Non-Abelian confinement in $\mathcal{N} = 2$ supersymmetric QCD

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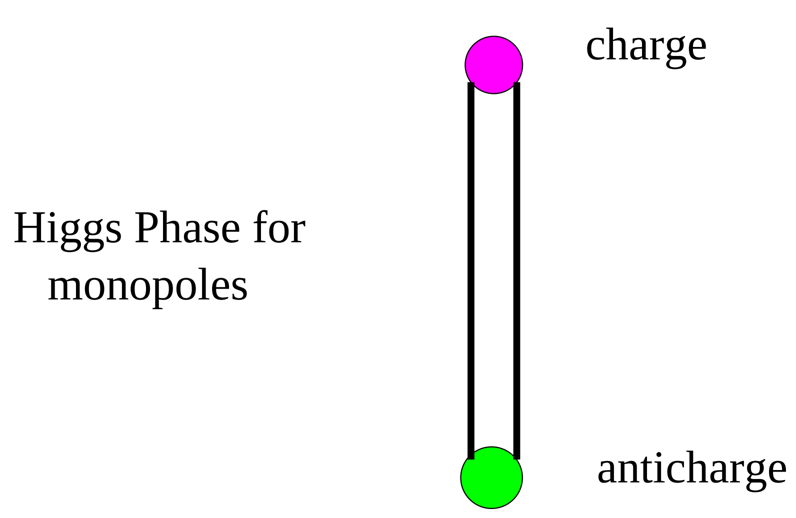
1 Introduction

Standard scenario of quark confinement:

Nambu, Mandelstam and 't Hooft 1970's:

Confinement is a dual Meissner effect upon condensation of monopoles.

Monopoles condense → electric Abrikosov-Nielsen-Olesen flux tubes are formed → electric charges are confined



Seiberg and Witten 1994 : Abelian confinement in $\mathcal{N}=2$ QCD

Cascade gauge symmetry breaking:

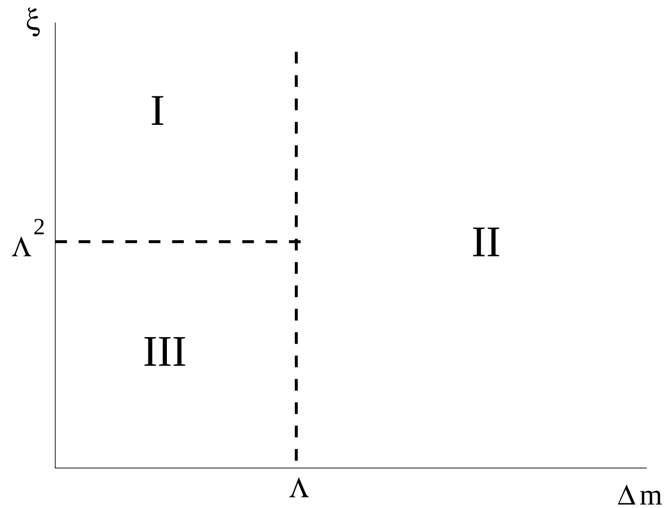
- SU(N)$\rightarrow$ U(1)$^{N-1}$ VEV's of adjoint scalars
- U(1)$^{N-1}$ $\rightarrow$ 0 (or discrete subgroup) VEV's of quarks/monopoles

At the last stage Abelian Abrikosov-Nielsen-Olesen flux tubes are formed.

$$\pi_1(U(1)^{N-1}) = \mathcal{Z}^{N-1}$$

What about non-Abelian confinement?

In both QCD or $\mathcal{N}=1$ supersymmetric QCD there are no adjoint fields



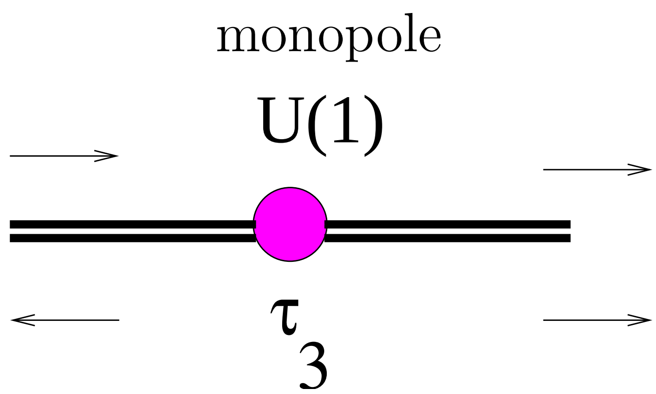
Non-Abelian setup:

$\mathcal{N}=2$ QCD with $U(N)$ gauge group and $N_f > N$ fundamental flavors (quarks).
Fayet-Iliopoulos term ξ.

Three different regimes separated by crossovers (= CMS)
I:

- N scalar quarks condense with VEV's $\sim \sqrt{\xi}$.
  $U(N)$ gauge theory with N_f quarks at weak coupling
- non-Abelian strings which confine monopoles

Example in $U(2)$



III:

- Gauge theory with dual gauge group

$$U(\tilde{N}) \times U(1)^{(N-\tilde{N})}$$

and N_f light dyons, $\tilde{N} = N_f - N$
(with *weight*-like electric charges)

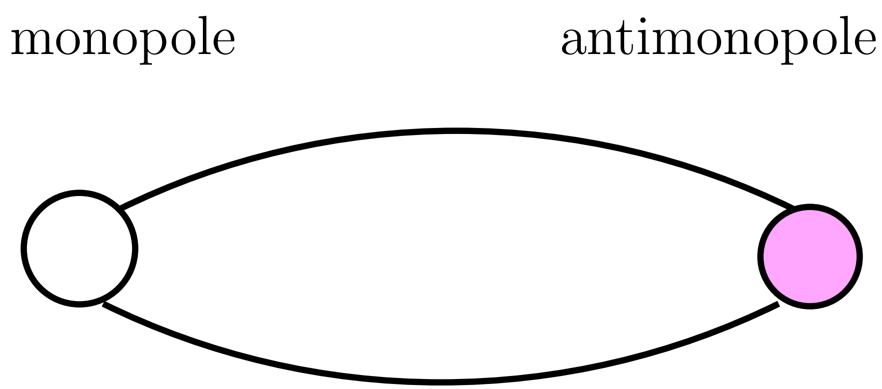
- non-Abelian strings which

still confine **monopoles**

(with *root*-like electric charges)

New mechanism of non-Abelian confinement at strong coupling (small ξ):

- No confinement of color-electric charges. Quarks and gauge bosons are Higgs-screened
- Screened quarks and gauge bosons decay into monopole-antimonopole pairs at CMS. They form stringy mesons.



2 Bulk theory with $N < N_f < 2N$

$\mathcal{N} = 2$ QCD with gauge group $U(N) = SU(N) \times U(1)$ and
N_f flavors of fundamental matter – quarks

+

Fayet-Iliopoulos term of U(1) factor

The bosonic part of the action

$$
\begin{aligned}
S &= \int d^4x \left[\frac{1}{4g_2^2} \left(F^a_{\mu\nu} \right)^2 + \frac{1}{4g_1^2} \left(F_{\mu\nu} \right)^2 + \frac{1}{g_2^2} \left| D_\mu a^a \right|^2 + \frac{1}{g_1^2} \left| \partial_\mu a \right|^2 \right. \\
&+ \left. \left| \nabla_\mu q^A \right|^2 + \left| \nabla_\mu \bar{\tilde{q}}^A \right|^2 + V(q^A, \tilde{q}_A, a^a, a) \right] .
\end{aligned}
$$

Here

$$
\nabla_\mu = \partial_\mu - \frac{i}{2} A_\mu - i A^a_\mu T^a .
$$

The potential is

$$
\begin{aligned}
V(q^A,\tilde{q}_A,a^a,a) &= \frac{g_2^2}{2}\left(\frac{i}{g_2^2}\,f^{abc}\bar{a}^b a^c+\bar{q}_A\,T^a q^A-\tilde{q}_A T^a\,\bar{\tilde{q}}^A\right)^2 \\
&+ \frac{g_1^2}{8}\left(\bar{q}_A q^A-\tilde{q}_A\bar{\tilde{q}}^A-N\xi\right)^2 \\
&+ 2g_2^2\left|\tilde{q}_A T^a q^A\right|^2+\frac{g_1^2}{2}\left|\tilde{q}_A q^A\right|^2 \\
&+ \frac{1}{2}\sum_{A=1}^{N}\left\{\left|(a+\sqrt{2}m_A+2T^a a^a)q^A\right|^2\right. \\
&+ \left.\left|(a+\sqrt{2}m_A+2T^a a^a)\bar{\tilde{q}}^A\right|^2\right\}\,.
\end{aligned}
$$

Region I. Large ξ

$\boxed{\text{Vacuum}}$

Adjoint fields:

$$\langle \frac{1}{2}\, a + T^a\, a^a \rangle = -\frac{1}{\sqrt{2}} \begin{pmatrix} m_1 & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & m_N \end{pmatrix},$$

Quarks

$$\langle q^{kA} \rangle \;=\; \sqrt{\xi} \begin{pmatrix} 1 & \ldots & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 1 & 0 & \ldots & 0 \end{pmatrix}, \qquad \langle \bar{\tilde{q}}^{kA} \rangle = 0,$$

$$k \;=\; 1, ..., N \qquad A = 1, ..., N_f \,,$$

In the equal mass limit $\mathrm{U}(N)_{\mathrm{gauge}} \times \mathrm{SU}(N_f)_{\mathrm{flavor}}$ is broken down to

$$\mathrm{SU}(N)_{C+F} \times \mathrm{SU}(\tilde{N})_F \times \mathrm{U}(1)\,,$$

where $\tilde{N} = N_f - N$.

Quarks and gauge fields fill following representations of the global group:

$$(1,1) \quad (N^2-1,1) \quad (\bar{N},\tilde{N}) \quad (N,\bar{\tilde{N}})$$

3 Non-Abelian bulk duality

Region III. Small ξ and small Δm_{AB} $(\ll \Lambda)$

- First go to the Coulomb branch at $\xi = 0$ in the region II at weak coupling
- Then use Seiberg-Witten curve to go to small Δm_{AB}

We get theory of non-Abelian dyons and dual gauge fields with

$$U(\tilde{N}) \times U(1)^{(N-\tilde{N})}$$

gauge group and N_f non-Abelian dyons and $(N - \tilde{N})$ Abelian dyons.

The non-Abelian gauge factor $U(\tilde{N})$ is not broken by adjoint VEV's in the equal mass limit because this theory is not asymptotically free and stays at weak coupling

Argyres Plesser Seiberg:
$SU(\tilde{N}) \times U(1)^{(N-\tilde{N})}$ was identified at the root of the baryonic branch in SU(N) theory with massless quarks

$\boxed{\text{Vacuum}}$

$$\langle U(\tilde{N})\ adjoints\rangle = -\frac{1}{\sqrt{2}}\begin{pmatrix} m_{N+1} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & m_{N_f} \end{pmatrix}$$

Dyons

$$\langle D^{lA}\rangle \quad = \quad \sqrt{\xi}\begin{pmatrix} 0 & \dots & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & 1 \end{pmatrix}, \qquad \langle \bar{\tilde{D}}\rangle = 0,$$

$$\langle D^{ii}\rangle = \sqrt{\xi}, \qquad l = 1, ..., \tilde{N} \qquad i = \tilde{N}+1, ..., N \qquad A = 1, ..., N_f\ ,$$

$$(1, ..., N)|_{I,II} \quad \Rightarrow \quad (N+1, ..., N_f, \tilde{N}+1, ..., N)|_{III}$$

In the equal mass limit the global group is broken to

$$\mathrm{SU}(N)_F \times \mathrm{SU}(\tilde{N})_{C+F} \times \mathrm{U}(1)$$

Now dyons and dual gauge fields fill following representations of the global group:

$$III: \qquad (1,1) \quad (1,\tilde{N}^2-1) \quad (\bar{N},\tilde{N}) \quad (N,\bar{\tilde{N}})$$

Recall that quarks and gauge bosons of the original theory are in

$$I: \qquad (1,1) \quad (N^2-1,1) \quad (\bar{N},\tilde{N}) \quad (N,\bar{\tilde{N}})$$

(N^2-1) of $\mathrm{SU}(N)$ and $(\tilde{N}^2-1)$ of $\mathrm{SU}(\tilde{N})$

are different states

What is the physical nature of (N^2-1) adjoints in the region III?

Consider W-boson and move from the region I to the region II and then to the region III along Coulomb branch at $\xi = 0$.

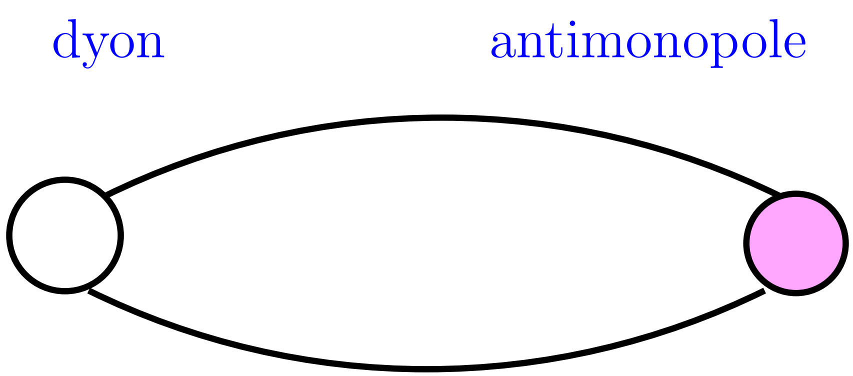
$SU(N)_{C+F} \to U(1)^N$

W-boson is charged with respect to $U(1)^N$

It decay to monopole and antimonopole (dyon with root-like electric charges) at CMS

Now switch on small ξ

At $\xi \neq 0$ monopoles/dyons are confined and cannot move apart



In the region III (N^2-1) of SU(N) are stringy mesons formed by by pairs of monopoles and antimonopoles connected by two strings

Thus stringy mesons formed by by pairs of monopoles and antimonopoles connected by two strings can be in the adjoint representation of the flavor global group.

Then a monopole confined on the non-Abelian string should be in the fundamental representation ??????

We approach this problem studying two dimensional effective low energy theory on the world sheet of the confining string

4 Non-Abelian strings

In search for non-Abelian confinement non-Abelian strings were suggested in $\mathcal{N} = 2$ U(N) QCD

Hanany, Tong 2003

Auzzi, Bolognesi, Evslin, Konishi, Yung 2003

Shifman Yung 2004

Hanany Tong 2004

Z_N Abelian string: Flux directed in the Cartan subalgebra, say for $SO(3) = SU(2)/Z_2$

$$flux \sim \tau_3$$

Non-Abelian string : $\boxed{\text{Orientational zero modes}}$

Rotation of color flux inside $SU(N)_{C+F}$.

5 $CP(N-1)$ model on the string

Consider first $N_f = N$ ($\tilde{N} = 0$)

Z_N solution breaks $SU(N)_{C+F}$ down to $SU(N-1) \times U(1)$ Thus the orientational moduli space is

$$\frac{\mathrm{SU}(N)}{\mathrm{SU}(N-1) \times \mathrm{U}(1)} \sim \mathrm{CP}(N-1)$$

String moduli: translational and orientational

Make them t, z-dependent

Gauge theory formulation of $\mathcal{N} = (2,2)$ supersymmetric $CP(N-1)$ model, $e^2 \to \infty$

$$\begin{aligned} S_{\mathrm{CP}(N-1)} &= \int d^2x \left\{ |\nabla_\alpha n^A|^2 + \frac{1}{4e^2} F^2_{\alpha\beta} + \frac{1}{e^2} |\partial_\alpha \sigma|^2 \right. \\ &+ \left. 2\,|\sigma + \frac{m_A}{\sqrt{2}}|^2 |n^A|^2 + \frac{e^2}{2} \left(|n^A|^2 - 2\beta \right)^2 \right\}, \end{aligned}$$

where

$$\nabla_\alpha = \partial_\alpha - iA_\alpha, \qquad \alpha = 1, 2,$$

while σ is a complex scalar field, superpartner of A_α. The condition

$$n^*_A n^A = 2\beta$$

is implemented in the limit $e^2 \to \infty$.

The coupling constant β is given by

$$\beta = \frac{2\pi}{g_2^2}(\xi)$$

Gauge field can be eliminated:

$$A_k = -\frac{i}{4\beta}\, \bar{n}_A \overleftrightarrow{\partial_\alpha}\, n^A \qquad \sigma = 0$$

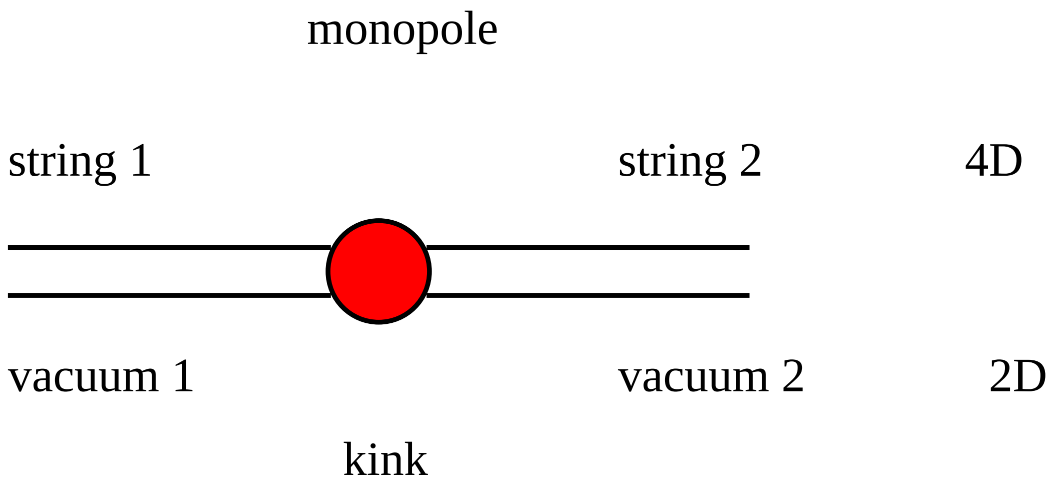
Number of degrees of freedom $= 2N - 1 - 1 = 2(N-1)$

In 2D $CP(N-1)$ model on the string we have

N vacua = N Z_N strings

and kinks interpolating between these vacua

Kinks = confined monopoles



6 World sheet duality, $N_f > N$

Large ξ, region I

Effective low energy theory on the world sheet of the non-Abelian string

$N_f > N$: semilocal non-Abelian strings

n^P – orientations

ρ^K – sizes

$\mathcal{N} = (2,2)$ supersymmetric U(1) gauge theory in the limit $e^2 \to \infty$

$$
\begin{aligned}
S &= \int d^2x \left\{ |\nabla_\alpha n^P|^2 + |\tilde{\nabla}_\alpha \rho^K|^2 + \frac{1}{4e^2} F^2_{\alpha\beta} + \frac{1}{e^2} |\partial_\alpha \sigma|^2 \right. \\
&+ \left. 2\left|\sigma + \frac{m_P}{\sqrt{2}}\right|^2 \left|n^P\right|^2 + 2\left|\sigma + \frac{m_K}{\sqrt{2}}\right|^2 \left|\rho^K\right|^2 + \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - 2\beta\right)^2 \right\},
\end{aligned}
$$

$$
P = 1, ..., N\,, \qquad K = N+1, ..., N_f\,, \qquad \tilde{\nabla}_k = \partial_k + iA_k\,.
$$

With respect to the U(1) gauge field the fields n^P and ρ^K have charges +1 and -1, respectively.

Small ξ, region III

We have $\mathrm{U}(\tilde{N})$ theory with $N_f = N + \tilde{N}$ flavors

$\tilde{N}$ – orientations $\tilde{\rho}^K$

N – sizes $\tilde{n}^P$

$$
\begin{aligned}
S_{\rm dual} &= \int d^2x \left\{ |\nabla_\alpha \tilde{\rho}^K|^2 + |\tilde{\nabla}_\alpha \tilde{n}^P|^2 + \frac{1}{4e^2} F^2_{\alpha\beta} + \frac{1}{e^2}\, |\partial_\alpha \sigma|^2 \right. \\
&+ \left. 2\left|\sigma + \frac{m_P}{\sqrt{2}}\right|^2 \left|\tilde{n}^P\right|^2 + 2\left|\sigma + \frac{m_K}{\sqrt{2}}\right|^2 \left|\tilde{\rho}^K\right|^2 + \frac{e^2}{2}\left(|\tilde{\rho}^K|^2 - |\tilde{n}^P|^2 - 2\tilde{\beta}\right)^2 \right\},
\end{aligned}
$$

$$
P = 1, ..., N\,, \qquad K = N+1, ..., N_f\,,
$$

Coupling constant

$$4\pi\tilde{\beta}(\xi) = \frac{8\pi^2}{\tilde{g}_2^2}(\xi) = (N - \tilde{N}) \ln \frac{\Lambda}{\tilde{g}\sqrt{\xi}} \gg 1$$

Both bulk and world sheet dual theories have the same β-functions with the first coefficient $(\tilde{N} - N) < 0$. They are both not asymptotically free, therefore the coupling constant $\tilde{\beta}$ is positive at $\Lambda \gg \sqrt{\xi}$

In the original sigma model in the region I

$$4\pi\beta(\xi) = \frac{8\pi^2}{g_2^2}(\xi) = (N - \tilde{N}) \ln \frac{g\sqrt{\xi}}{\Lambda} \gg 1$$

Thus

$$\tilde{\beta} = -\beta.$$

The dual theory can be interpreted as a continuation of the original sigma model to negative values of the coupling constant β.

7 Mirror description

Hori and Vafa, 2000

Exact superpotential

$$W_{\rm mirror} = -\frac{\Lambda}{4\pi}\left\{\sum_{P=1}^{N} X_P - \sum_{K=N+1}^{N_f} Y_K - \sum_{P=1}^{N}\frac{m_P}{\Lambda}\ln X_P + \sum_{K=N+1}^{N_f}\frac{m_K}{\Lambda}\ln Y_K\right\}$$

supplemented by the constraint

$$\prod_{P=1}^{N} X_P = \prod_{K=N+1}^{N_f} Y_K .$$

Kink masses:

$$M^{\rm BPS} = 2\left|\mathcal{W}_{\rm mirror}^{{\rm CP}(N-1)}(l=1) - \mathcal{W}_{\rm mirror}^{{\rm CP}(N-1)}(l=0)\right|$$

Consider

$$m_P \sim m_{P'}, \qquad m_K \sim m_{K'}, \qquad m_P - m_K \sim \Delta m,$$

where $P, P' = 1, ..., N$ and $K, K' = N+1, ..., N_f$

We are interested in the region of small mass differences:

$$|\Delta m_{PP'}| \sim |\Delta m_{KK'}| \lesssim \tilde{\Lambda}_{LE} \ll |\Delta m| \ll \Lambda\,,$$

where

$$\tilde{\Lambda}_{\mathrm{LE}}^{\tilde{N}} \equiv \frac{(\Delta m)^N}{\Lambda^{N-\tilde{N}}}, \qquad \tilde{m} \equiv \frac{1}{\tilde{N}} \sum_{K=N+1}^{N_f} m_K$$

Two types of kinks:

K-kinks, number $= \tilde{N}$

$$M_K^{\mathrm{BPS}} \approx \left| \frac{N-\tilde{N}}{2\pi}\, \tilde{\Lambda}_{LE} \left(e^{\frac{2\pi}{\tilde{N}}\,i} - 1 \right) - i\,(m_K - \tilde{m}) \right|$$

P-kinks, number $= N$

$$M_P^{\mathrm{BPS}} \approx \left| \frac{N-\tilde{N}}{2\pi}\, \tilde{\Lambda}_{LE} \left(e^{\frac{2\pi}{\tilde{N}}\,i} - 1 \right) - i\,(m_P - \tilde{m}) \right|$$

In the limit

$$m_P = m_{P'}, \qquad m_K = m_{K'}, \qquad m_P - m_K = \Delta m,$$

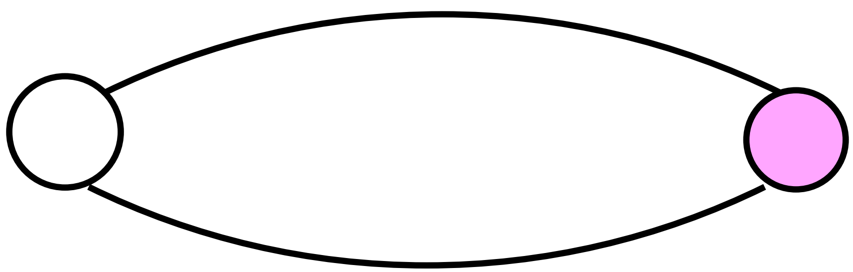
(where $P, P' = 1, ..., N$ and $K, K' = N+1, ..., N_f$) and small ξ (region III) global

$$\mathrm{SU}(N) \times \mathrm{SU}(\tilde{N})_F \times \mathrm{U}(1)$$

group restores. We have $N_f = N + \tilde{N}$ kinks in $(N, 1) + (1, \tilde{N})$ representation.

Kinks = confined monopoles

Stringy mesons



are in

$$(1, 1) \quad (N^2 - 1, 1) \quad (1, \tilde{N}^2 - 1) \quad (\bar{N}, \tilde{N}) \quad (N, \bar{\tilde{N}})$$

representations of global group.

8 Conclusions

- Two dimensional results confirm the bulk picture:

  Stringy mesons are in the adjoint representation of the global group.

- In both original and dual theories confined states are monopoles

- Non-Abelian confinement = Higgs screening + decay on CMS + magnetic string formation

  Abelian confinement = formation of *electric* strings connecting quarks

- Constituent quark = monopole