Condensates, Correlators and Holography

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Plan of the talk

- AdS/CFT Basics
- Symmetries: Break or Save?
- QCD Models: bottom-up and top-down models
- Holography and Condensates:
  - Kinetic coefficients
  - Two-point correlators

My talk is intended to serve as a pedagogical introduction to the talks by Peter Kopnin and Alexander Krikun in this session as well.

The Conjecture

The AdS/CFT or Maldacena conjecture states the equivalence (also referred to as duality) between the following theories *Polyakov 1998, Maldacena 1998, Witten 1998*:

- Type IIB superstring theory on $AdS_5 \times S^5$, where both AdS_5 and S^5 have the same radius R, the 5-form F_5^+ has integer flux $N_c = \int_{S^5} F_5^+$, string coupling is g_s
- SYM in 4 dimensions, with gauge group $SU(N_c)$ and Yang-Mills coupling g_{YM} in its superconformal phase

with the following identification of the parameters of the theory

$$g_s = g_{YM}^2,$$

$$R^4 = 4\pi g_s N_c \alpha'^2,$$

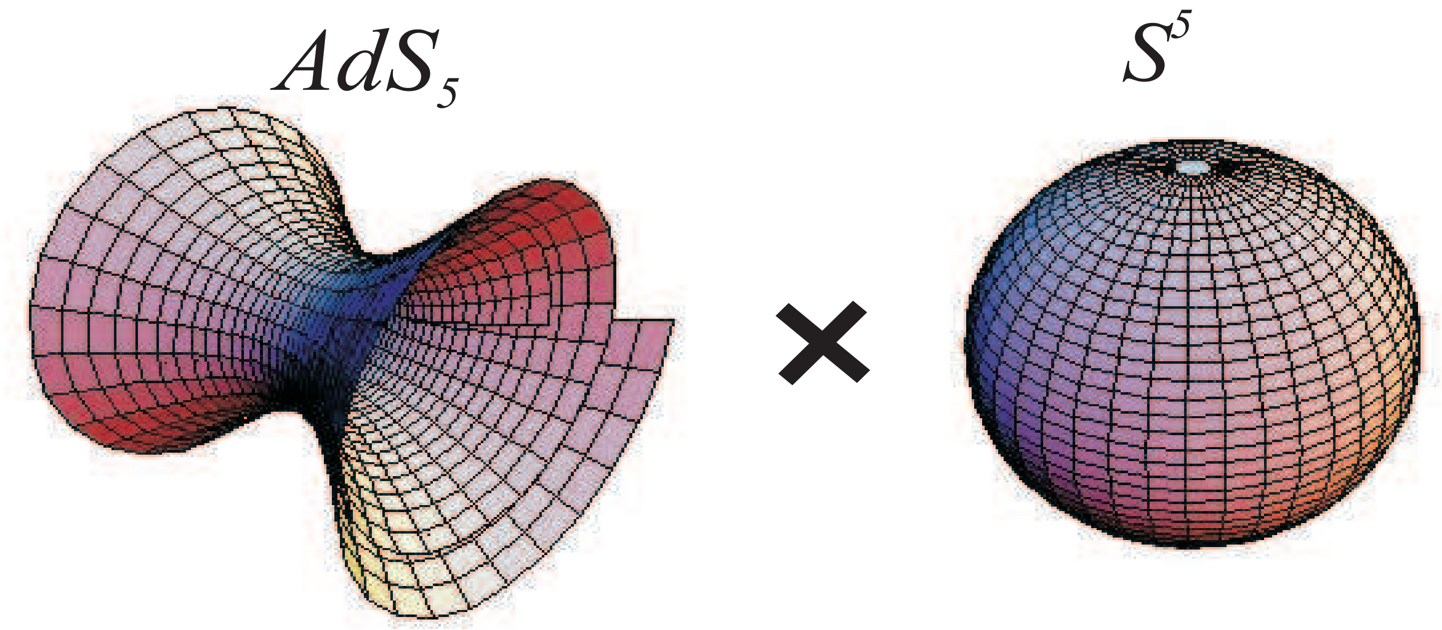
and the axion expectation value in AdS equals the SYM instanton angle

$$\langle C \rangle = \theta_I$$

Equivalence includes a precise map between the fields on the supergravity side and the local gauge invariant operators on the $\mathcal{N} = 4$ SYM side.

A Cartoon of $AdS_5 \times S^5$

Here we pictorially show what our geometry looks like:



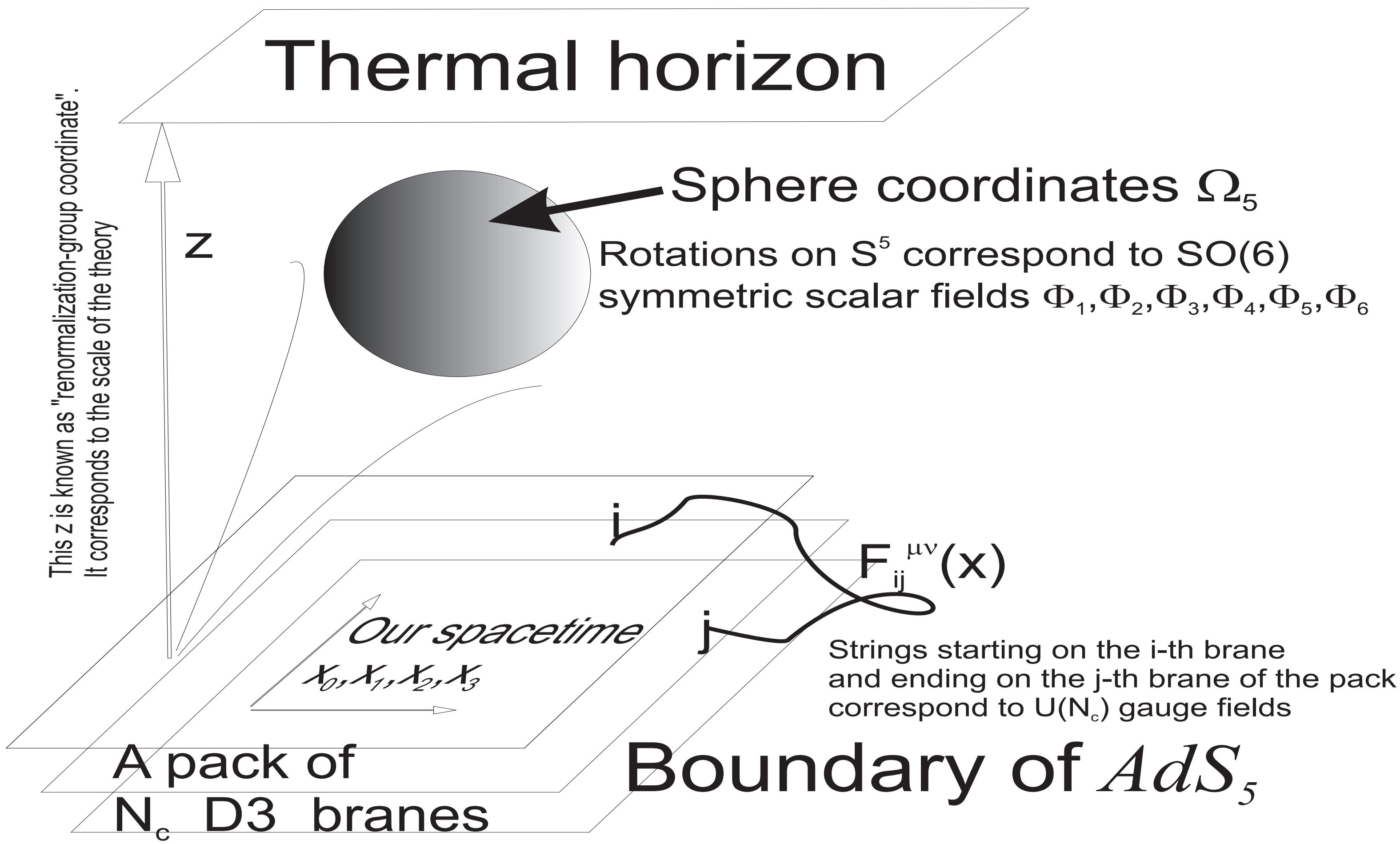
The geometry of AdS space can be described by the following embedding:

$$-Y_0^2 - Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

thus a pseudo-orthogonal group $SO(2,4)$ with signature $--++++$ can act upon it.

Why Do We Need Such Geometry?

The geometry is designed in such a way that its isometry group would **coincide** with the internal and Lorentzian symmetry of the field theory *[Maldacena 1998]*.



Basics on AdS/CFT correspondence

For review see *D'Hoker, Freedman[2002]*, *Gubser et al. [1999]*. The AdS/CFT conjecture is:

$$Z_{SYM}[J] = Z_{string}[\Phi]$$

where partition function $Z_{SYM}[J]$ is calculated in presence of four-dimensional currents J, coupled to some operators O

$$Z_{SYM}[J] \equiv \langle e^{-(S+\sum JO_J)} \rangle,$$

and $Z_{string}[\Phi_{\partial AdS}]$, where the boundary value of each bulk field is related to the corresponding current

$$\Phi^J_{\partial AdS} \sim J.$$

Some examples of correspondence:

- Dilaton field ϕ in supergravity is dual to operator $\mathrm{tr}\, F^2$ in SYM.
- Graviton field $h_{\mu\nu}$ is dual to energy-momentum current $T_{\mu\nu}$ in SYM.

Identification of Sources

In other words, the general case of a multiparticle correlator the Main Prescription is:if we have

S_{5d} – the five-dimensional action,
$\mathcal{O}_i(x)$ – operators,
$\Phi_i(z,x)$ – fields,
$\Phi_{\mathrm{c}\,i}(z,x)$ are classical solutions for EOM on $\Phi_i(z,x)$
$\phi_i(z,x)$ – fluctuations above them
$\int d^4x \sum_i \mathcal{O}_i(x)\phi_i(0,x)$ – interaction term

then the multi-point correlator is

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta}{\delta\phi_1(0,x_1)}\cdots\frac{\delta}{\delta\phi_n(0,x_n)} S_{5d}[\Phi_{\mathrm{c}\,i}(z,x)+\phi_i(z,x)]|_{z\to 0,\phi_i(z,x)=0}$$

This definition for correlator will be used here and in the talks by P.Kopnin and A.Krikun.

Examples in this Session

- Two-point functions $\langle \mathrm{tr}\,F^2(x)\,\mathrm{tr}\,F^2(0)\rangle$ in theories with condensates – my talk.
- One-point function of vector $U(1)$ current $\langle \vec{J}(\mu,B)\rangle$ – talk by P.Kopnin on chiral magnetic effect
- Four-point correlator of vector current – talk by A.Krikun.

(Non)-Normalizable Modes

- If bulk field $\phi_\Delta(z)$ is the counterpart of an operator $\dim(\hat{O}) = \Delta$ then the mode $\phi \sim z^{4-\Delta}$ is known as the "non-normalizable" mode, the mode $\phi \sim z^\Delta$ as the "normalizable" mode. "Normalizability" is understood as convergence of action in the UV.
- Normalizable mode *in a classical solution* is useful for calculating one-point functions (condensates).
- Non-normalizable modes *in Green functions* are useful for all higher-order functions.

Example: ϕ is the dilaton, $e^{-\phi}$ corresponds to gluon field strength $\operatorname{tr} G^2$. The dual action is

$$S = \int d^4x \int \frac{dz}{z^3} \phi'^2 \stackrel{\text{EOM}}{=} \frac{1}{z^3} \phi \phi' \Big|_{z \to 0},$$

let the dilaton classical solution be $e^{-\phi_c} = a + bz^4$. One-particle correlator is

$$\langle \operatorname{tr} G^2 \rangle = \frac{\delta S}{\delta \phi} = K(z) \frac{1}{z^3} \partial_z \phi_c = (a + bz^4) \cdot \frac{1}{z^3} \cdot 4bz^3 = 4b.$$

Here $K(z)$ is the field ϕ bulk-to-boundary propagator; since it is normalized to $K(z) = 1$ we don't need to know it. If we need $\langle \operatorname{tr} G^2(x) \operatorname{tr} G^2(0) \rangle$, we have to write down $\langle \operatorname{tr} G^2(x) \operatorname{tr} G^2(0) \rangle = K(x,z) \partial_z K(x,z)|_{z \to 0}$, where the normalizable mode cannot enter since ϕ_c is gone from this expression. In other words, normalizable mode is the *value* of the operator VEV, whereas non-normalizable mode is the *source* of the operator.

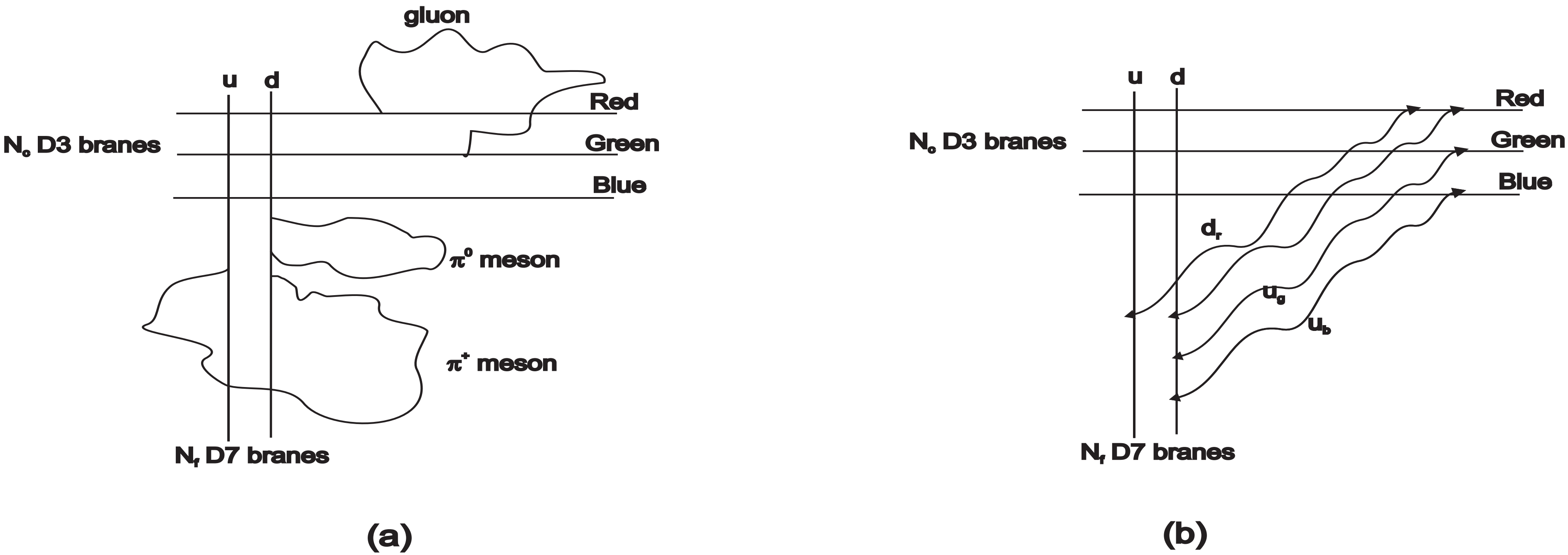
Realistic Model-Building

The original theory is $\mathcal{N} = 4$ SYM and is therefore completely unrealistic. How to cure that?

- Introduce condensates. Then you will end up in the same theory but over a different vacuum.
- Deform the metric. This might kill the conformal symmetry (e.g. add a logarithmic term to the dilaton). By deformations of the background you can obtain an $\mathcal{N} = 2$, $\mathcal{N} = 1$ or $\mathcal{N} = 0$ theories.
- Add some matter in fundamental reps (action on branes). This will cut the supersymmetries by half.
- Make the matter massive.
- Switch on the temperature. At high temperatures all theories are approximately conformal, thus comparison between them makes more sense.

AdS/CFT with flavours

For review see: *Aharony, [2002]*; *Mateos [2007]*. General idea of introducing flavour into AdS/CFT is illustrated in the two pictures below:



We require $N_c \gg N_f$, for otherwise the stack of N_f $D7$ branes will deform the metric essentially. This is known as "quenched approximation".

Flavoured Models on the Market

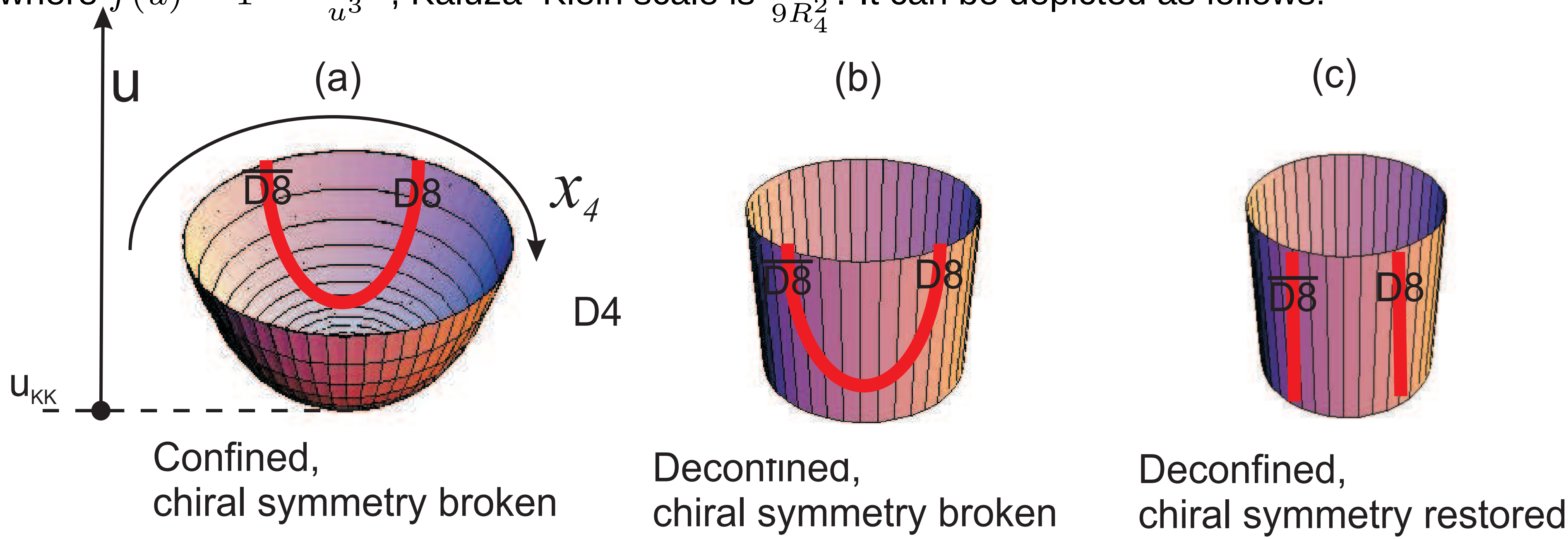
- Top-down models:
  - Sakai-Sugimoto: embedding of one/two D8 branes into metric, generated by D4 metric.
  - D3/D7: embedding of one D7 brane into metric, generated by D3 brane.
- Bottom-up models: (AdS/QCD) five-dimensional action with adjoint degrees of freedom (scalars, pseudoscalars, vectors, pseudovectors and dilaton):
  - Soft-wall
  - Hard-wall

Sakai—Sugimoto Model

There are N_c $D4$ branes, compactified on a circle of radius R_4 *[Sakai,Sugimoto 2005]*. A pack of N_f $D8$ branes are located in a point on this circle, as well as a pack of $\overline{D8}$ branes in an opposite point. The metric of the background is given by

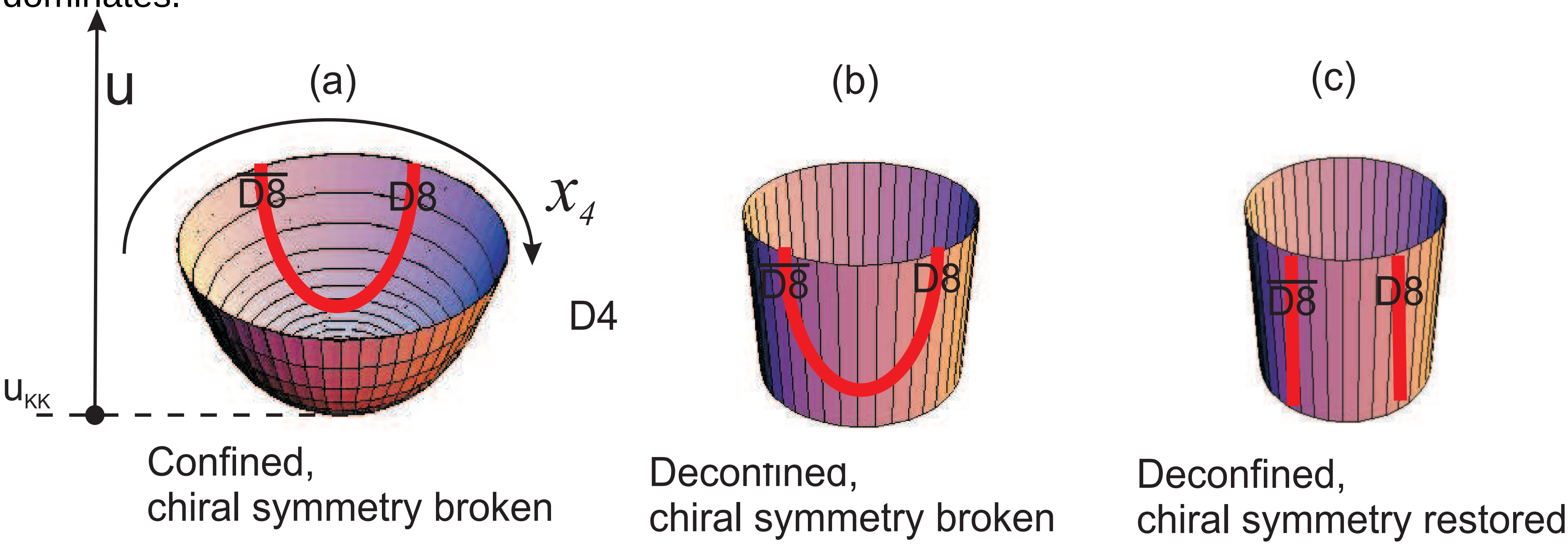
$$ds^2 = u^{\frac{3}{2}}\left(-dx_0^2 + d\mathbf{x}^2 + f(u)dx^4\right) + u^{-\frac{3}{2}}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

where $f(u) = 1 - \frac{u_{KK}^3}{u^3}$, Kaluza–Klein scale is $\frac{4}{9R_4^2}$. It can be depicted as follows:



Phase Transitions

Curvature of $D8$ brane is an order parameter for the chiral symmetry transition in the deconfined phase. At non-zero temperature there are two possible backgrounds. For $T < \frac{1}{2\pi R_4}$ the dominant background is given by (a), wheer gluons are confinend and chiral symmetry broken. For $T > \frac{1}{2\pi R_4}$ the dominant background is a cylinder, and the two possible embeddings of the $D8$ branes are shown in fig. (b) and (c). For $T < 0.154/L$, where L is asymptotic distance between $D8$-ends, a U-shaped (b) embedding dominates. At larger temperatures, configuration (c) with noth chiral symmetry and confinement broken dominates.



D3/D7 model

$D7$ brane does not change the metric in the quenched approximation. The dynamics of the brane in an external electromagnetic field is described by a Dirac—Born—Infeld action

$$S_{D7} = \mu_7 \int d^8\xi \sqrt{\det_{\alpha,\beta}\left(2\pi\alpha' B_{\alpha\beta} + g_{\mu\nu}\frac{\partial X^\mu}{\partial\xi^\alpha}\frac{\partial X^\nu}{\partial\xi^\beta}\right)} + \int d^8\xi C_4 \wedge F \wedge B$$

The $D3/D7$ model is advantageous for making it possible to introduce $\langle\overline{\psi}\psi\rangle$ condensate into the theory together with quark mass via brane embedding function $w(\rho)$:

$$X^8 + iX^9 = w(\rho)e^{i\phi},$$

where ρ is the holographic coordinate in the sense of the metric $ds^2 = \frac{dr^2}{r^2} + r^2 dx_4^2 + R^2 d\Omega_5^2$, $\rho^2 = r^2 - (X^8)^2 - (X^9)^2$, here

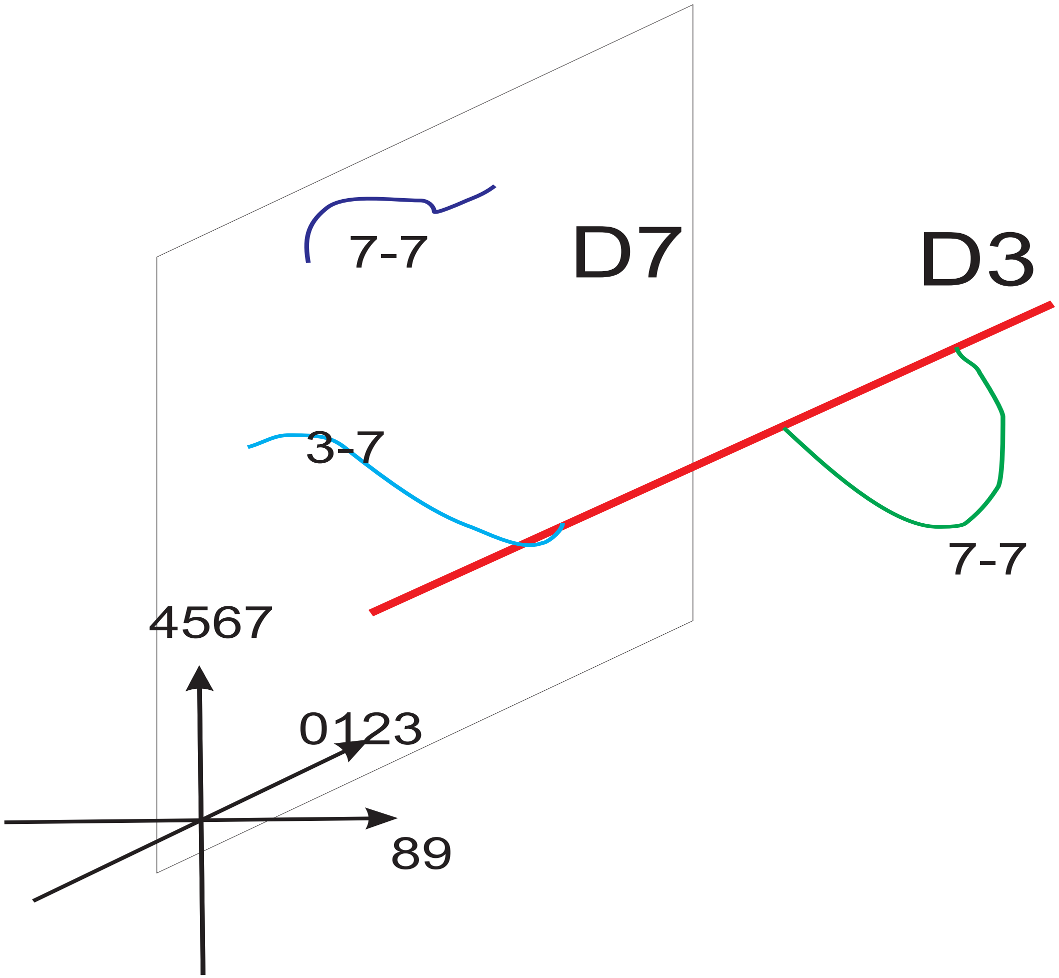
$$w(\rho)|_{\rho\to\infty} = a + \frac{b}{\rho^2},$$

where mass and condensate are respectively

$$m = a, \quad \langle\overline{\psi}\psi\rangle = 2b$$

D3-D7 Spectrum

Shown is the schematic geometry of $D3/D7$ model. The coordinates $8, 9$ describe the chiral dynamics of the model.



7-7 $U(N_f)$ theory
3-7 chiral multiplet $\overline{Q}$
7-3 chiral multiplet Q
3-3 $\mathcal{N}$=4 vector multiplet

AdS/QCD 5D models

Will be used as main calculation tool in the talks by A.Krikun and P.Kopnin.

Basically there are two types of AdS/QCD models. They have the same action

$$S = S_{YM}[L] + S_{YM}[R] + S_{CS}[L] - S_{CS}[R] + S_{sc}[L-R]$$

$$\begin{aligned}
S_{YM}[A] &= -\frac{1}{8g_5^2}\int e^{-\phi} F\wedge *F, \\
S_{CS}[A] &= -\frac{\cdot N_c}{24\pi^2}\int A\wedge F\wedge F - \frac{1}{2}A\wedge A\wedge A\wedge F + \frac{1}{10}A\wedge A\wedge A\wedge A\wedge A, \\
S_{sc}[L-R] &= \int d^4x \frac{dz}{z^3}\int e^{-\phi}\left[(D^\mu X)^\dagger D^\mu X + \frac{3}{z^5}|X|^2\right] + \\
&\quad + \frac{N_c}{24\pi^2}\mathrm{tr}\int d^4x dz \partial_\mu A_\nu \partial_\lambda A_\rho \frac{\partial_\alpha \pi}{f_\pi}\epsilon^{\mu\nu\lambda\rho\alpha}
\end{aligned}$$

- Hard-wall model: in the IR a boundary is introduced at some $z = z_m$, where Dirichlet boundary conditions are imposed.
- Soft-wall model: a non-trivial dilaton field $\phi = \lambda z^2$ is switched on; at $z \to \infty$ finiteness of action is required.

Liu-Tseytlin-Ghoroku Background

Consider now the following self-dual finite-temperature background: **dilaton** is

$$e^{\phi} = 1 + \frac{q}{\pi^4 T^4} \log\left(\frac{1}{1 - r^4 \pi^4 T^4}\right),$$

axion is related to dilaton in the same way as in zero-temperature Liu-Tseytlin background

$$C = e^{-\phi} - 1,$$

and the metric is

$$ds^2 = R^2\left(\frac{1 - r^4\pi^4 T^4}{r^2} dt^2 + \frac{dx_3^2}{r^2} + \frac{dr^2}{r^2(1 - r^4\pi^4 T^4)}\right) + R^2 d\Omega_5^2,$$

Physics Behind

This background is very interesting for modelling conformal $\mathcal{N} = 4$ theory with condensates

$$\langle \mathrm{tr}\, F^{+2} \rangle = q, \;\; \langle \mathrm{tr}\, F^{-2} \rangle = 0.$$

Thus this is "the softest possible" non-trivial symmetry breaking: we are just moved in a different self-dual vacuum, yet the theory is still conformal, its coupling being constant in the UV and β-function zero. Yet its IR properties are different from those of SYM over trivial vacuum; the coupling constant has got an IR singularity. The bulk theory action is

$$S = \int d^{10}x \sqrt{g} \left(R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} e^{2\phi} (\partial_\mu C)^2 - \frac{1}{2}|F_5|^2 \right).$$

We shall calculate two-point correlation functions in this background, establish dilatation Ward identities, and calculate some of the kinetic coefficients for a theory holographically dual to this background.

Field-operator correspondence

Fluctuations of the fields on the bulk couple to the operators $\mathrm{tr}\, F^2, \mathrm{tr}\, F\tilde{F}, T_{\mu\nu}$ on the boundary. The gauge field part of the boundary action is normalized as

$$S_{4d} = \frac{1}{2g_{YM}^2} \int d^4x \left(\mathrm{tr}\, F^2 - \frac{i\theta}{16\pi^2} \mathrm{tr}\, F\tilde{F}, \right)$$

with non-trivial condensates switched on:

$$\mathrm{tr}\, F^2 = \mathrm{tr}\, F\tilde{F} = N_c \frac{q}{\pi^2}$$

Fluctuation terms are defined as $\phi = \phi_c + \varphi, \quad C = C_0 + \xi, \quad g = g_{0\mu\nu} + h_{\mu\nu}.$ The interaction term is

$$S_{int} = \int d^4x \left[\frac{1}{2} T_{\mu\nu} \bar{h}^{\mu\nu} - e^{-\phi_c} \left(\bar{\varphi}\, \mathrm{tr}\, F^2 + \bar{\xi}\, \mathrm{tr}\, F\tilde{F} \right) \right],$$

which, after introduction of $F^{\pm} = \frac{F \pm \tilde{F}}{2}, \;\; \eta^{\pm} = \eta \pm \xi$, becomes

$$S_{int} = \int d^4x \left[\frac{1}{2} T_{\mu\nu} \bar{h}^{\mu\nu} - e^{-\phi_c} \left(\bar{\eta}^{+}\, \mathrm{tr}\, F^{+2} + \bar{\eta}^{-}\, \mathrm{tr}\, F^{-2} \right) \right].$$

Fluctuations of fields

Choose the gauge $h_{5\mu} = 0$, $k^\mu h_{\mu\nu} = 0$, $u^\mu h_{\mu\nu} = 0$, where $k = (\omega, 0, 0, k)$, constant vector u is $u = (1, 0, 0, 0)$. We work with five fields:

$$\bar{\Phi}_i = (\eta^+, \bar{h}_{11} + \bar{h}_{22}, \bar{h}_{11} - \bar{h}_{22}, \bar{h}_{12}, \eta^-),$$

coupled to operators

$$O_i = \left(\frac{1}{g_s^2} \operatorname{tr} F^{+2}, \frac{1}{8} T^\mu_\mu, \frac{3}{8} T_{11} - \frac{1}{8} T_{22} - \frac{1}{8} T_{33} - \frac{1}{8} T_{00}, T_{xy}, \frac{1}{g_s^2} \operatorname{tr} F^{-2} \right)$$

and what we calculate is the matrix of correlators

$$M_{ij} = \langle O_i O_j \rangle|_{(p)} = \frac{\delta^2 S_{full}}{\delta \bar{\Phi}_i(p) \delta \bar{\Phi}_j(-p)}.$$

Additional action terms

The standard wisdom: take the action of the type

$$S_{bulk} = \int d^4x dz \phi'^2 g^{zz} \sqrt{g}$$

and project onto the boundary as

$$S_{boundary} = \int d^4x \phi \phi' g^{zz} \sqrt{g}|_{z \to 0}.$$

In terms of bulk-to-boundary Green functions $G(x, z)$ correlator is $\langle O(x) O(0) \rangle = G(x, z) \partial_z G(0, z)|_{z=0}$. In our case two difficulties arise:

- The correct boundary term should be supplemented by the Gibbons–Hawking term, which makes globally diffeomorphism invariant the theory defined on manifold with boundary.
- The bilinear action of fields' fluctuations is non-diagonal, this means that we shall be dealing with a matrix of Green functions rather than with separately-treatable ones.

Mixing of modes

Let us define Green function matrix. Namely, if field Φ_i has a bulk solution $\Phi_i(z)$, satisfying $z^{\delta_i}\Phi_i(z)|_{z\to 0} = \bar{\Phi}_i$, (for our fields $\delta_i = (0,0,2,2,0)$) then by definition

$$K_{ij}(z) = \frac{\delta\Phi_j(z)}{\delta\bar{\Phi}_i}.$$

We establish the correct boundary term. The full action of our bulk theory is actually

$$S_{full} = S_{10d} + S_{div} + S_{4d},$$

where the Gibbons–Hawking term

$$S_{4d} = -2\partial_z \int d^4x\sqrt{-g_4} - c\int d^4x\sqrt{-g_4},$$

here $g_4 = \det(g_{ij}),\ \ i = 0,1,2,3$. Another piece is the full divergence term

$$S_{div} = \frac{3}{2}\partial_\mu W^\mu,$$

the vector W^μ found in textbooks is $W^\mu = \sqrt{-g}\left(g^{\alpha\beta}\delta\Gamma^\mu_{\alpha\beta} - g^{\alpha\mu}\delta\Gamma^\beta_{\alpha\beta}\right)$, where $\delta\Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta}(g+h) - \Gamma^\mu_{\alpha\beta}(g)$.

Second variation of action

Consider now the second variation of these actions in fluctuation fields; denoted these second-order expressions as $S^{(2)}_{10d}$, $S^{(2)}_{div}$, $S^{(2)}_{4d}$ respectively. They contain both fields and their derivatives. The two-point correlator is then

$$\langle O_i O_j \rangle = K_{ik} \frac{\partial^2 \mathcal{L}}{\partial \Phi'_k \partial \Phi'_m} \partial_z K_{jm} + K_{ik} \frac{\partial^2 S^{(2)}_{4d}}{\partial \Phi_k \partial \Phi'_m} \partial_z K_{jm} + K_{ik} \frac{\partial^2 S^{(2)}_{4d}}{\partial \Phi_k \partial \Phi_m} K_{jm}$$

here $\mathcal{L}$ is Lagrangian density of the bulk action:

$$S_{bulk} = S^{(2)}_{10d} + S^{(2)}_{div} = \int dz\, \mathcal{L}.$$

The above structure is obvious, since the bulk action is

$$\delta^2 S_{bulk} = \frac{\delta \Phi_m(z)}{\delta \bar{\Phi}_j} \frac{\delta^2 S_{bulk}}{\delta \Phi_m \delta \Phi_k} \frac{\delta \Phi_k(z)}{\delta \bar{\Phi}_i}, \text{ where}$$

$$\delta^2 S_{bulk} = \int dz \left[\frac{\partial^2 L}{\partial \Phi'_m \partial \Phi'_k} \partial_z \delta \Phi_m \partial_z \delta \Phi_k + \frac{\partial^2 L}{\partial \Phi_m \partial \Phi'_k} \delta \Phi_m \partial_z \delta \Phi_k + \frac{\partial^2 L}{\partial \Phi_m \partial \Phi_k} \delta \Phi_m \delta \Phi_k \right].$$

How to vary the action

Taking into account that Green functions of field fluctuations by definition satisfy equations:

$$\left[-\partial_z \frac{\partial^2 L}{\partial \Phi'_m \partial \Phi'_k}\partial_z + \frac{\partial^2 L}{\partial \Phi_m \partial \Phi'_k}\partial_z + \frac{\partial^2 L}{\partial \Phi_m \partial \Phi_k}\right]\delta\Phi_k(z) = 0,$$

one sees that the only contribution of S_{bulk} into the correlator will be:

$$\delta^2 S_{bulk} = \delta\Phi_m(z)\frac{\partial^2 L}{\partial \Phi'_m \partial \Phi'_k}\partial_z \delta\Phi_k(z).$$

Now remembering the definition of Green function matrix $K_{mj} = \frac{\delta \Phi_m(z)}{\delta \bar{\Phi}_j}$, we arrive exactly at previous slide. Hawking-Gibbons term contributes the following:

$$\delta^2 S_{4d} = \frac{\partial^2 S_{4d}}{\partial \Phi'_m \partial \Phi_k}\partial_z \delta\Phi_m \delta\Phi_k + \frac{\partial^2 S_{4d}}{\partial \Phi_m \partial \Phi_k}\delta\Phi_m \delta\Phi_k.$$

The action S_{4d} contains no more than one derivative term, which is due to normal differentiating of extrinsic curvature, thus $\frac{\partial^2 L}{\partial \Phi'^2} = 0$.

Correlators at $\omega \neq 0$

At finite frequency ω the matrix of correlators becomes after some algebra:

$$\left(\begin{array}{ccccc} -4q & -2q & 0 & 0 & -2q + \frac{1}{8}\log(2\epsilon\omega \\ -2q & -\frac{1}{32}\log(2\epsilon\omega)\omega^4 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{32}\log(2\epsilon\omega)\omega^4 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{8}\log(2\epsilon\omega)\omega^4 & 0 \\ -2q\frac{1}{8}\log(2\epsilon\omega)\omega^4 & 0 & 0 & 0 & 0 \end{array}\right.$$

One of the most interesting physical implication of this correlator matrix comes from the $\langle T_{xy} T_{xy} \rangle$ element. It is proportional to $\frac{\eta}{s}|_{T=0}$, and here we observe its independence of q. This fact is not trivial from dimensional considerations, since we do possess another dimensionful parameter, ω.

Summary of results at $\omega \neq 0$

We write out the most interesting elements of correlator matrix we can establish low-energy theorems. After due normalization we have

$$
\begin{aligned}
\int d^4x \left\langle \frac{\mathrm{tr}\, F^{+2}}{g^2} T \right\rangle &= -4 \left\langle \frac{\mathrm{tr}\, F^{+2}}{g^2} \right\rangle \\
\int d^4x \left\langle \frac{\mathrm{tr}\, F^{-2}}{g^2} T \right\rangle &= 0 \\
\int d^4x \left\langle \frac{\mathrm{tr}\, F^2}{g^2} \frac{\mathrm{tr}\, F^2}{g^2} \right\rangle &= -\frac{1}{2} \frac{1}{4\pi^2} \left\langle \frac{\mathrm{tr}\, F^2}{g^2} \right\rangle \\
\int d^4x \left\langle \frac{\mathrm{tr}\, F^2}{g^2} \frac{\mathrm{tr}\, F\tilde{F}}{g^2} \right\rangle &= -\frac{1}{4} \frac{1}{4\pi^2} \frac{\langle \mathrm{tr}\, F^2 \rangle}{g^2} \\
\int d^4x \left\langle \frac{\mathrm{tr}\, F\tilde{F}}{g^2} \frac{\mathrm{tr}\, F\tilde{F}}{g^2} \right\rangle &= 0
\end{aligned}
$$

Kubo formula

Shear viscosity is extracted from correlators according to the Kubo formula:

$$\eta = \lim_{w\to 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(t,x), T_{xy}(0,0)] \rangle.$$

Similar correlator – transport coefficient relations apply to other correlators, e.g. bulk viscosity

$$\zeta + \frac{4}{3}\eta = \lim_{w\to 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{xx}(t,x), T_{xx}(0,0)] \rangle.$$

Viscosity coefficient

from $\langle T_{xy} T_{xy} \rangle, \ \langle T_{xx} T_{xx} \rangle$ we thus establish:

$$\frac{\eta}{s}(q,\omega)|_{T=0} = \frac{1}{4\pi}, \ \ \zeta = 0.$$

This result is very important, since *apriori* it isunclear if condensate affects viscosity or not. Many people nowadays are trying to experimentally and theoretically establish the validity of $\frac{1}{4\pi}$ bound(which by some is thought as general theorem in holography); my contribution is that viscosity is condensate-independent.

Conclusions

- Low energy theorems

$$\int d^4x \langle \hat{O}(x) T(0) \rangle = -\mathrm{dim}(\hat{O}) \langle \hat{O} \rangle$$

  work in holography with condensates.
- Interesting "sum rules" established for gluon field strength operators, e.g. $\int d^4x \langle \mathrm{tr}\, F^2(x)\, \mathrm{tr}\, F^2(0) \rangle \sim \langle \mathrm{tr}\, F^2(0) \rangle$
- Relations $\frac{\eta}{s} = \frac{1}{4\pi},\ \zeta = 0$ survive in theories with temperature and condensate.