left[1 - D^2\right]^{-(d-2)/2}$$

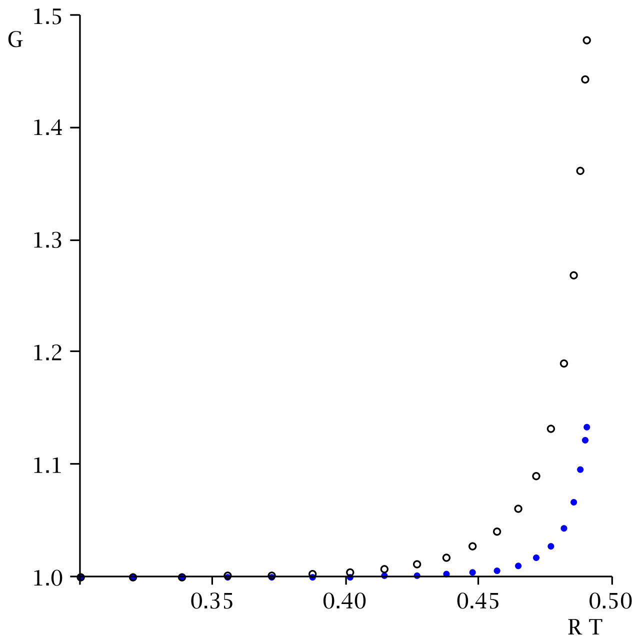
where

$$D_{pk} = -\left[(-1)^k + (-1)^p\right](RT)^{k+p+2}\,\frac{p}{p+b}\,\frac{(p+k+1)!}{(p+1)!\,k!}\,\zeta(p+k+2)$$

with $b = (\varepsilon_1 + \varepsilon_2)/(\varepsilon_1 - \varepsilon_2)$.

A. Monin, MBV, 2008

Critical temperature $T\ell_c = 1$ (at larger $T$ the bounce does not fit on the cylinder).



- Low temperature expansion

$$\mathcal{K} = 1 + (d-2)\,\frac{\pi^8}{450}\,\left(\frac{\varepsilon_1-\varepsilon_2}{3\varepsilon_1-\varepsilon_2}\right)^2\,\left(\frac{\ell_c T}{2}\right)^8 + O(T^{12})$$

Why $T^8$?

To answer one needs to consider the breaking of the string in collisions of the Goldstone bosons.

Thermal catalysis = additional contribution due to breaking of the string in collisions of thermally excited Goldstone bosons.

• Consider the process ($n$ Goldstone bosons $\to$ broken string $+\ X$).
Let $z(x,t)$ be the field of (one polarization of) the Goldstone bosons (a shift by $z(t,x)$ in a transverse direction). Consider low-energy $\omega$ — expansion in derivatives of $z$. $z = const$ (overall shift of the string) and $\partial z$ =const (overall fixed rotation) cannot enter an amplitude. Essential terms start with $\partial^2 z \propto \omega^2$, and there is $1/\sqrt{\omega}$ factor per each particle due to normalization. $\Rightarrow$ each boson's energy enters as $\omega^{3/2}$ in the amplitude, i.e. $\omega^3$ in the probability.

Start with the minimal process $n = 2$. The (dimensionless) probability $W_2$ of breaking string in two-particle collisions can depend only on the invariant $s = 4\omega_1\omega_2$. At small $s$ $W_2 = C\, s^3 + \ldots$. In a thermal state with the distribution function for the bosons

$$n(\omega) = \frac{1}{\exp(\omega/T) - 1}$$

the contribution $\delta\gamma$ of the two-particle process to the thermal decay rate is

$$\delta\gamma = (d-2) \int n(\omega_1) n(\omega_2) W_2(4\omega_1\omega_2) \frac{d\omega_1 d\omega_2}{(2\pi)^2} = (d-2)\, C\, \frac{16\pi^6}{225}\, T^8$$

Compare with the $T^8$ term in $\mathcal{K} \Rightarrow$ determine $C$.

$$W_2 = \gamma_0\, R^2 \left[\frac{\pi^2}{32}\left(\frac{\varepsilon_1 - \varepsilon_2}{3\varepsilon_1 - \varepsilon_2}\right)^2 R^6\, s^3 + \ldots\right]$$

• Higher terms of the $T$ expansion for $\mathcal{K}$ are contributed by higher terms in $s$ in $W_2$ and also by $W_n$ with higher $n$. In order to untangle we introduce a negative chemical potential $\mu$ for Goldstone bosons. Denote $\nu = -\mu > 0$, so that

$$n(\nu, \omega) = \frac{1}{\exp[(\omega + \nu)/T] - 1}$$

The $\zeta(q) = \sum_{n=1}^{\infty} n^{-q}$ is replaced in the integrals by $\mathrm{Li}_q(e^{-\nu/T}) = \sum_{n=1}^{\infty} e^{-n\nu/T} n^{-q}$.

$$\mathcal{K}(\nu, T) = \mathrm{Det}\left[1 - \mathcal{D}^2(\nu, T)\right]^{-(d-2)/2}$$

$$[\mathcal{D}(\nu, T)]_{pk} = -\left[(-1)^k + (-1)^p\right](RT)^{k+p+2}\,\frac{p}{p+b}\,\frac{(p+k+1)!}{(p+1)!\,k!}\,\mathrm{Li}_{p+k+2}\left(e^{-\nu/T}\right)$$

Two parameters, $\nu$ and $T$ allow to untangle the energy dependence from the dependence on the number of Goldstones.

A. Monin, MBV, 2009

- Operationally the number of Goldstones = the power of $\mathcal{D}$ in

$$\mathcal{K}(\nu, T) = \exp\left\{-\frac{d-2}{2}\,\mathrm{Tr}\ln\left[1-\mathcal{D}(\nu,T)^2\right]\right\} =$$

$$1+\frac{d-2}{2}\,\mathrm{Tr}\left[\mathcal{D}(\nu,T)^2\right]+\frac{d-2}{4}\,\mathrm{Tr}\left[\mathcal{D}(\nu,T)^4\right]+\frac{(d-2)^2}{8}\,\left\{\mathrm{Tr}\left[\mathcal{D}(\nu,T)^2\right]\right\}^2+O\left(\mathcal{D}^6\right)$$

  - Only even powers of $\mathcal{D}$ are present $\Rightarrow$ the string is destroyed only in collisions of *even* number of Goldstone bosons. We do not understand the deep reason for this .... This property is not true for the destruction of a metastable wall by the Goldstones: odd terms are also present.
  - Only the Goldstones with the same polarization destroy the string in binary collisions ($W_2 \propto (d-2)$). For quartic collisions a cross-talk between different polarizations is present (the term $\propto (d-2)^2$).
  - All the probabilities $W_n$ are smooth functions of energy. The catalysis factor $\mathcal{K}$ blows up at the critical $T$ not because some processes become large, but because infinitely many processes become important simultaneously.

The term $\frac{d-2}{2}\,\mathrm{Tr}\,[\mathcal{D}(\nu,T)^2]$ contains the full contribution of binary collisions at any $s$ (satisfying though $s \ll \varepsilon_1$ for the whole approach to be applicable).

$$W_2(s) = 8\,\pi^2\,\gamma_0\,R^2\,\left[\Phi_b(\sqrt{s}\,R)\right]^2$$

with

$$\Phi_b(x) = \frac{x}{2}\sum_{p=1}^{\infty}\frac{1}{p+b+1}\,\frac{x^{2p}}{(p-1)!\,(p+1)!}$$

In particular at $b=1$ (corresponding to $\varepsilon_2 = 0$) one has $\Phi_1(x) = I_3(x)$ (the third Bessel function). In this case

$$W_2(s) = 2\,\pi^2\,\gamma_0\,\ell_c^2\,\left[I_3\left(\frac{\sqrt{s}\,\ell_c}{2}\right)\right]^2$$

At $\sqrt{s}\ell_c \gg 1$ (at any $b$) $W_2 \propto \gamma_0\,\exp(\sqrt{s}\ell_c)$ — the known semiclassical behavior — tunneling at energy $E = \sqrt{s}$ rather than at $E = 0$.

• Metastable wall decay

Spontaneous:

$$\gamma_W^{(0)} = \frac{\mathcal{C}}{\varepsilon^{7/3}} \exp\left(-\frac{16\,\pi\,\sigma_R^3}{3\,\varepsilon^2}\right)$$

$\sigma_R$ - renormalized tension of the boundary.

Thermal:

Long expression (Monin, MBV '09). At low $T$ reduces to

$$\gamma_W(T) = \left[1 + 12\,(d-3)\,\zeta^2(5)\,(R_W\,T)^{10} + \ldots\right]\gamma_W^{(0)}$$

with $R_W = 2\sigma/\varepsilon$.

Stimulated by collisions of Goldsotone bosons:

Effective length $\lambda$ (a 2+1 dim analog of cross section): $\lambda(s)$ with $s = 2\omega_1\omega_2\,(1+\cos\theta)$.

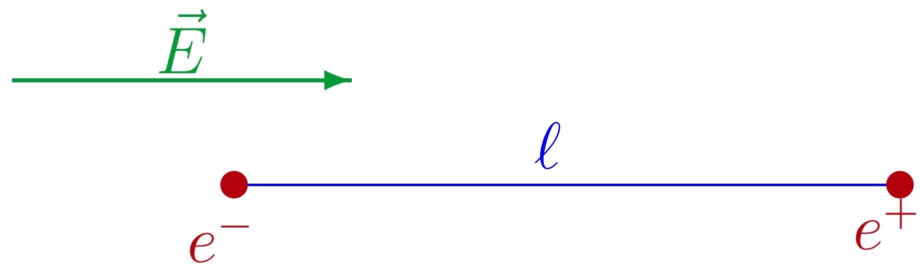
The $T^{10}$ thermal behavior corresponds to low energy $s^3$ behavior of $\lambda$:

$$\lambda = \frac{d-3}{5}\,\pi^2\,\gamma_W^{(0)}\,s^3\,R_W^{10} + \ldots$$

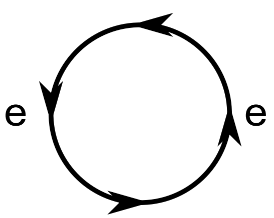
At $\sqrt{s}\,R_W \gg 1$ reduces to

$$\lambda \sim \exp\left(-\frac{16\,\pi\,\sigma^3}{3\,\varepsilon^2} + 4\sqrt{s}\,\frac{\sigma}{\varepsilon}\right)$$

- Schwinger pair creation




Work $= eE\,\ell \quad \Rightarrow \quad$ critical distance $\ell_c = 2m/(eE)$. Similar to string break.
Tunneling at $\ell < \ell_c$. (Fritz Sauter 1929)




$$\frac{\Gamma}{V} = \frac{(eE)^2}{4\pi^3} \exp\left(-\frac{\pi\, m^2}{eE}\right)$$

(Schwinger 1951)

$E_{crit} \sim m^2/e \sim 10^{(16)}$ V/cm. People discuss approaching (within a factor of $10^{-3}$?) such field strength by colliding femtosecond pulses from lasers.

• Recently suggested (Gies, Dunne, Schutzhold '08 - '09) that and additional photon beam should stimulate pair creation (effectively lower the barrier).

Can be viewed as attenuation of the photon beam intensity $\propto \exp(-\kappa\, L)$ over length $L$.

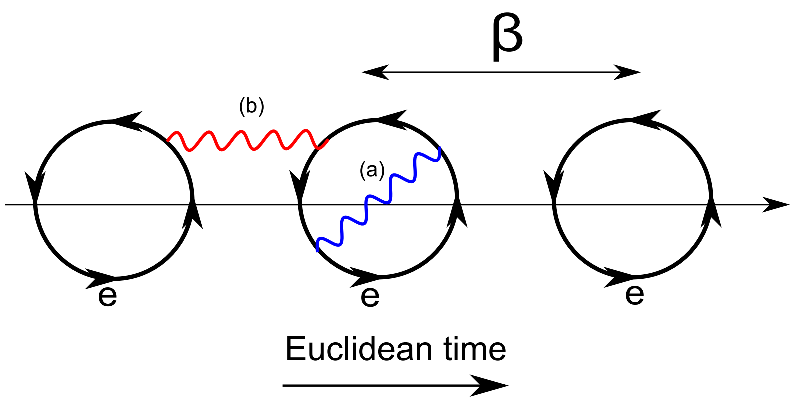
$$\kappa = -\frac{1}{\omega}\mathrm{Im}\Pi$$

$\Pi$ - vacuum polarization in external field ($\pi(k) = \mathrm{Im}\Pi|_{k^2=0} \neq 0$).
$\Pi$ is known in a complicated integral form (Dittrich & Gies 2000). A semiclassical calculation of $\kappa$ is much simpler.

$\mathrm{Im}\Pi$ depends on $k_\perp = \omega\, \sin\theta$ (due to the Lorentz invariance of the bounce in (t,x)). The relevant combination, Keldysh parameter, $\gamma_\theta = \omega\, \sin\theta\, m/(eE)$.
$\omega/m \ll 1$, but $\omega R = \omega m/(eE)$ arbitrary.

Separate behavior for different polarization of the photon w.r.t. $\vec{E}$: $\pi_\parallel$, $\pi_\perp$.

• Consider the Schwinger process in a thermal bath
and consider the effect of the interaction of the current in the loop(s).




Type (a): self interaction — $T$ independent rad. correction to the rate
$\Delta S = -\pi\alpha$ (Affleck, Alvarez, Manton 1982).
Type (b): thermal effect. (A sophomore exercise in interaction of currents). $\Delta S$ per period:

$$-\frac{e^2}{2}\sum_{n=1}^{+\infty}\left(\frac{1-2(RT/n)^2}{\sqrt{1-(2\,RT/n)^2}}-1\right) = -\frac{e^2}{2}\sum_{p=2}^{\infty} 2^{2p-1}(RT)^{2p}\,\frac{(p-1)\,\Gamma(p-1/2)}{\sqrt{\pi}\,\Gamma(p+1)}\,\zeta(2p)$$

Thermal factor in the rate: $\exp(-\Delta S) = 1 - \Delta S + \ldots$
$A_\mu$ is in the (t,x) plane $\Rightarrow$ only the parallel polarization is relevant.

Thermal effect $=$ the effect of photons in the bath. In the order $e^2$ one photon contribution $= \Gamma_{1\gamma} = (-\Delta S)\,\Gamma$.

Write general expression for $\kappa_{\|}$ as an expansion

$$\kappa_{\|}(\vec{k}) = \frac{1}{\omega}\sum_{p=2}^{\infty} C_p(\omega\,\sin\theta)^{2p-2}$$

Then

$$\frac{\Gamma_{1\gamma}}{V} = \int \frac{d^3\,k}{(2\pi)^3}\,\frac{\kappa_{\|}(\vec{k})}{e^{\omega/T}-1} = \sum_{p=2}^{\infty} C_p\,\frac{2}{(2\pi)^2}\,T^{2n}\,\Gamma(2p)\,\frac{\sqrt{\pi}\Gamma(p)}{\Gamma(p+1/2)}\,\zeta(2p)$$

Compare with the $T$ expansion for $\Delta S$ and find the coefficients $C_p$.

$$\kappa_{\|}(\vec{k}) = \frac{2\alpha\,m^2}{\omega}\,\exp\left(-\frac{\pi\,m^2}{eE}\right)\sum_{n=1}^{\infty}\frac{\Gamma(n+1/2)}{\sqrt{\pi}\,(n-1)!\,n!\,(n+1)!}\,\gamma_\theta^{2n}$$

$$= \frac{2\alpha\,m^2}{\omega}\,\exp\left(-\frac{\pi\,m^2}{eE}\right)\left[I_1(\gamma_\theta)\right]^2$$

$I_1$ - Bessel.

- Alternative calculation. Semiclassical $\Pi(k)$.

Euclidean $\Pi$:

$$\Pi_{\mu\nu}(x) = \langle j_\mu(x) j_\nu(0) \rangle$$

Effective action $S[\gamma]$ for electron on trajectory $\gamma$ ($X_\mu(s)$):

$$S = \oint_\gamma \left( m\sqrt{\dot{X}_\mu^2} - eA_\mu^{\text{ext}} \dot{X}_\mu \right) ds$$

$$\pi_{\mu\nu}(x) \equiv \text{Im}\Pi_{\mu\nu}(x) = -\frac{(eE)^3}{8\pi^3} \exp\left[ -\frac{\pi m^2}{eE} \right] \int \langle j_\mu(x) j_\nu(0) \rangle_{x_B} \, d^4 x_B$$

$x_B$ - position of the bounce, and

$$j_\mu^\gamma(x) = e \oint_\gamma \dot{X}_\mu(s) \delta^4(x - X(s)) \, ds$$

$\pi_{\|}$ :

Currents on the stationary trajectory: only $j_a$ with $a = 0, 1$.

$$j_a(x) = e \oint \dot{X}_a(s)\, \delta^4(x - X(s))\, ds = e n_a(\theta)\delta(r - R)\delta(y)\delta(z)$$

$n_a$ - tangential unit vector to a circle, $r_a = (t, x)$.

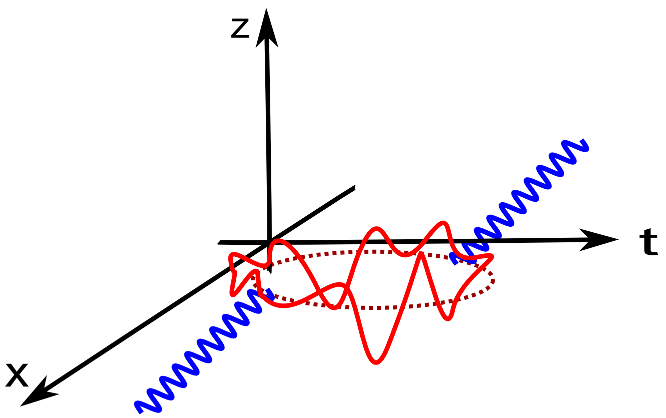
$$\int d^4 x_B\, j_a^B(x) j_b^B(0) = e^2\, \frac{4R^2 r_a r_b - \delta_{ab} r^4}{r^3 \sqrt{4R^2 - r^2}}\, \theta(2R - r)\, \delta(y)\delta(z)$$

$$\pi_{ab}(q) = \int d^4 x\, \pi_{ab}(x)\, e^{iqx} = -2\alpha m^2 I_1^2 \left(\sqrt{-q^2}\, R\right) \left(\delta_{ab} - \frac{q_a q_b}{q^2}\right) \exp\left(-\frac{\pi m^2}{eE}\right)$$

$q^2 = q_0^2 + q_1^2 \Rightarrow$ in Minkowski $-q^2 \to \omega^2 - k_{\|}^2 = k_{\perp}^2 = \omega^2 \sin^2\theta$.

$\pi_\perp$ :

Fluctuations of the trajectory in $\perp$ direction: $X_\mu(s) = X_a(s) + \xi_\mu(s)$.




Standard Gaussian integral:

$$\Pi_\perp = \langle j_3(x) j_3(0)\rangle = e^2 \int \mathcal{D}\xi \, ds_1 ds_2 \dot{\xi}_3^B(s_1)\dot{\xi}_3^B(s_2)\delta^4\left(x - X^B(s_1)\right)\delta^4\left(X^B(s_2)\right) e^{-S_B - \delta S[\xi]}$$

Result

$$\pi_\perp = -\frac{\alpha}{\pi} eE \left(1 - I_0^2(\gamma_\theta)\right) \exp\left[-\frac{\pi m^2}{eE}\right]$$

$\pi_\perp / \pi_\parallel \sim eE/m^2$.

• Additional contribution to $\pi_\perp$: $\pi_\perp^m$ from the magnetic moment.
Suppressed by $\omega^2/m^2$ rather than $eE/m^2$.
Magnetic moment interaction (Lorentz generalization of $-\vec{\mu}\cdot\vec{B}$):

$$\mathcal{L}_{int} = \frac{1}{4m}\,\epsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}f_\lambda j_\sigma^\gamma \to -\frac{1}{2m}\,A_\nu\partial_\mu\epsilon_{\mu\nu\lambda\sigma}f_\lambda j_\sigma^\gamma$$

$f_\mu$ - Lorentz boost of unit vector in the direction of $e$ polarization.
"Magnetic" current: $j_\nu^m = -\frac{1}{2m}\,\epsilon_{\mu\nu\lambda\sigma}f_\lambda\,\partial_\mu j_\sigma^\gamma$.

$$\langle j_\nu^m(x) j_\rho^m(0)\rangle = \frac{1}{4m^2}\,\varepsilon_{\mu\nu\lambda\sigma}\,f_\lambda\,\varepsilon_{\chi\rho\xi\tau}\,f_\xi\,\partial_\mu\,\partial_\chi\,\langle j_\sigma^\gamma(x) j_\tau^\gamma(0)\rangle$$

Result:

$$\pi_\perp^m = \frac{1}{2}\,(\pi_{22}+\pi_{33}) = -\frac{1}{8m^2}\,f_\mu^2\,q^2\pi_\parallel = -\frac{1}{8m^2}\,q^2\,\pi_\parallel(q^2)$$

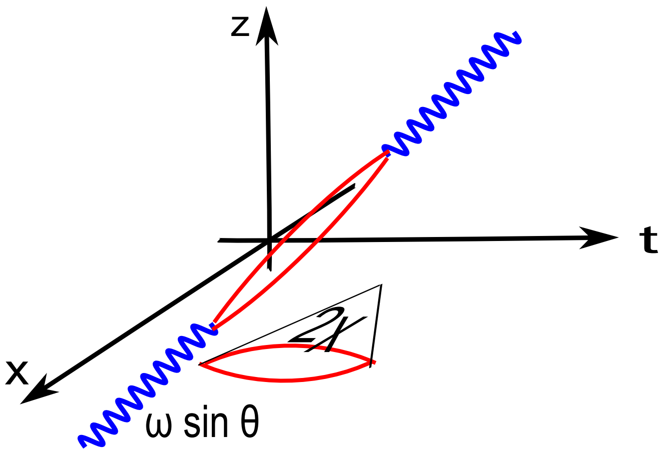
• High energy: $\omega \sin\theta$ not small compared to $m$.

Lorentz boost in the direction of $\vec{E}$, so that $k = (\omega \sin\theta, 0, 0, \omega \sin\theta)$.

The energy $\omega \sin\theta$ is transferred to tunneling - lowers the barrier.

The momentum conservation: $p_\perp(e^+) + p_\perp(e^-) = \omega \sin\theta$ - increases the barrier.

Euclidean space trajectory gets 'tilted'. Time duration T; $z$ 'duration': Z.




Action to extremize ($\varphi$ - polar angle in the (t,x) projection of the loop):

$$S = m \int \sqrt{{x'}_\mu^2} d\varphi - e \int A_\mu {x'}_\mu d\varphi - T\,\omega \sin\theta + Z\,\omega \sin\theta$$

$x' = dx/d\varphi$. Result: $\pi_{\|,\perp} \sim$

$$\exp\left[\gamma_\theta - \left(\frac{2m^2}{eE} + \frac{eE}{2m^2}\gamma_\theta^2\right) \arctan \frac{2m^2}{eE\,\gamma_\theta}\right] = \exp\left\{-\frac{2m^2}{eE}\left[(1+x^2)\arctan\frac{1}{x} - x\right]\right\}$$

$x = \omega \sin\theta/(2m)$. Need $x \sim m^2/(eE)$ to remove the suppression.

- Main lesson:

Extracting the probabilities of multiparticle (2+) processes from the thermal rate looks a lot simpler than calculating multipoint (4+) correlators.

For a one-particle-induced rate both approaches look workable. The direct one possibly allows for some extra features.