5d Models of Mesons and Baryons

Andrea Wulzer

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Yes, but only in the large–N_c expansion:

('t Hooft, Veneziano, Coleman, Witten ...)

- Meson couplings scale like $1/\sqrt{N_c}$
- Meson masses scale like N_c^0
- Mesons come in infinite towers

Weakly interacting theory of an ∞ number of mesons.

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Weak coupling: $$g_\rho \simeq 4\pi/\sqrt{N_c} \to 0$$

$1/N_c$ is the only known candidate coupling for a dual theory

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Attractiveness of the 5d models:

1. "Holographic" implementation of χ_{SB} automatically leads to KK towers of vector (and scalar) mesons.
2. Extremely predictive framework (description of ρ, ω, a_1, ρ' ...)
3. Easy bookkeping of $1/N_c$ factors:
$$1/\sqrt{N_c} \sim \text{5d coupling}$$
4. 5d models are valid effective theories:
$$\Lambda_5/m_\rho \to \infty \text{ for } N_c \to \infty$$

Main Limitation:

Absence of high spin states and of Regge phenomenology
Higher spins above the 5d cutoff, no known large–N_c reason

String models could do better, but we have no candidate

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Where are the Baryons?

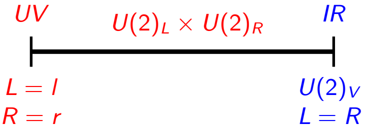
Baryons are solitons (skyrmions) at large–N_c (Witten)

- size $\sim N_c^0/\Lambda_{QCD}$
- mass $\sim N_c\Lambda_{QCD}$

Are there skyrmions in the 5d model?

The Model:

$$Z_{QCD} = \int \mathcal{D}\mathbf{L}_M \mathcal{D}\mathbf{R}_M e^{iS_5[\mathbf{L},\mathbf{R}]}$$



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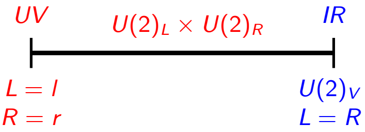

The 5d action is: $S_5 = S_{CS} + S_g$

$$S_{CS} = -i\frac{N_c}{24\pi^2}\int\left[\omega_5(\mathbf{L}) - \omega_5(\mathbf{R})\right] ,$$ reproduces the QCD Anomaly

$$S_g = -\int \frac{M_5}{2}\left\{\mathrm{Tr}\left[L_{MN}L^{MN}\right] + \frac{\alpha^2}{2}\widehat{L}_{MN}\widehat{L}^{MN} + \{L \leftrightarrow R\}\right\} + \dots .$$

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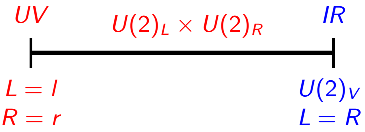

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Metric: $ds^2 = a(z)^2\,(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$, $a(z) = \frac{L}{z}$, $z \in [0, L]$

At two derivatives order we have 3 parameters: M_5, L, α.

Some interesting features:

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- Vector Meson Dominance holds in our model.
- Decay constants and meson couplings all scale like:

$$F_i \sim \sqrt{M_5}, \quad g_i \sim 1/\sqrt{M_5}$$

  Correct N_c scalings if $\alpha, L \sim N_c^0$ and $M_5 \sim N_c$.
- Automatic Zweig Rule for $m_\omega = m_\rho$ only, $F_\omega = \alpha F_\rho$.
- "Modified" KSFR relation: $m_\rho^2 \simeq 3 g_{\rho\pi\pi}^2 F_\pi^2$ (exp=2.1).

Relations valid at the leading $1/M_5$ $(1/N_c)$ order.

Though $N_c = 3$, what if we compare with real hadrons?

Comparison with experiments:

	Experiment	AdS_5	Deviation
m_ρ	775	824	+6%
m_{a_1}	1230	1347	+10%
m_ω	782	824	+5%
F_ρ	153	169	+11%
F_ω/F_ρ	0.88	0.94	+7%
F_π	87	88	+1%
$g_{\rho\pi\pi}$	6.0	5.4	−10%
L_9	$6.9 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$	−10%
L_{10}	$-5.2 \cdot 10^{-3}$	$-6.2 \cdot 10^{-3}$	−12%
$\Gamma(\omega \to \pi\gamma)$	0.75	0.81	+8%
$\Gamma(\omega \to 3\pi)$	7.5	6.7	−11%
$\Gamma(\rho \to \pi\gamma)$	0.068	0.077	+13%
$\Gamma(\omega \to \pi\mu\mu)$	$8.2 \cdot 10^{-4}$	$7.3 \cdot 10^{-4}$	−10%
$\Gamma(\omega \to \pi ee)$	$6.5 \cdot 10^{-3}$	$7.3 \cdot 10^{-3}$	+12%

Fit with 3 parameters to 14 observables, Total RMS Error of 11%.

Observables selected to have $< 10\%$ experimental error.

Skyrmions in 4 and 5d

The Skyrme's Idea

Static Goldstones: $U(\mathbf{x}) : \; S^3 \; \rightarrow \; SU(2) \; \sim \; S^3 \, , \quad \Pi_3(S^3) = Z$

$$B \; = \; \frac{1}{24\pi^2} \int d^3x \, \epsilon^{ijk} \mathrm{Tr} \left[U\partial_i U^\dagger \; U\partial_j U^\dagger \; U\partial_k U^\dagger \right] \; \in Z \, .$$

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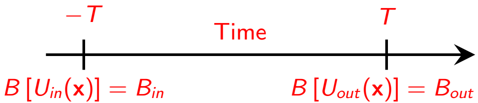
B is exactly conserved topological charge:



$\Rightarrow \quad B_{out} = B_{in}$

$B\left[1 + i\,\pi\right] = 0; \quad B\left[U_{Skyrme}\right] = 1;$ Skyrmions are Baryons

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However, Skyrme's idea cannot be implemented in the σ–model:

$$S_2 = \int \frac{F_\pi^2}{4} \mathrm{Tr}\left\{\partial_\mu U \partial^\mu U^\dagger\right\} \quad \rightarrow \quad E_2[\rho] \propto F_\pi^2\, \rho$$

$$S_4 \supset \int \frac{F_\pi^2}{\Lambda^2} \mathrm{Tr}\left\{\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\right]^2\right\} \quad \rightarrow \quad E_4[\rho] \propto F_\pi^2/\Lambda^2\, 1/\rho$$

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Skyrmion's size is $\rho_S \propto 1/\Lambda$. All operators contribute the same.

Skyrmions are UV–dominated, crucially depend on resonances

The 5d Skyrmion

(with A. Pomarol and G. Panico)

Topological charge: $$B = \frac{1}{32\pi^2}\int_0^L d^4x\, \epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \mathrm{Tr}\left[L^{\hat{\mu}\hat{\nu}} L^{\hat{\rho}\hat{\sigma}} - R^{\hat{\mu}\hat{\nu}} R^{\hat{\rho}\hat{\sigma}}\right]$$

Can be shown equivalent to Skyrme's.

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Can be shown equivalent to Skyrme's.

The energy is: $E = M_5 \int_0^L d^4x \, \left[F^2 + \gamma L \, \epsilon^{\cdots} \widehat{A} F F \right]$

For $\gamma << 1$ expand around an instanton of size ρ:

$$E(\rho) \; = \; 8\pi^2 M_5 \left[1 + \frac{\rho}{2L} + \gamma^2 \frac{L^2}{\rho^2} \right]$$

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Consistent with large–N_c: baryon's size must be finite

In AdS/QCD, this comes because of the N_c in the global anomaly

What about the 5d cutoff?

From standard NDA considerations:

$$\Lambda_5 \simeq (16\pi^2) M_5 \left(\frac{4}{N_c}\right)^{2/3} = m_\rho \, \mathcal{O}(N_c^{1/3})$$

The CS lowers the cutoff, but still well defined expansion

We have $\rho_S \sim L \sim 1/m_\rho$, expansion parameter:

$$\frac{1}{\rho_S \Lambda_5} \to 0 \quad \text{for} \quad N_c \to \infty$$

5d models provide Calculable Implementation of Skyrme's idea!!

Static Properties of the Nucleons:

	Experiment	AdS_5	Deviation
M_N	940 MeV	1130 MeV	$+20\%$
μ_S	0.44	0.34	-30%
μ_V	2.35	1.79	-31%
g_A	1.25	0.70	-79%
$\sqrt{\langle r^2_{E,S}\rangle}$	0.79 fm	0.88 fm	$+11\%$
$\sqrt{\langle r^2_{E,V}\rangle}$	0.93 fm	∞	
$\sqrt{\langle r^2_{M,S}\rangle}$	0.82 fm	0.92 fm	$+12\%$
$\sqrt{\langle r^2_{M,V}\rangle}$	0.87 fm	∞	
$\sqrt{\langle r^2_{A}\rangle}$	0.68 fm	0.76 fm	$+12\%$
μ_p/μ_n	-1.461	-1.459	$+0.1\%$

Significantly better agreement than original Skyrme model.

What changes with pion masses? in progress with O. Domenech, G. Panico

Holography forces to introduce 5d scalar

	Experiment (MEV)	AdS_5 (MEV)	Deviation
m_π	135 MeV	134 MeV	0.6%
$m_{\pi(1300)}$	1300 MeV	1230 MeV	5.6%
m_ρ	775 MeV	783 MeV	1.0%
m_ω	782 MeV	783 MeV	0.1%
$m_{a_1(1260)}$	1230 MeV	1320 MeV	7.6%
$m_{a_0(980)}$	980 MeV	1040 MeV	6.5%
$m_{f_0(980)}$	980 MeV	1040 MeV	6.5%
f_π	92 MeV	89 MeV	3.6%
f_ρ	153 MeV	149 MeV	2.7%
f_ω	140 MeV	149 MeV	6.4%
$g_{\rho\pi\pi}$	6.0	4.89	22.7%
$g_{\omega\pi\gamma}$	0.72	0.71	1.1%
$g_{\rho\pi\gamma}$	0.22	0.24	7.9%
$g_{\omega\rho\pi}$	15.0	15.6	3.7%
RMSE			7.7%

Static Properties of the Nucleons (very preliminary)

	Experiment	AdS_5	Deviation
M_N	940 MeV	$\sim$ 1070 MeV	$\sim$ 14%
μ_S	0.44	0.38	16%
μ_V	2.35	$\sim$ 1.2	$\sim$ 100%
g_A	1.25	$\sim$ 0.6	$\sim$ 100%
$\sqrt{\langle r^2_{E,S}\rangle}$	0.79 fm	0.82 fm	4%
$\sqrt{\langle r^2_{E,V}\rangle}$	0.93 fm	0.97 fm	4%
$\sqrt{\langle r^2_{M,S}\rangle}$	0.82 fm	0.84 fm	3%
$\sqrt{\langle r^2_{M,V}\rangle}$	0.87 fm	0.87 fm	0.5%
$\sqrt{\langle r^2_{A}\rangle}$	0.68 fm	$\sim$ 0.6 fm	$\sim$ 13%

Conclusions

- 5d models mimic several expected features of large–N_c QCD
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- 5d models mimic several expected features of large–N_c QCD
- Provide a valid and "economic" description of QCD hadrons
- Motivated by Bottom–Up phenomenological considerations:

  A relation with AdS/CFT? (seems unlikely)
- Calculable Skryme model is automatically implemented

In Progress

The 5d model should be improved in several aspects:

1. Add quark masses to the 5d skyrmion
   (almost done with O. Domenech and Giuliano Panico)
2. Include $U(1)_A$ anomaly and η' mass
   (few literature, some ideas with G.Panico)

The Sakai–Sugimoto Model

Equivalent to ours in the meson sector.

Only differs by the shape of the warp factor.

Perturbative Field Theory regime for $N_c, \lambda \to \infty$ ($M_{St} \sim \sqrt{\lambda}/L$).

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Perturbative Field Theory regime for $N_c, \lambda \to \infty$ ($M_{St} \sim \sqrt{\lambda}/L$).

The CS is subleading in $1/\lambda$: $\gamma \sim 1/(M_{St} L)^2 \sim 1/\lambda$.

For $\gamma \ll 1$ the 5d skyrmion becomes small instanton:

$$E(\rho) \sim \rho^2/L^2 + \gamma^2 L^2/\rho^2 \qquad \Rightarrow \qquad \rho \sim \sqrt{\gamma} L \sim 1/M_{St}$$

Strong indication of non–calculability.