A cosmological model based on scale invariance and unimodular gravity

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Problems in "standard" cosmology

- Horizon problem, flatness problem, relic problem,...
  $\longrightarrow$ *Inflation*
- Missing matter
  $\longrightarrow$ *Dark matter*
- Accelerated expansion of the universe
  $\longrightarrow$ *Dark energy*
- Cosmological constant problem (from qft)
  $\longrightarrow$ *?*

There is enough motivation to modify something!

Guiding idea

- In the SM and GR there are many different mass scales:
  - $m_{Higgs}, \langle h \rangle$
  - $M_P \sim \frac{1}{\sqrt{G}}$
  - Λ_{QCD}, ...
- Question 1: Can we construct a theory in which all these scales are related, induced dynamically?
- Question 2: Is it possible to construct a theory that doesn't contain such absolute scales at all?

Construction of the model

Introduce a new scalar field χ.

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{SM[M\to 0]}} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi,\chi) + \left(\bar{N}_I i\gamma^\mu \partial_\mu N_I - h_{\alpha I}\,\bar{L}_\alpha N_I \tilde{\varphi} - f_I \bar{N}_I{}^c N_I \chi + \mathrm{h.c.}\right) ,$$

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{1}{2}\zeta^2\chi^2\right)^2 + \beta(\chi^2 - \chi_0^2)^2 ,$$

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger\varphi\right)\frac{R}{2} .$$

$\chi_0 \neq 0$, $\beta > 0 \Rightarrow$ Induced gravity (Zee (1979), Smolin (1979))

The case for scale invariance

- The model is somewhat artificial.
  - Terms $m_H^2\varphi^\dagger\varphi$, $m_N\bar{N}_I^{\,c}N_I$ and M_P^2 are absent.
  - Terms $\chi_0^2\chi^2$ and χ_0^4 are present.
- Try to construct a theory with no dimensional constants at all $\rightarrow \chi_0 = 0$.
- No dimensional parameters $\rightarrow$ dilatational symmetry

$$\phi(x) \rightarrow \sigma^n \phi(\sigma x)$$
$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(\sigma x)$$

The case for a flat direction

$$V(\varphi, \chi) = \frac{\lambda}{4}\left(h^2 - \zeta^2\chi^2\right)^2 + \beta\chi^4$$

- The ground state should break the dilatational symmetry.
- "Only possibility" is to have a flat direction $\rightarrow \beta = 0$.

$$V(\varphi, \chi) = \frac{\lambda}{4}\left(h^2 - \zeta^2\chi^2\right)^2$$

  - Ground state is infinitely degenerate.
  - Physics doesn't depend on field values at the minimum.

Interim balance

Model based on scale invariance + flat direction

- Horizon problem, flatness problem, monopole problem,...
  $\rightarrow$ *Inflation due to roll down to the potential minimum*
- Missing matter
  $\rightarrow$ *Dark matter given by sterile neutrinos*
- Cosmological constant problem
  $\rightarrow \Lambda = 0$, *due to scale invariance and flat direction*
- Accelerated expansion of the universe
  $\rightarrow$ *Problem is not adressed by this model!*
  We need another ingredient.

Unimodular gravity

- UG mainly appeared in the context of the cosmological constant problem

$$\mathcal{L} = \sqrt{-g}(-M_P^2 R + \Lambda)$$

- Idea is to constrain variables by $\det(g_{\mu\nu}) = -1$

$$\mathcal{L} \to \mathcal{L}_{UG} = -M_P^2 \hat{R} + \Lambda$$

- Equations of motion are

$$\hat{R}_{\mu\nu} - \frac{1}{4}\hat{R}\hat{g}_{\mu\nu} = 0$$

$\rightarrow \Lambda$ *doesn't appear.*

- But apply $\hat{\nabla}^\mu$ to equations $\Rightarrow \partial_\nu \left(\frac{\hat{R}}{4}\right) = 0$
  $\Rightarrow \frac{\hat{R}}{4} = constant \equiv \Lambda_0$
- Reinsert this into equations to get

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} + \Lambda_0 \hat{g}_{\mu\nu} = 0$$

  $\rightarrow$ *Integration constant acts as a cosmological constant.*
- These are the equations for GR with a cosmological constant Λ_0 in the coordinate frame where $\det g_{\mu\nu} = -1$.
- Hence, the result

$$\hat{\mathcal{L}} = -M_P^2 \hat{R} \quad \leftrightarrow \quad \mathcal{L} = \sqrt{-g}(-M_P^2 R + \Lambda_0)$$

(Van der Bij (1982), Zee (1985),...)

UG plus other fields

- General result for UG in combination with non-gravitational fields.

$$\mathcal{L}_{UG} = \mathcal{L}(\hat{g}_{\mu\nu}, \phi, \partial\phi) \leftrightarrow \mathcal{L}_{GR} = \sqrt{-g}(\mathcal{L}(g_{\mu\nu}, \phi, \partial\phi) + \Lambda_0)$$

Result doesn't depend on the nature of the fields and on the way they couple to gravity.

UG in our model

- Replace GR by UG, scalar and gravity parts become

$$\mathcal{L}_{UG} = -\frac{1}{2}(\xi_\chi \chi^2 + \xi_h h^2)\hat{R} + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}(\partial h)^2 - V(h,\chi)$$

- Theory is still scale invariant.
- Has the same classical solutions as

$$\mathcal{L}_{GR} = \sqrt{-g}\Big(-\frac{1}{2}(\xi_\chi \chi^2 + \xi_h h^2)R + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}(\partial h)^2 - V(h,\chi) - \Lambda_0\Big)$$

  Λ_0 depends on initial conditions and is not really part of the action.
- Classically it's enough to analyse $\mathcal{L}_{GR}$ for different values of Λ_0.

UG in our model

- Change to the Einstein frame

$$\tilde{g}_{\mu\nu} = (\xi_\chi \chi^2 + \xi_h h^2) M_P^{-2} g_{\mu\nu}$$

- Action in Einstein frame

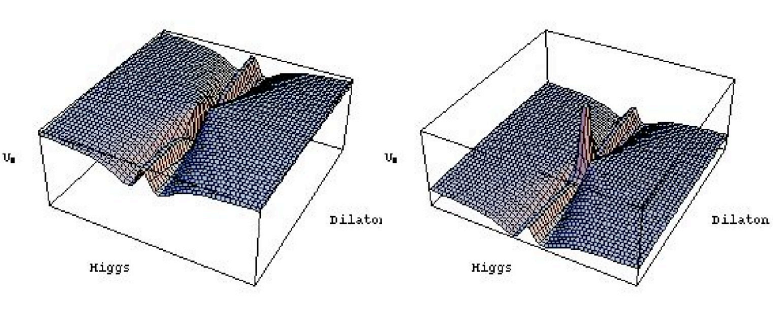
$$\mathcal{L}_E = \sqrt{-\tilde{g}} \left(-M_P^2 \frac{\tilde{R}}{2} + K - U_E(h, \chi) \right)$$

$$U_E(h, \chi) = \frac{M_P^4}{(\xi_\chi \chi^2 + \xi_h h^2)^2} \left(V(h, \chi) + \Lambda_0 \right)$$

$K =$ kinetic term

- K is non diagonal, but positive definite for $\xi_\chi, \xi_h > 0$.

$$U_E(h,\chi) = \frac{M_P^4}{(\xi_\chi \chi^2 + \xi_h h^2)^2}\left(\frac{\lambda}{4}\left(h^2 - \zeta^2\chi^2\right)^2 + \Lambda_0\right)$$



$\Lambda_0 < 0$ $\qquad\qquad$ $\Lambda_0 > 0$

We will suppose that $0 < \Lambda_0 \lesssim M_P^4$

Inflation and reheating

- $U_E(h,\chi) = \frac{M_P^4}{(\xi_\chi \chi^2 + \xi_h h^2)^2}\left(V(h,\chi) + \Lambda_0\right)$
- Chaotic inflation $\rightarrow$ initially χ and h away from valley and $\chi, h > M_P$ $\rightarrow$ First term in potential dominates.
- For $\xi_h \gg 1$,$\xi_\chi \ll 1$ and $\chi \sim h$ dynamics dominated by h $\rightarrow \chi \sim const$.
- Inflation due to roll down of h into the valley.
- Reheating due to oscillations of h in the valley.
- $\xi_h \sim 700 - 20000$ produces correct perturbation spectrum and a reheating temperature of $T_{rh} \sim 10^{13} GeV$. (Bezrukov and Shaposhnikov)

Late time evolution

- Fields are trapped in the valley $h \sim \zeta\chi$.
  $\rightarrow$ Second term of potential dominates.
- $U_E(h,\chi) \sim M_P^4 \frac{\Lambda_0}{(\xi_\chi \chi^2 + \xi_h h^2)^2}$
- Good phenomenology can be obtained for
  $\lambda \sim 1$ and $\zeta \lll 1 \quad \Rightarrow \quad h \lll \chi$.
- Define dilaton field η along the valley and ϕ perpendicular to it.
- Equations of motion

$$\ddot{\eta} + 3H\dot{\eta} + \frac{dU_\eta}{d\eta} \approx 0$$

$$\ddot{\phi} + 3H\dot{\phi} + m_h^2 \phi \approx 0$$

Late time evolution

The late time evolution of the universe is described by

$$\ddot{\eta} + 3H\dot{\eta} + \frac{dU_\eta}{d\eta} = 0$$

$$H^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\eta}^2 + U_\eta + \frac{C_\gamma}{a^4} + \frac{C_M}{a^3}\right)$$

$$U_\eta = \frac{\Lambda_0}{\xi_\chi^2}\exp\left(-\frac{\gamma\eta}{M_P}\right)$$

$$\gamma = \frac{4}{\sqrt{6 + \frac{1}{\xi_\chi}}}$$

$\rightarrow$ well-studied model (quintessence field,...)

- $\gamma > \sqrt{3} \rightarrow$ scaling solutions, no dark energy
- $\gamma < \sqrt{3} \rightarrow$ no scaling solutions, Ω_η keeps growing (Wetterich (1988), Ferreira and Joyce (1998))

Dark energy

- For $\gamma < \sqrt{3}$ thawing quintessence (Ferreira and Joyce (1998), Caldwell(2005)) $\rightarrow$ dark energy
- Field frozen by Hubble friction at early times, then slow roll-down
- Result for this scenario (Ferreira and Joyce (1998))

$$1 + \omega = \frac{\gamma^2}{3}\left[\frac{1}{\sqrt{\Omega_\eta}} - \frac{1}{2}\left(\frac{1}{\Omega_\eta} - 1\right)\log\frac{1+\sqrt{\Omega_\eta}}{1-\sqrt{\Omega_\eta}}\right]^2$$

- Identify $\Omega_\eta = \Omega_{DE}$
- WMAP data $\rightarrow -0.04 < 1 + \omega < 0.2$ and $\Omega_{DE} \approx 0.73$ $\Rightarrow \gamma < 1.11 < \sqrt{3}$ (ok for thawing scenario)
- $0 < \xi_\chi < 0.16$
- Fate of the universe in this model
  $\Omega_\eta \rightarrow 1, \quad \omega \rightarrow \frac{\gamma^2}{3} - 1, \quad a(t) \propto t^{\frac{2}{\gamma^2}}$
  (Ferreira, Joyce)

Interim balance

Model based on scale invariance + flat direction **+ UG**

- Horizon problem, flatness problem, relic problem,...
  $\rightarrow$ *Inflation due to roll down to the potential minimum*
- Missing matter
  $\rightarrow$ *Dark matter given by sterile neutrinos*
- Cosmological constant problem
  $\rightarrow$ $\Lambda = 0$ *since we require a flat direction*
- Accelerated expansion of the universe
  $\rightarrow$ **Dark energy due to UG with** $\Lambda_0 > 0$

Quantum aspects

- Results remain valid on quantum level if
  - Quantum theory is exactly scale invariant
  - Dilaton remains exactly massless (flat direction)
  - Initial conditions of UG give dark energy
- But, in all common theories dilatation symmetry is anomalous
  $\rightarrow \partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a}$
- Can we construct a scale invariant quantum theory?

Standard renormalization

- Theory is continued to $d = 4 - 2\epsilon$ dimensions

$$S = \int dx^d \left\{ \frac{1}{2} \left[(\partial_\mu \chi)^2 + (\partial_\mu h)^2 \right] - \lambda \left(h^2 - \zeta^2 \chi^2 \right)^2 \right\}$$

  - Fields have mass dimension $1 - \epsilon$
  - λ has dimension 2ϵ
  - Action is no longer scale invariant.
- Define finite dimensionless coupling λ_R

$$\lambda = \mu^{2\epsilon} \left[\lambda_R + \sum_{n=0}^{\infty} \frac{a_n}{\epsilon^n} \right] \qquad \mu \text{ has dimensions of mass}$$

- Fix a_n such that renormalized Green's functions are finite to all orders of perturbation theory.

Standard renormalization

- One loop effective potential along the flat direction in $\overline{\text{MS}}$ scheme

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2}\left[\log\frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2}\right]$$

$$m_H^2(\chi) = 2\lambda\zeta^2(1+\zeta^2)\chi^2$$

- Dilatation anomaly appears because regularization breaks the symmetry.

Scale invariant procedure

- Continuation to d-dimensions in a scale invariant way.
- Instead of μ introduce a combination of the fields with the correct mass dimension

$$\mu^{2\epsilon} \rightarrow \left(\xi_\chi \chi^2 + \xi_h h^2\right)^{\frac{\epsilon}{1-\epsilon}}$$

One-loop effective potential

- Denote $\omega^2 = \xi_\chi \chi^2 + \xi_h h^2$.
- Write the classical potential in d dimensions

$$U = \frac{\lambda_R}{4} \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[h^2 - \zeta_R^2 \chi^2\right]^2$$

- Introduce counter terms

$$U_{cc} = \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[A h^2 \chi^2 \left(\frac{1}{\bar{\epsilon}} + a\right) + B\chi^4 \left(\frac{1}{\bar{\epsilon}} + b\right) + C h^4 \left(\frac{1}{\bar{\epsilon}} + c\right)\right]$$

- We have parameters A, B, C, a, b, c.

One-loop effective potential

- A, B, C are chosen to cancel the divergences.
- One loop correction is automatically scale invariant.
- $U_1(h, \chi)$ does not automatically have a flat direction.
- Fix b, c such that $W_1(\zeta_R) = W_1'(\zeta_R) = 0$.
  $\rightarrow$ Flat direction

Effective Higgs potential

One loop potential for $\chi = \chi_0$, $v \equiv \zeta_R \chi_0$ and $\zeta_R \lll 1$

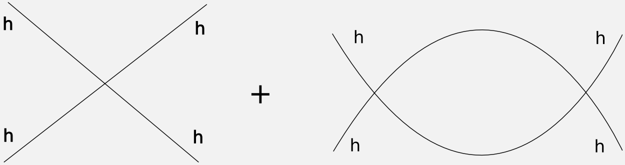
$$
\begin{aligned}
U_1 = \quad & \frac{m^4(h)}{64\pi^2}\left[\log\frac{m^2(h)}{v^2} + \mathcal{O}\left(\zeta_R^2\right)\right] \\
+ \quad & \frac{\lambda_R^2}{64\pi^2}\left[C_0 v^4 + C_2 v^2 h^2 + C_4 h^4\right] + \mathcal{O}\left(\frac{h^6}{\chi_0^2}\right)
\end{aligned}
$$

where $m^2(h) = \lambda_R(3h^2 - v^2)$.

- First term is standard effective potential for the theory with h only and the potential $U = \frac{\lambda}{4}\left(h^2 - \zeta^2\chi_0^2\right)^2$.
- Additional terms give redefinitions of coupling, mass and vacuum energy.
- Corrections to the Higgs mass proportional to $v^2 \propto \zeta_R^2\chi_0^2$ $\rightarrow$ No problem of stability of the Higgs mass.

Running coupling

- Higgs-Higgs scattering at $v \ll \sqrt{s} \ll \chi_0$ $(\zeta_R \lll 1)$



$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2}\left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \mathrm{const}\right] + \mathcal{O}\left(\zeta_R^2\right)$$

- Coupling runs in standard way.

Conclusions

- Constructed a scale invariant cosmological model.
  - All Scales are generated dynamically.
  - Inflation similar to Higgs inflation.
  - Sterile Neutrino as DM candidate.
  - Dark energy due to unimodular gravity.
  - $\omega > -1$
- Constructed a class of scale invariant quantum theories.
  - Dilatation symmetry is spontaneously broken.
  - Standard running of coupling constants.
  - No problem of stability of the Higgs mass.
- Classical results could remain valid in full quantum theory.

Some remaining tasks and questions

- Detailed analysis of inflation.
- The model doesn't adress the question about the big difference between the electroweak and the Planck scale.
- Can one do an experiment to detect effects of the dilaton?
- Are the scale invariant theories unitary, ... ?

Thank You.