

Test of QCD through hadron form factor measurements at large momentum transfer

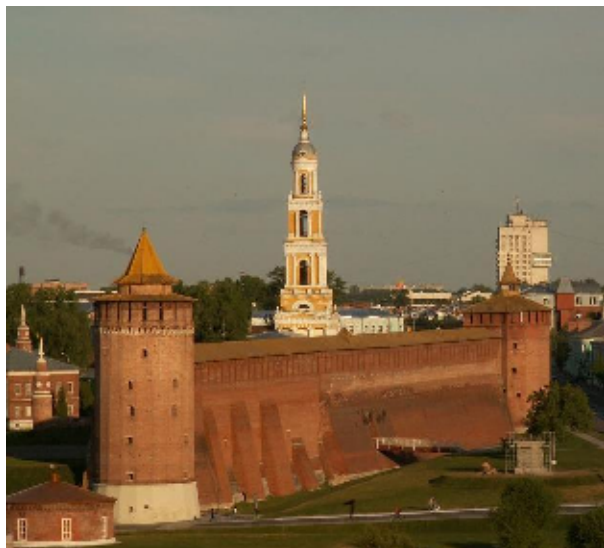
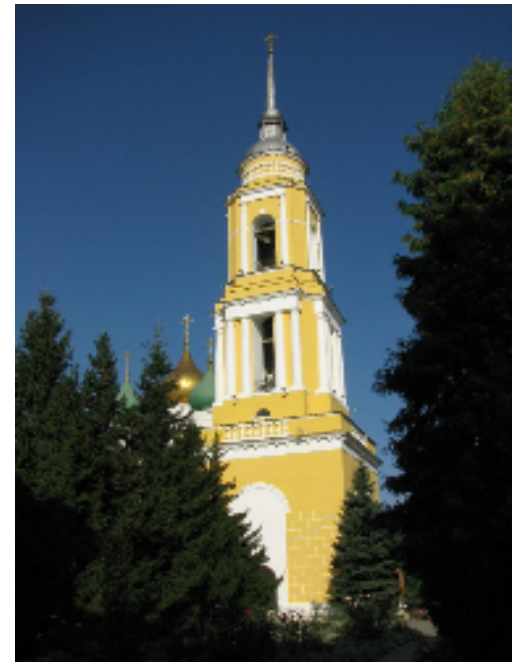
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Egle Tomasi-Gustafsson
IRFU, SPhN-Saclay, and
IN2P3- IPN Orsay France



QUARKS-2010
16th International Seminar on High Energy Physics
Kolomna, Russia, 6-12 June, 2010.

PLAN

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- *Introduction*
- *What is new?*
 - *Space-like region : new data at JLab*
 - *Rosenbluth separation (unpolarized scattering)*
 - *Recoil proton polarization*
(A.I. Akhiezer and M.P. Rekalo in 1968)
 - *Time-like region*
 - *Plans at BES-III*
 - *Plans at PANDA*
- *Global description in a wide kinematical region*
 - *Asymptotics*
 - *Counting rules*

Hadron Electromagnetic Form factors

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The Nobel Prize in Physics 1961

"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"



Robert Hofstadter

1/2 of the prize

USA

Stanford University
Stanford, CA, USA

Characterize the **internal structure of a particle** ($\neq$ point-like)

Elastic form factors contain information on the **hadron ground state**.

In a P- and T-invariant theory, the EM structure of a particle of spin S is defined by **$2S+1$ form factors**.

Neutron and proton form factors are different.

Deuteron: 2 structure functions, but 3 form factors.

Playground for theory and experiment

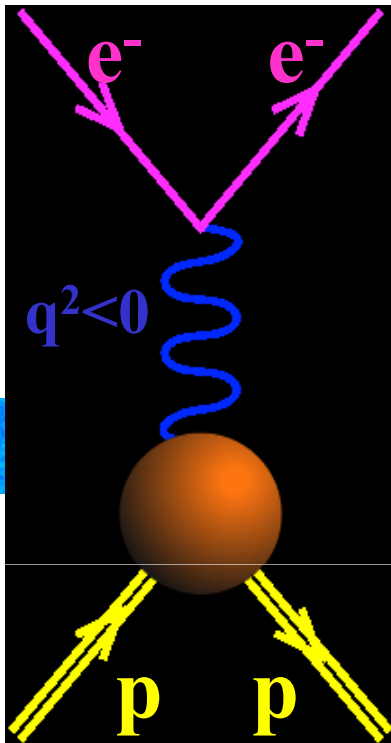
at low q^2 probe **the size of the nucleus**,
at high q^2 test **QCD scaling**

Electromagnetic Interaction

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The electron vertex is known, γ_μ

The interaction is carried by a virtual photon of mass q^2

The proton vertex is parametrized in terms of FFs: Pauli and Dirac F_1, F_2

$$\Gamma_\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M} F_2(q^2)$$

or in terms of Sachs FFs:

$$GE = F_1 - \tau F_2, \quad GM = F_1 + F_2, \quad \tau = -q^2/4M^2$$

What about high order radiative corrections?

Crossing Symmetry

Scattering and annihilation channels:

- Described by the same amplitude :

$$|\overline{\mathcal{M}}(e^\pm h \rightarrow e^\pm h)|^2 = f(s, t) = |\overline{\mathcal{M}}(e^+ e^- \rightarrow \bar{h} h)|^2,$$

- function of two kinematical variables, s and t

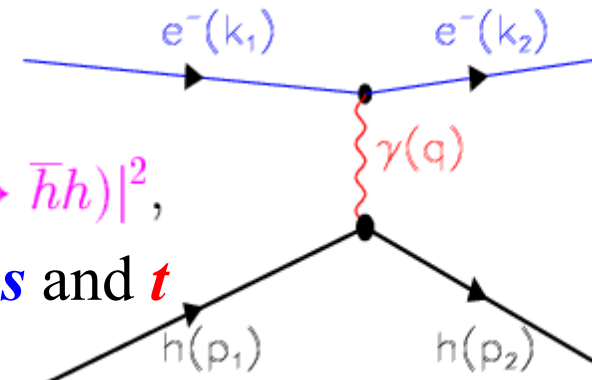
$$s = (k_1 + p_1)^2$$

$$t = (k_1 - k_2)^2$$

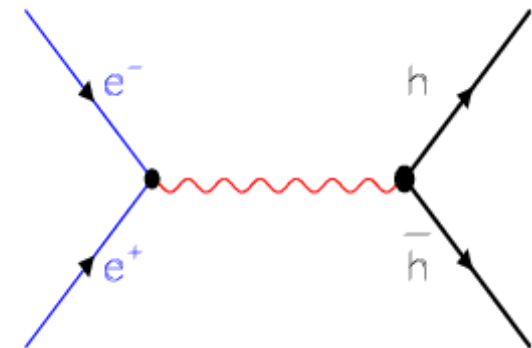
- which scan different kinematical regions

$$\cos^2 \tilde{\theta} = 1 + \frac{st + (s - M^2)^2}{t(\frac{t}{4} - M^2)} \rightarrow 1 + \frac{ctg^2 \frac{\theta}{2}}{1 + \tau}$$

$$e^- + h \rightarrow e^- + h$$



$$e^- + e^+ \rightarrow \bar{h} + h$$



Analyticity

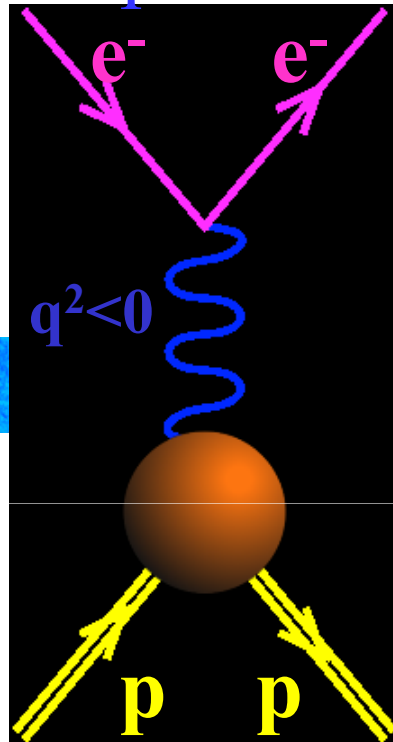
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Space-like



$$GE(0)=1$$

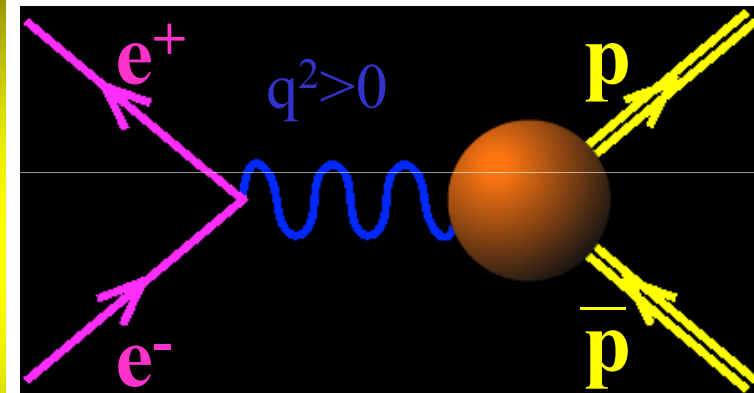
$$GM(0)=\mu_p$$

FFs are real

Unphysical region
 $p+\bar{p} \leftrightarrow e^+ + e^- + \pi$

Asymptotics
 - QCD
 - analyticity

Time-like



FFs are complex

$$e+p \rightarrow e+p$$

$$q^2=4m_p^2$$

$$GE=GM$$

$$p+\bar{p} \leftrightarrow e^+ + e^-$$

q^2

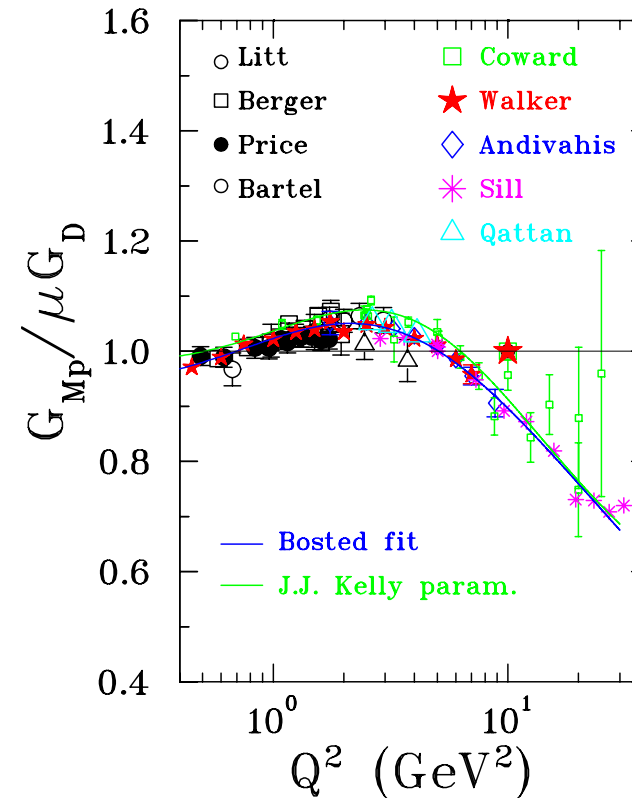
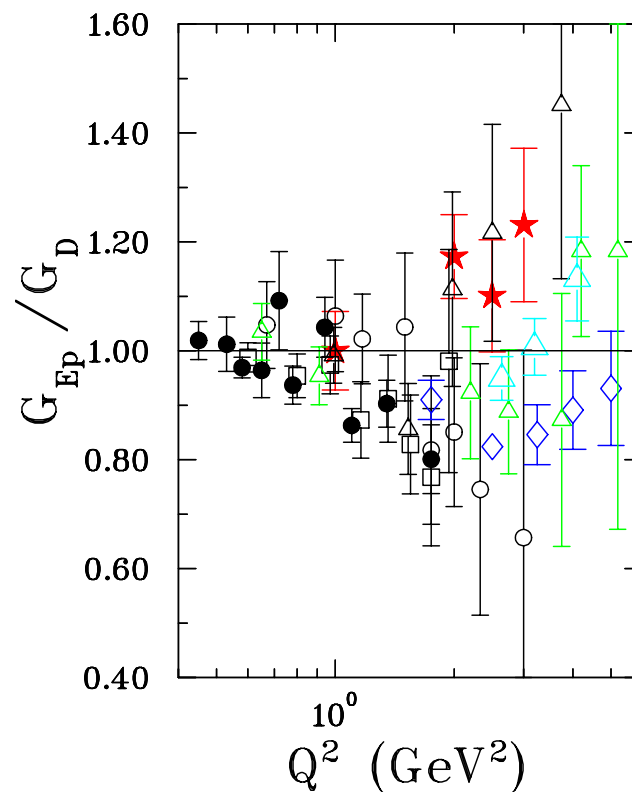
Proton Form Factors ...before

Dipole approximation: $G_D = (1 + Q^2/0.71 \text{ GeV}^2)^{-2}$

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Rosenbluth separation/ Polarization observables

Dipole Approximation

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- Classical approach

- Nucleon FF (in the Breit system) are Fourier transform of the charge or magnetic distribution.

$$\overrightarrow{\gamma^*(\mathbf{q}_B)} \quad \overleftarrow{P_1(\mathbf{q}_B/2)} \quad \overrightarrow{P_2(\mathbf{q}_B/2)}$$

Dipole approximation: $G_D = (1 + Q^2/0.71 \text{ GeV}^2)^{-2}$

- *The dipole approximation corresponds to an exponential density distribution.*

- $\rho = \rho_0 \exp(-r/r_0),$
- $r_0^2 = (0.24 \text{ fm})^2, \langle r^2 \rangle \sim (0.81 \text{ fm})^2$
 $\leftrightarrow m_D^2 = 0.71 \text{ GeV}^2$

Dipole Approximation and pQCD

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Dimensional scaling

$$- F_n(Q^2) = C_n [1/(1+Q^2/m_n^2)]^{n-1},$$

• $m_n^2 = n\beta^2$, <quark momentum squared>

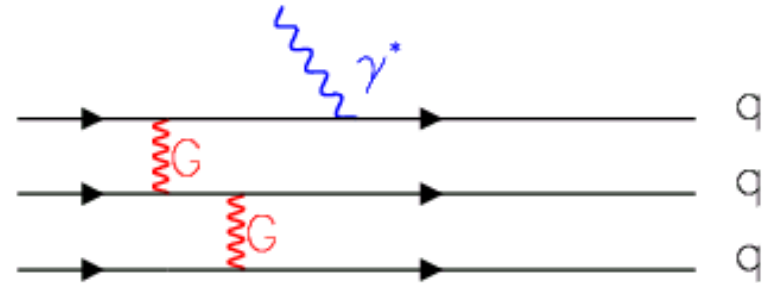
• n is the number of constituent quarks

– Setting $\beta^2 = (0.471 \pm 0.010) \text{ GeV}^2$ (fitting pion data)

• pion: $F_\pi(Q^2) = C_\pi [1/(1+Q^2/0.471 \text{ GeV}^2)]^1$,

• nucleon: $F_N(Q^2) = C_N [1/(1+Q^2/0.71 \text{ GeV}^2)]^2$,

• deuteron: $F_d(Q^2) = C_d [1/(1+Q^2/1.41 \text{ GeV}^2)]^5$



V. A. Matveev, R. M. Muradian, and A. N. Tavkhelidze (1973), Brodsky and Farrar (1973), Politzer (1974), Chernyak & Zhitnisky (1984), Efremov & Radyuskin (1980)...

The Rosenbluth separation

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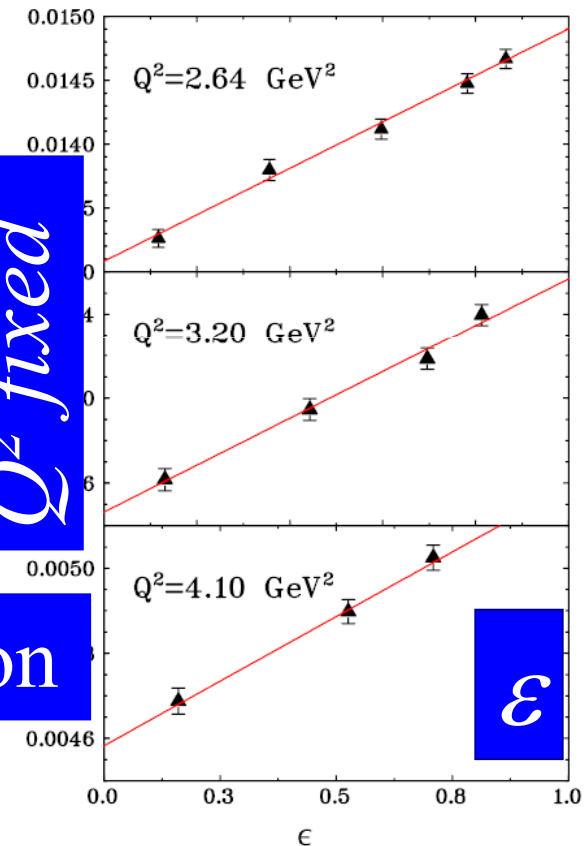


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{Mott} \frac{1}{(1+\tau)} \left(G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2) \right)$$

$$\varepsilon = \left(1 + 2(1+\tau) \tan^2 \left(\frac{\theta_e}{2} \right) \right)^{-1}, \quad \tau = \frac{Q^2}{4M^2}$$

$$\sigma_R = \varepsilon G_E^2 + \tau G_M^2$$

Q^2 fixed



Linearity of the reduced cross section

→ $\tan^2 \theta_e$ dependence

→ Holds for 1γ exchange only

PRL 94, 142301 (2005)

The polarization method (1967)

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SOVIET PHYSICS - DOKLADY

VOL. 13, NO. 6

DECEMBER, 1968

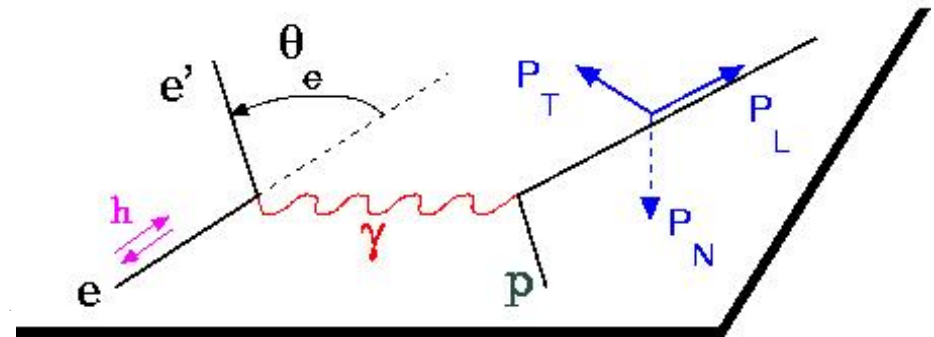
PHYSICS

POLARIZATION PHENOMENA IN ELECTRON SCATTERING BY PROTONS IN THE HIGH-ENERGY REGION

Academician A. I. Akhiezer* and M. P. Rekalo

Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR
Translated from Doklady Akademii Nauk SSSR, Vol. 180, No. 5,
pp. 1081-1083, June, 1968
Original article submitted February 26, 1967

$$s_2 \frac{d\sigma}{d\Omega_R} = 4p_2 \frac{(s \cdot q)}{1 + \tau} \Gamma(\theta, \varepsilon_1) \left[\tau G_M (G_M + G_E) - \frac{1}{4\varepsilon_1} G_M (G_E - \tau G_M) \right],$$



The polarization induces a term in the cross section proportional to $G_E G_M$
Polarized beam and target or
polarized beam and recoil proton polarization

The polarization method (exp)

Transferred polarization is:

C. Perdrisat et al, JLab-GEp collaboration

$$P_n = 0$$

$$\pm h P_t = \mp h 2\sqrt{\tau(1+\tau)} G_E^p G_M^p \tan\left(\frac{\theta_e}{2}\right) / I_0$$

$$\pm h P_l = \pm h (E_e + E_{e'}) (G_M^p)^2 \sqrt{\tau(1+\tau)} \tan^2\left(\frac{\theta_e}{2}\right) / M / I_0$$

Where, $h = |h|$ is the beam helicity

$$I_0 = (G_E^p(Q^2))^2 + \frac{\tau}{\epsilon} (G_M^p(Q^2))^2$$

$$\Rightarrow \frac{G_E^p}{G_M^p} = - \frac{P_t}{P_l} \frac{E_e + E_{e'}}{2M} \tan\left(\frac{\theta_e}{2}\right)$$

The simultaneous measurement of P_t and P_l reduces the systematic errors

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Polarization experiments - Jlab

A.I. Akhiezer and M.P. Rekalo, 1967

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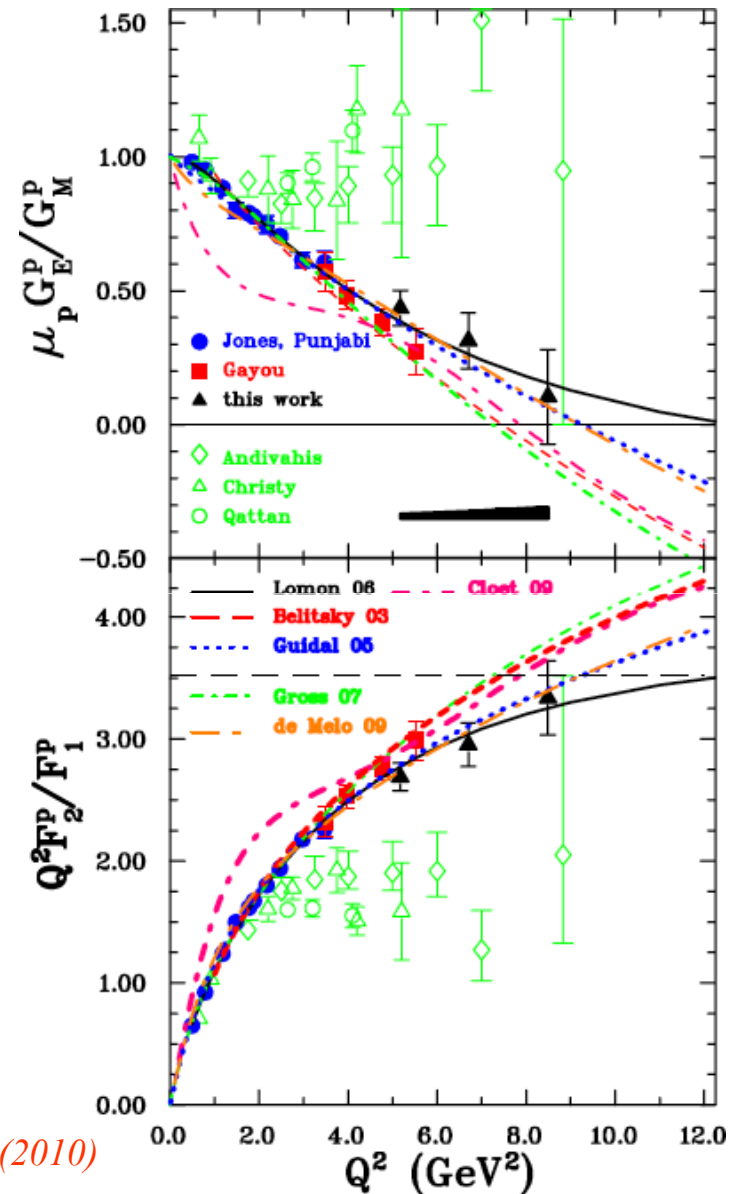
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Gep collaboration

- 1) "standard" **dipole function** for the nucleon magnetic FFs **GMp** and **GMn**
- 2) **linear deviation** from the dipole function for the electric proton FF **Gep**
- 3) **QCD scaling** not reached
- 3) **Zero crossing** of Gep?
- 4) **contradiction between polarized and unpolarized measurements**



A.J.R. Puckett et al, PRL (2010)

Issues

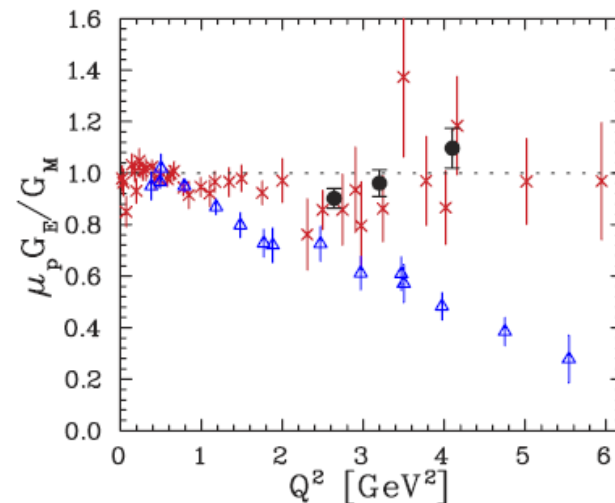
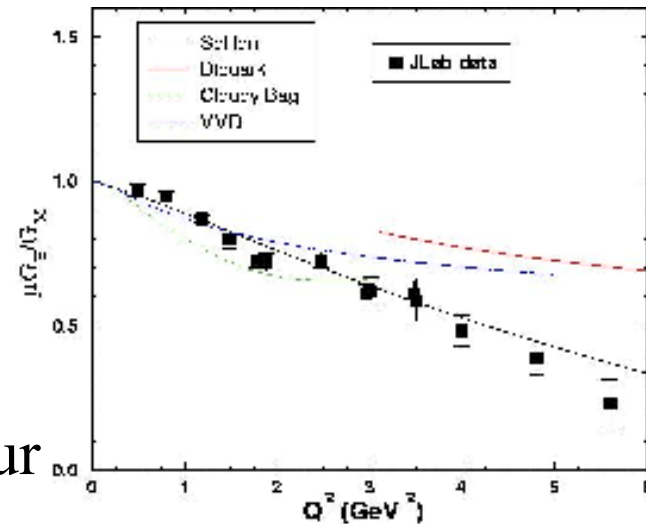
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- Some models (IJL 73, Di-quark, soliton..) predicted such behavior before the data appeared

BUT

- Simultaneous description of the four nucleon form factors...
- ...in the space-like and in the time-like regions
- Consequences for the light ions description
- When pQCD starts to apply?
- Source of the discrepancy



The nucleon form factors

E. T.-G., F. Lacroix, Ch. Duterte, G.I. Gakh, EPJA (2005)

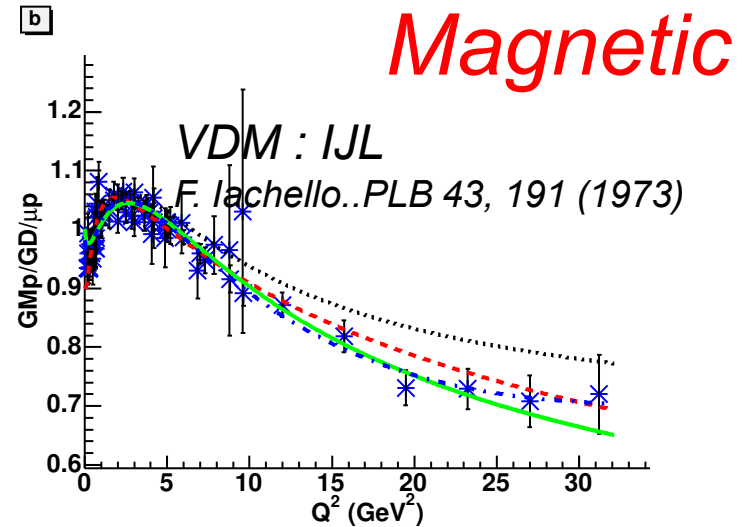
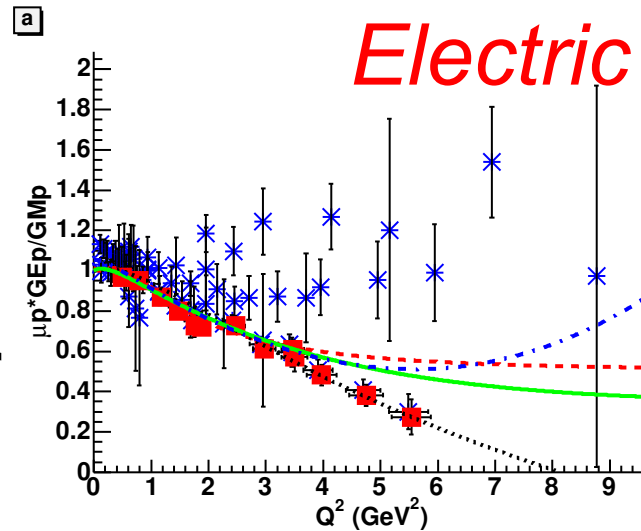
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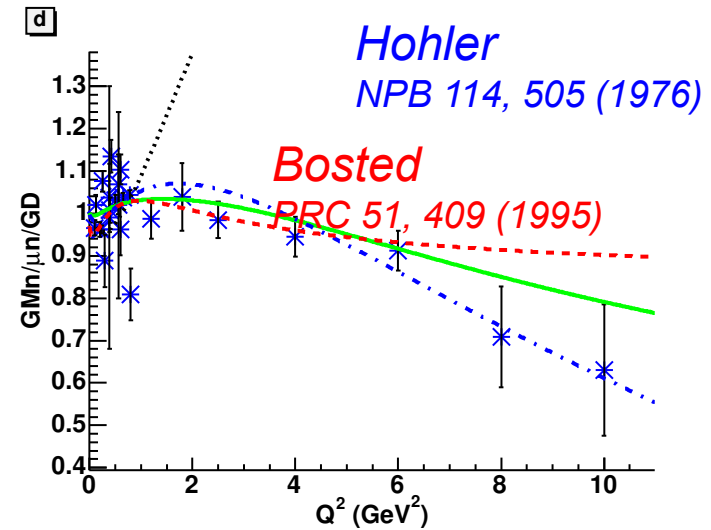
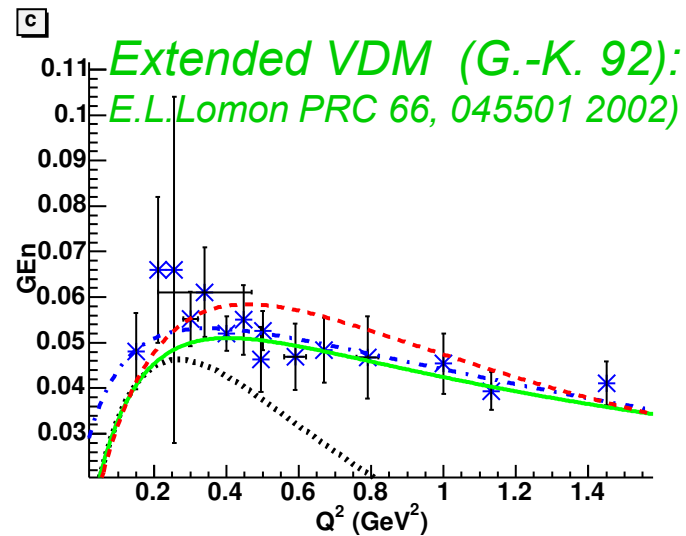
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proton



neutron



Models in T.L. region

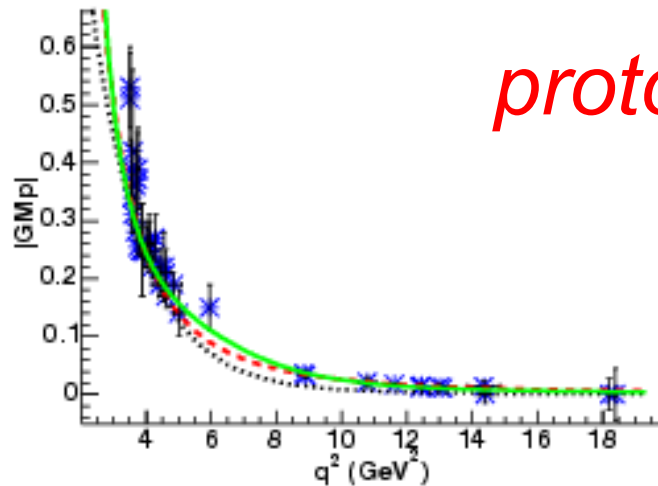
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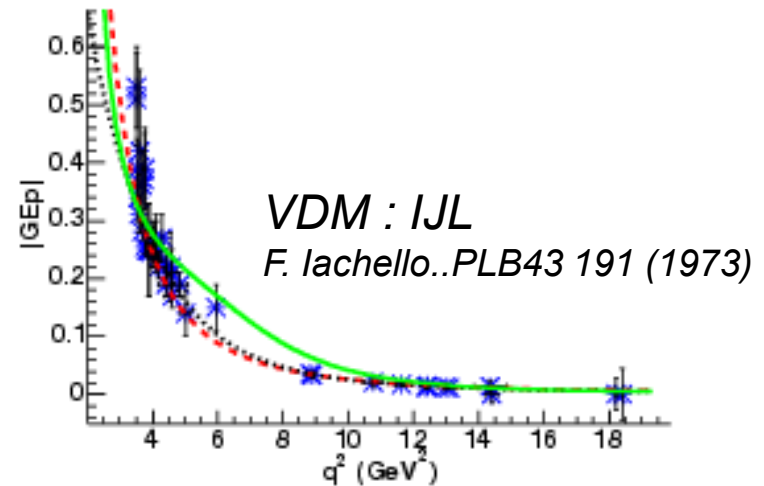


a



proton

b

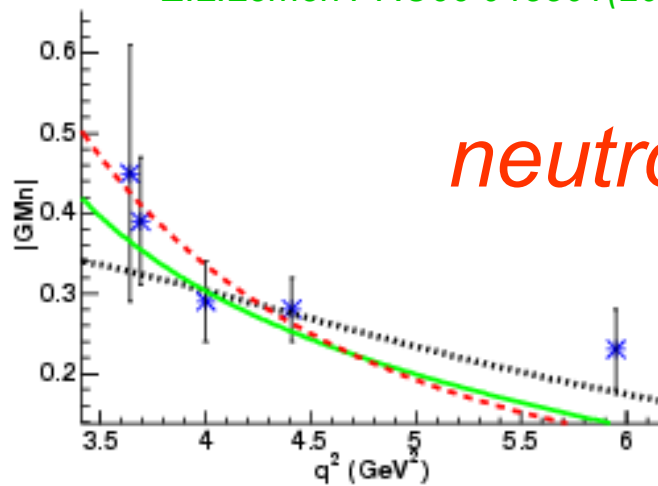


VDM : IJL

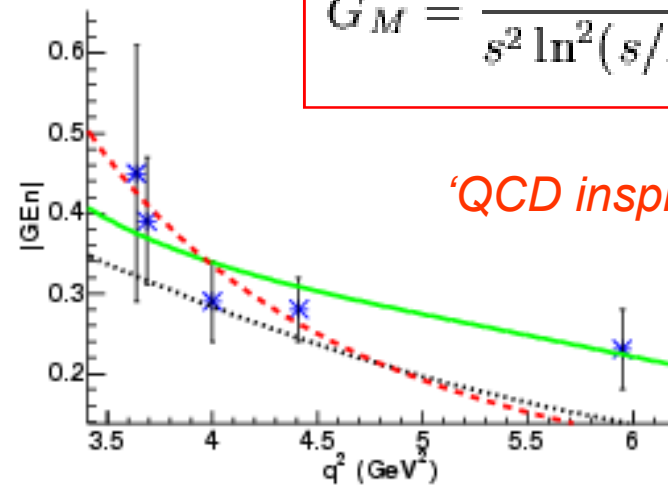
F. Iachello..PLB43 191 (1973)

Extended VDM (G.-K. 92):
E.L.Lomon PRC66 045501(2002)

c



neutron



'QCD inspired'

$$G_M = \frac{A}{s^2 \ln^2(s/\Lambda^2)}$$

E. T-G., F. Lacroix, C. Duterte, G.I. Gakh EPJA 2005

Time-like observables: $|G_E|^2$ and $|G_M|^2$.

- The cross section for $\bar{p} + p \rightarrow e^+ + e^-$ (1 γ -exchange):

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{8m^2\sqrt{\tau-1}} [\tau|G_M|^2(1 + \cos^2\theta) + |G_E|^2\sin^2\theta]$$

θ : angle between e^- and $\bar{p}$ in cms.

A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto, Il Nuovo Cimento XXIV, 170 (1962)

B. Bilenkii, C. Giunti, V. Wataghin, Z. Phys. C 59, 475 (1993).

G. Gakh, E.T-G., Nucl. Phys. A761,120 (2005).

As in SL region:

- Dependence on q^2 contained in FFs
- Even dependence on $\cos^2\theta$ (1γ exchange)
- No dependence on sign of FFs
- Enhancement of magnetic term

but TL form factors are complex!

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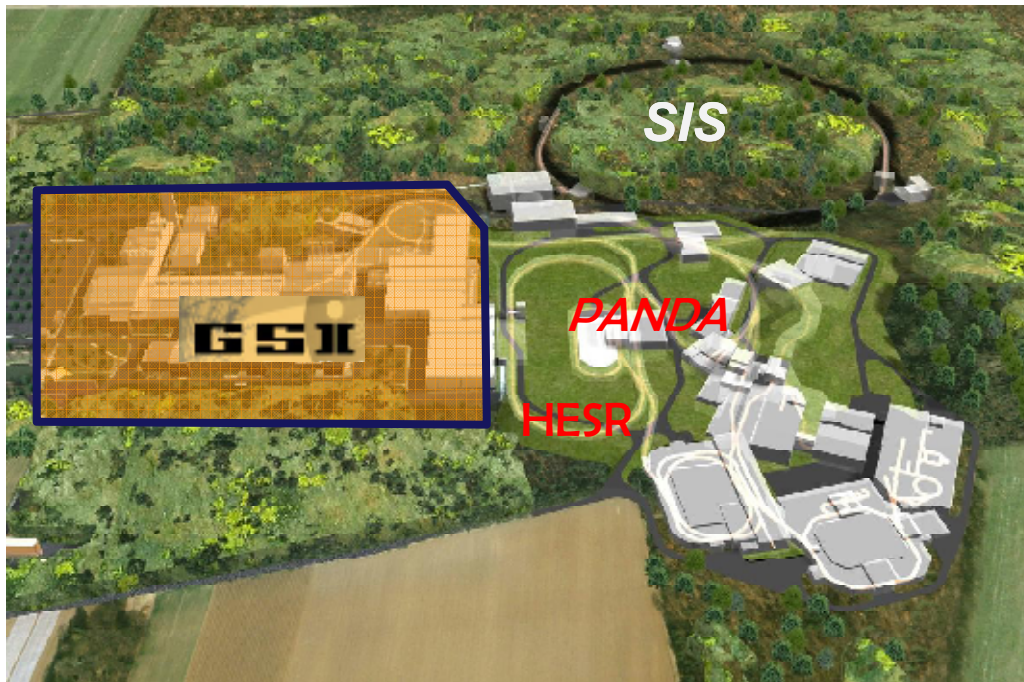


Proton-Antiproton Annihilation

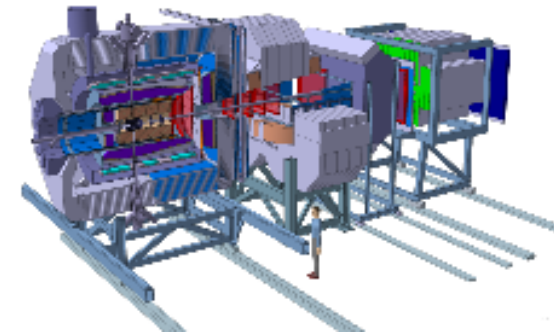
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panda



need of

- highest rates
- good resolution
- Good Particle Identification

Parameters of HESR

- Injection of p at 3.7 GeV
- Slow synchrotron (1.5-14.5 GeV/c)
- Storage ring for internal target operation
- Luminosity up to $L \sim 2 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$
- Beam cooling (stochastic & electron)

<http://www-panda.gsi.de/>

Physical Background

M. Sudol et al, arXiv:0907.4478 [nucl-ex]

- **3 body reactions**

→ kinematical constraints

→ PID

- **2 body reactions (hadrons)**

$\pi^+\pi^-$, K^+K^-

$$\bar{p} + p \rightarrow e^+ + e^-$$

$$\frac{\sigma_{\pi^+\pi^-}}{\sigma_{e^+e^-}} \sim 10^6$$

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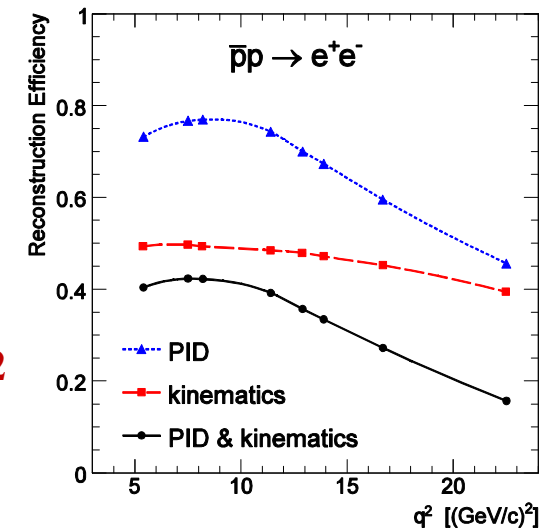
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High statistics GEANT4 simulations

- **< few % misidentified events / $\cos\theta_{CM}$ bin**

- **< 1% on the total cross section up to 16 GeV^2**



It is possible to discriminate e^+e^- from $\pi^+\pi^-$

Expected Results

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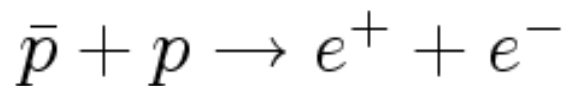
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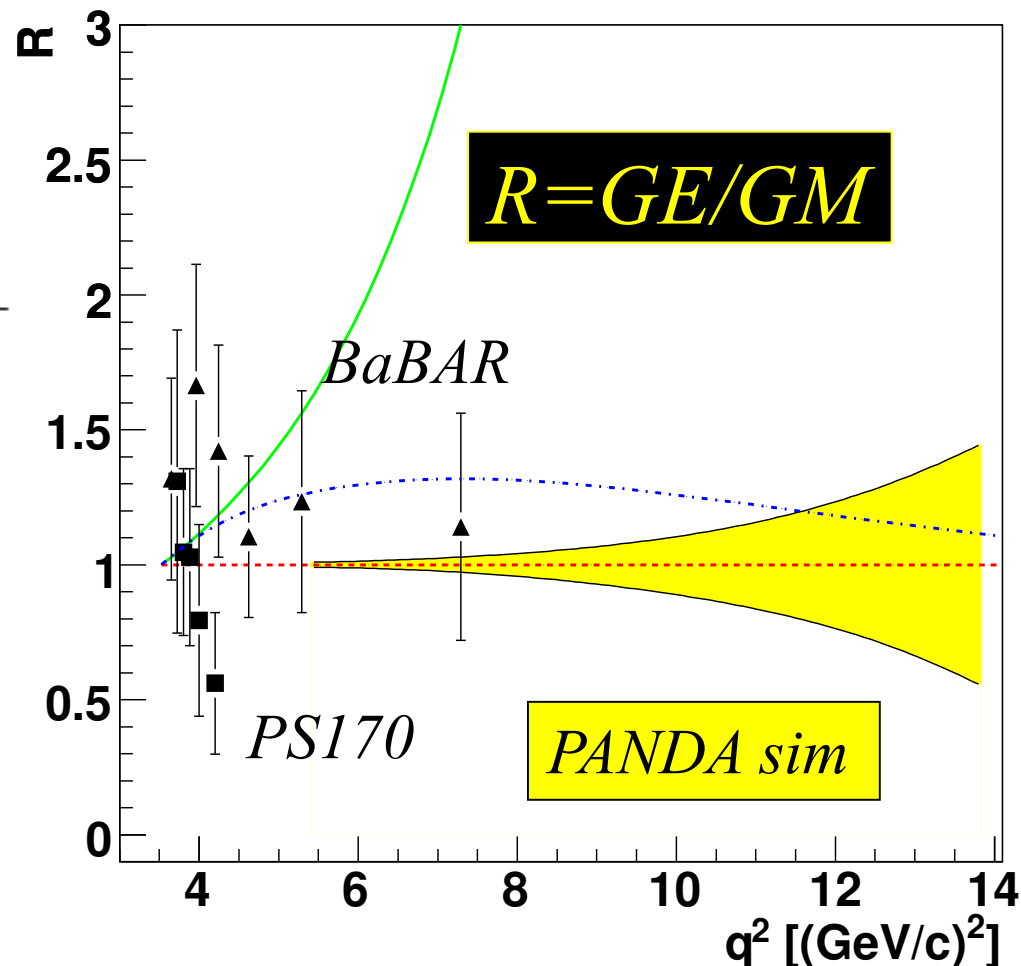
$$\mathcal{L} = 2 \cdot 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$$

100 jours



Individual
determination
of
GE and GM
up to large Q^2

*M. Sudol et al, EPJA 2010,
arXiv:0907.4478 [nucl-ex]*



$\bar{p} + p \rightarrow e^+ + e^-$ Angular Distributions

The form of the differential cross section:

$$\frac{d\sigma}{d(\cos \theta)} = \frac{\pi\alpha^2}{8m^2\sqrt{\tau-1}} [\tau |G_M|^2 (1 + \cos^2 \theta) + |G_E|^2 \sin^2 \theta]$$

is equivalent to:

$$\frac{d\sigma}{d(\cos \theta)} = \sigma_0 [1 + \mathcal{A} \cos^2 \theta]$$

Cross section at 90°

$$\sigma_0 = \frac{\alpha^2}{4q^2} \sqrt{\frac{\tau}{\tau-1}} \left(|G_M|^2 + \frac{1}{\tau} |G_E|^2 \right)$$

Angular asymmetry

$$\mathcal{A} = \frac{\tau |G_M|^2 - |G_E|^2}{\tau |G_M|^2 + |G_E|^2} = \frac{\tau - \mathcal{R}^2}{\tau + \mathcal{R}^2}$$
$$\mathcal{R} = |G_E|/|G_M|$$

E. T-G. and M. P. Rekalo, Phys. Lett. B 504, 291 (2001)

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Spin Observables

Analyzing power, A

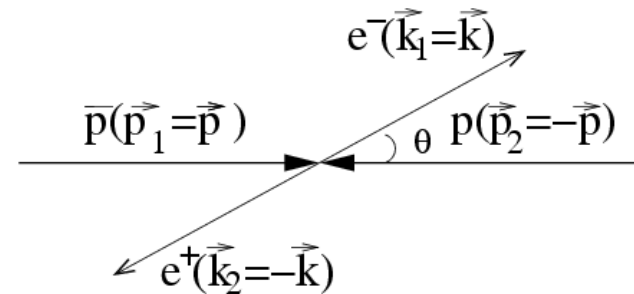
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$$\frac{d\sigma}{d\Omega}(P_y) = \left(\frac{d\sigma}{d\Omega}\right)_0 [1 + \mathcal{A}P_y],$$



$$\mathcal{A} = \frac{\sin 2\theta \operatorname{Im} G_E^* G_M}{D\sqrt{\tau}}, \quad D = |G_M|^2(1 + \cos^2 \theta) + \frac{1}{\tau}|G_E|^2 \sin^2 \theta$$

Double spin observables

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{xx} = \sin^2 \theta \left(|G_M|^2 + \frac{1}{\tau} |G_E|^2 \right) \mathcal{N},$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{yy} = -\sin^2 \theta \left(|G_M|^2 - \frac{1}{\tau} |G_E|^2 \right) \mathcal{N},$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{zz} = \left[(1 + \cos^2 \theta) |G_M|^2 - \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right] \mathcal{N},$$

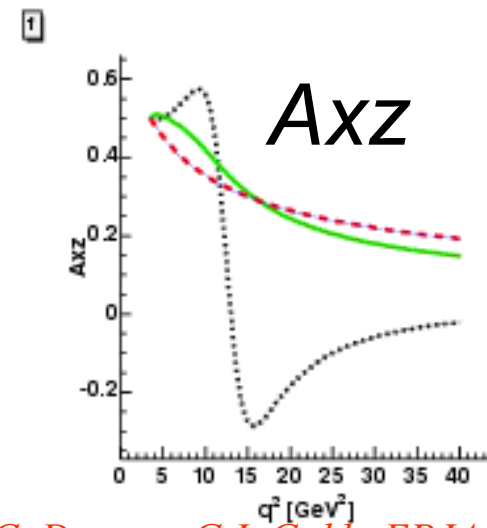
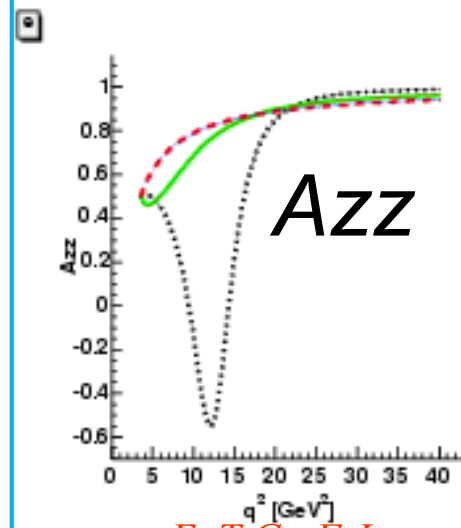
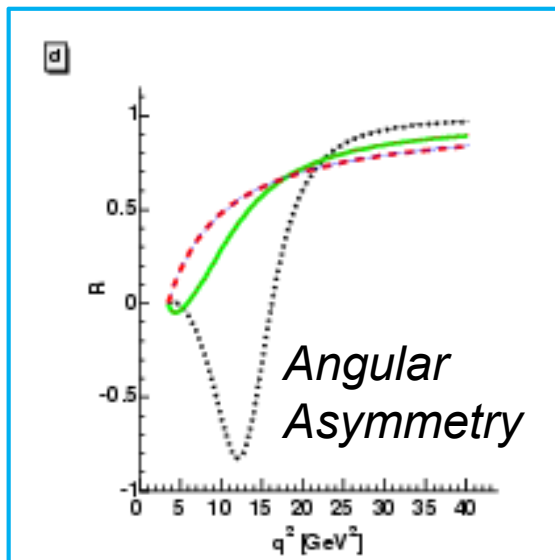
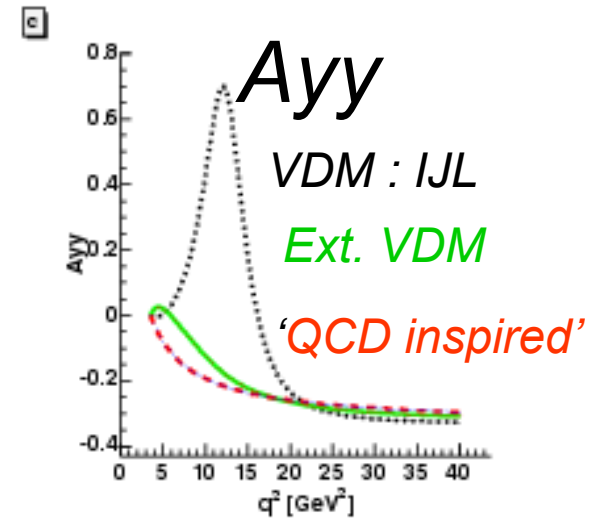
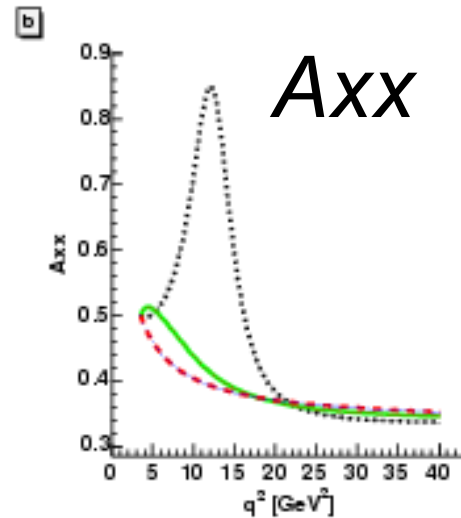
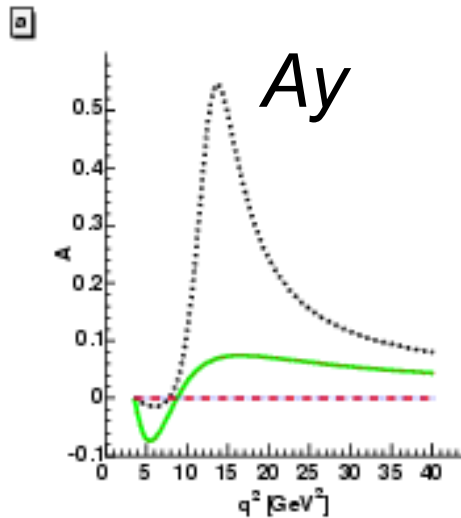
$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{xz} = \left(\frac{d\sigma}{d\Omega}\right)_0 A_{zx} = \frac{1}{\sqrt{\tau}} \sin 2\theta \operatorname{Re} G_E G_M^* \mathcal{N}.$$

Models in T.L. Region (polarization)

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E. T-G., F. Lacroix, C. Duterte, G.I. Gakh, EPJA 2005

Phragmén-Lindelöf theorem

Asymptotic properties for analytical functions

*If $f(z) \rightarrow a$ as $z \rightarrow \infty$
along a straight line,
and $f(z) \rightarrow b$ as $z \rightarrow \infty$
along another
straight line,
and $f(z)$ is regular
and bounded
in the angle between,
then $a=b$ and $f(z) \rightarrow a$
uniformly in the angle.*

$$\lim_{q^2 \rightarrow -\infty} F^{(SL)}(q^2) = \lim_{q^2 \rightarrow \infty} F^{(TL)}(q^2)$$

space-like *time-like*
 $(e^- + p \rightarrow e^- + p)$ $(e^+ + e^- \leftrightarrow \bar{p} + p)$

$$- F^{(TL)}(q^2) \rightarrow \text{real, if } q^2 \rightarrow \infty$$

$$\mathcal{F} = |Im(F_2/F_1)|/|Re(F_2/F_1)| = \Delta$$
$$|P_y| = \Delta \quad \Delta=0.05, 0.1$$
$$\mathcal{R} = |F_2/F_1|_{TL}/|F_2/F_1|_{SL} = 1 + \Delta$$

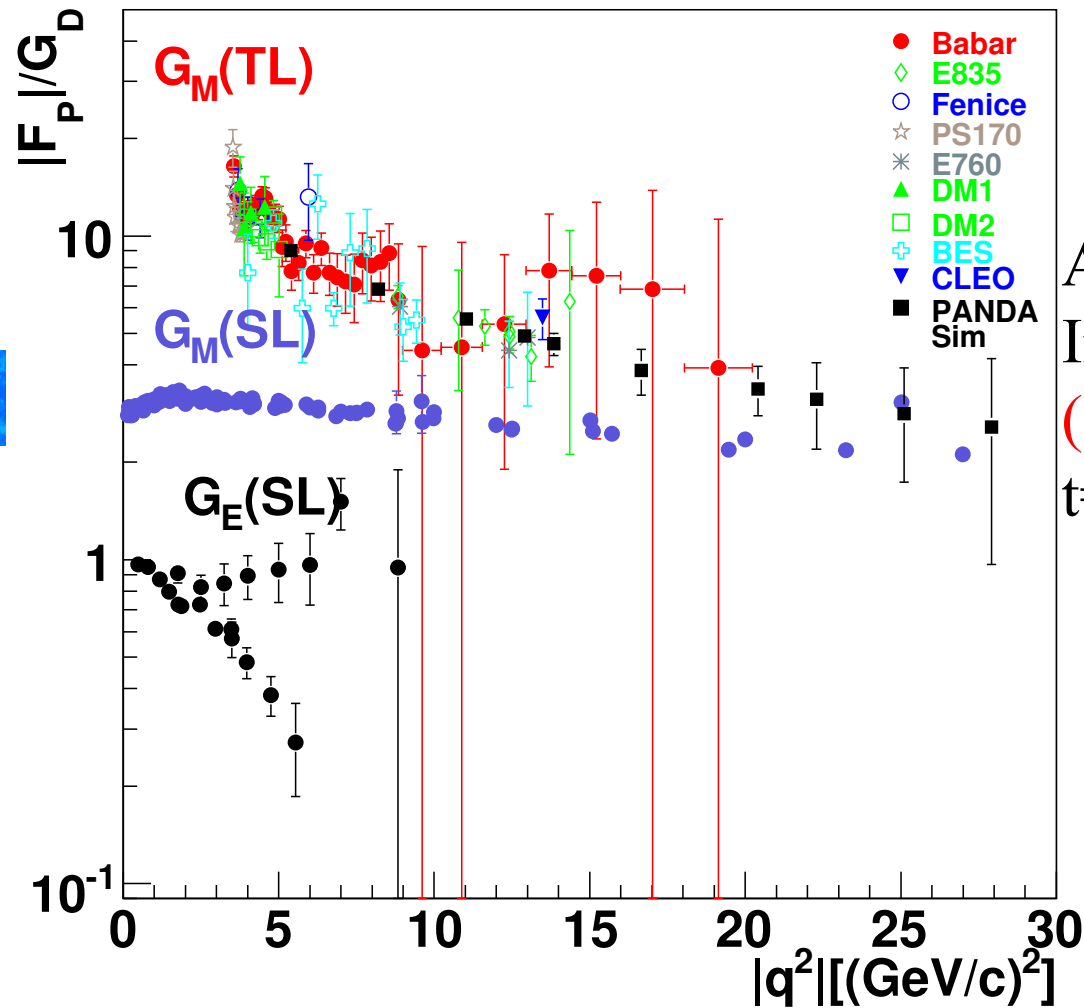
E. T-G. and G. Gakh, Eur. Phys. J. A 26, 265 (2005)

Phragmén-Lindelöf theorem

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Connection with QCD asymptotics?

Applies to NN and $\bar{N}N$ Interaction

(Pomeranchuk theorem)

$t=0$: not a QCD regime!

E. T-G. and M. P. Rekalo, Phys. Lett. B 504, 291 (2001)

E. T-G. e-Print: [arXiv:0907.4442](https://arxiv.org/abs/0907.4442) [nucl-th]

Exclusive processes: hadronic ratios

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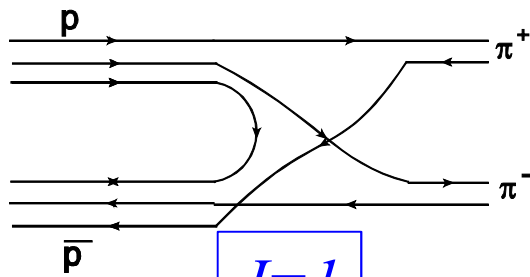
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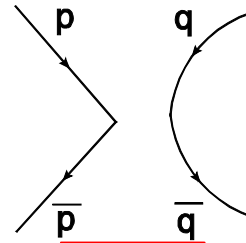


$$R_s = \frac{(p\bar{p})_{J=0} \rightarrow K\bar{K}}{(p\bar{p})_{J=0} \rightarrow \pi^+\pi^-}, \quad R_p = \frac{(p\bar{p})_{J=1} \rightarrow K\bar{K}}{(p\bar{p})_{J=1} \rightarrow \pi^+\pi^-}$$

Threshold region ($L=0$): $R_p \ll R_s \simeq 1$



Quark rearrangement



Excited vacuum

$$\frac{Y_{KK}}{Y_{\pi\pi}} = \frac{1/2}{3 + 1/2} \frac{\beta_K}{\beta_\pi} \left(\frac{\beta_s}{\beta_u} \right)^2 = 0.108$$

Experimental value at LEAR:

$$R = f(K^+K^-)/f(\pi^+\pi^-) = 0.108 \pm 0.007.$$

E.A. Kuraev, E.T-G, PRD 81,017501 (2010)

Conclusions

- Comparison of *scattering* and *annihilation* channels: model independent statements

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- Study of the *validity of one photon exchange approximation*: basic question for deriving information on the hadron structure : *in annihilation processes all FFs information is contained in a precise measurement of the angular distribution.*

- Determination of time-like form factors up to large q^2
Tests of *validity of pQCD and analyticity.*

Polarization observables?

- *basic program with anti-proton beams:*

Panda Physics Performance Report

e-Print: **arXiv:0903.3905** [hep-ex]

Symmetry relations (annihilation)

- Differential cross section at complementary angles:

The SUM cancels the 2γ contribution:

$$\frac{d\sigma_+}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta) = 2 \frac{d\sigma^{Born}}{d\Omega}(\theta)$$

The DIFFERENCE enhances the 2γ contribution:

$$\begin{aligned} \frac{d\sigma_-}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) = 4N \Big[(1 + x^2) \text{Re} G_M \Delta G_M^* + \\ + \frac{1 - x^2}{\tau} \text{Re} G_E \Delta G_E^* + \sqrt{\tau(\tau - 1)} x (1 - x^2) \text{Re} \left(\frac{1}{\tau} G_E - G_M \right) F_3^* \Big] \end{aligned}$$

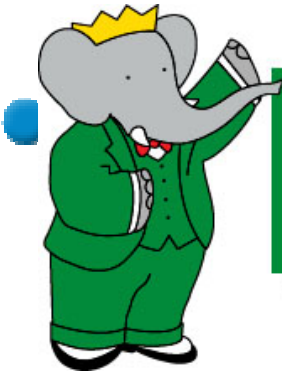
$$\tau = \frac{q^2}{4m^2}, \quad x = \cos\theta$$

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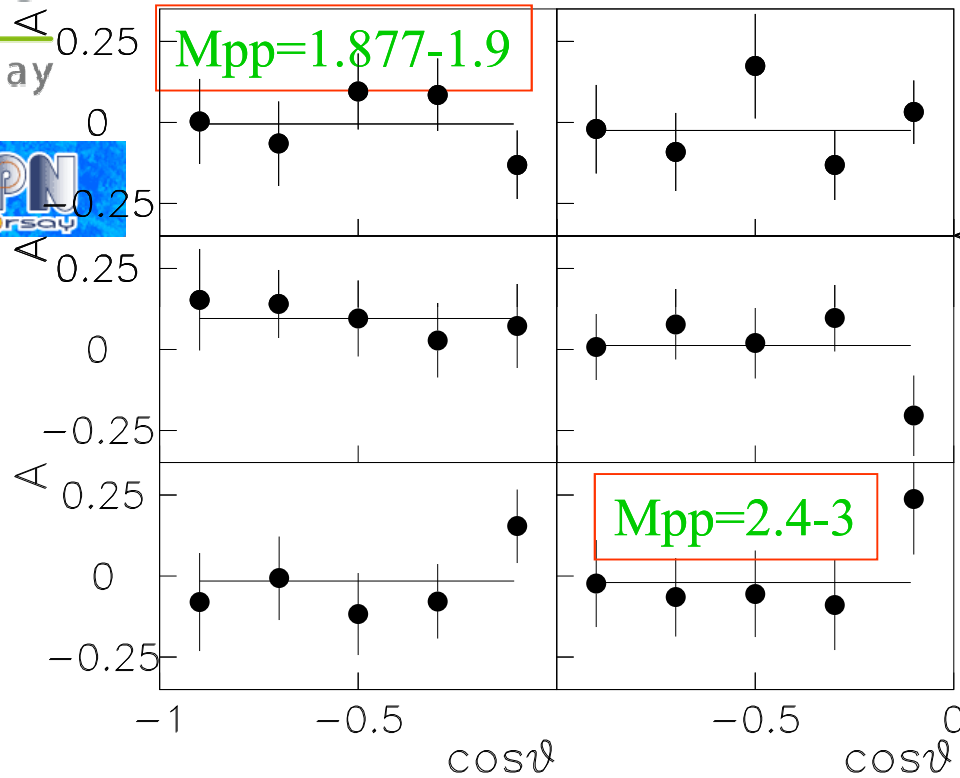
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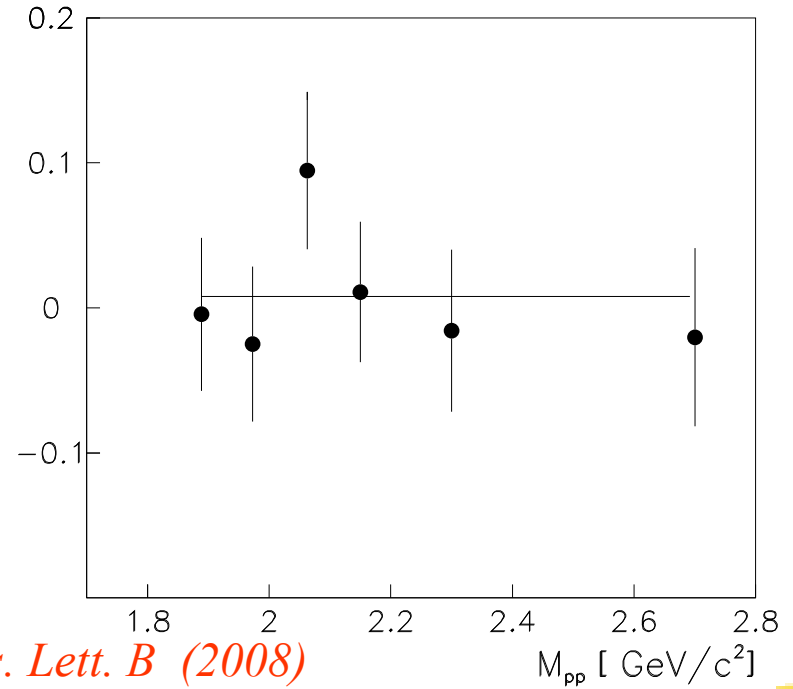
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$$\frac{d\sigma_-}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta)$$

$$A = 0.01 \pm 0.02$$



E. T.-G., E.A. Kuraev, S. Bakmaev, S. Pacetti, Phys. Lett. B (2008)

Rosenbluth separation

Contribution of the *electric term*

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ipn
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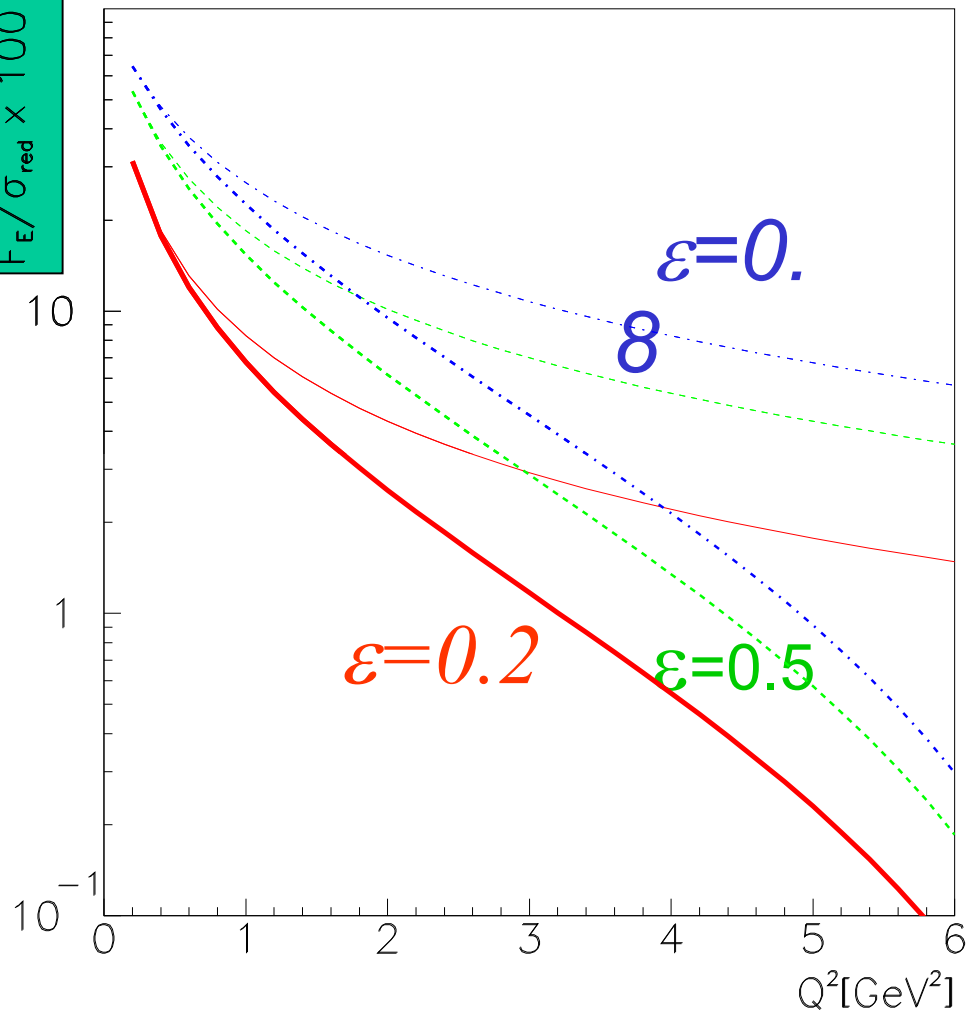
$$\epsilon = \frac{1}{1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2}},$$

$$\tau = \frac{q^2}{4m^2}$$

$$F_E / \sigma_{red} \times 100$$

$$\sigma_{red} = \tau G_{Mp}^2 + \epsilon G_{Ep}^2$$

...to be compared to the
absolute value of the
error on σ and to the size
and ϵ dependence of RC



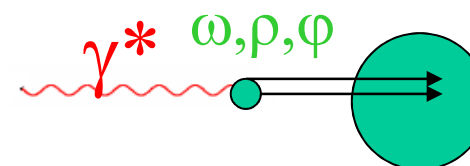
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Isoscalar and isovector FFs



$$F_1^s(Q^2) = \frac{g(Q^2)}{2} \left[(1 - \beta_\omega - \beta_\phi) + \beta_\omega \frac{\mu_\omega^2}{\mu_\omega^2 + Q^2} + \beta_\phi \frac{\mu_\phi^2}{\mu_\phi^2 + Q^2} \right],$$

$$F_1^v(Q^2) = \frac{g(Q^2)}{2} \left[(1 - \beta_\rho) + \beta_\rho \frac{\mu_\rho^2 + 8\Gamma_\rho \mu_\pi / \pi}{(\mu_\rho^2 + Q^2) + (4\mu_\pi^2 + Q^2)\Gamma_\rho \alpha(Q^2) / \mu_\pi} \right],$$

$$F_2^s(Q^2) = \frac{g(Q^2)}{2} \left[(\mu_p + \mu_n - 1 - \alpha_\phi) \frac{\mu_\omega^2}{\mu_\omega^2 + Q^2} + \alpha_\phi \frac{\mu_\phi^2}{\mu_\phi^2 + Q^2} \right],$$

$$F_2^v(Q^2) = \frac{g(Q^2)}{2} \left[(\mu_p - \mu_n - 1) \frac{\mu_\rho^2 + 8\Gamma_\rho \mu_\pi / \pi}{(\mu_\rho^2 + Q^2) + (4\mu_\pi^2 + Q^2)\Gamma_\rho \alpha(Q^2) / \mu_\pi} \right],$$

$$g(Q^2) = \frac{1}{(1 + \gamma e^{i\theta} Q^2)^2}$$

$$\alpha(Q^2) = \frac{2}{\pi} \sqrt{\frac{Q^2 + 4\mu_\pi^2}{Q^2}} \ln \left| \frac{\sqrt{(Q^2 + 4\mu_\pi^2)} + \sqrt{Q^2}}{2\mu_\pi} \right|$$

$$2F_i^p = F_i^s + F_i^v,$$

$$2F_i^n = F_i^s - F_i^v.$$