

Quasiattractor in models of new and chaotic inflation

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Universe inflation solves the problems:

- flatness
- homogeneity
- causality

Investigation of the Universe inflation by

quasiattractor method:

- Mathematical aspects
- Comparing with experiment

Mathematical aspects

Scalar field

$$S = \int dx^4 \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right\}$$

$$V = \frac{\lambda}{4} (\phi^2 - v^2)^2$$

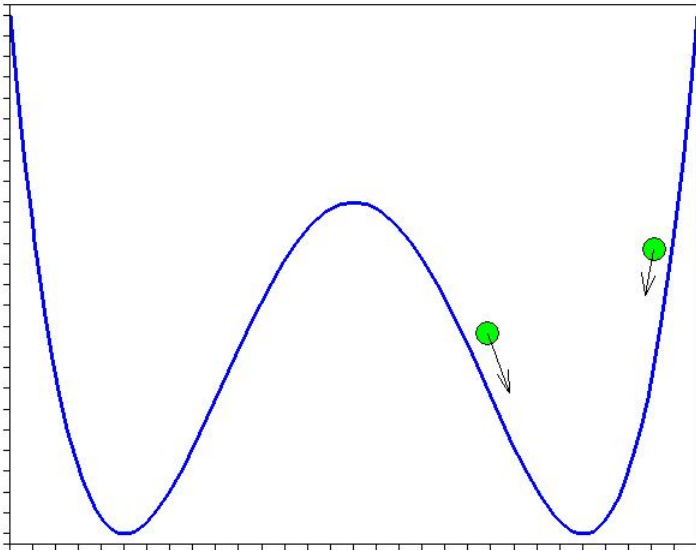
$$g_{\mu\nu} = \text{diag}(1, -a^2(t), -a^2(t), -a^2(t)), \quad N = \ln(a/a_{\text{init}}.)$$

The equations of motion

$$\ddot{\phi} = -3H\dot{\phi} - \lambda\phi(\phi^2 - v^2)$$

$$\dot{H} = -4\pi G\dot{\phi}^2$$

$$H^2 = \frac{4\pi G}{3} \left\{ \dot{\phi}^2 + \frac{1}{2}\lambda(\phi^2 - v^2)^2 \right\}$$

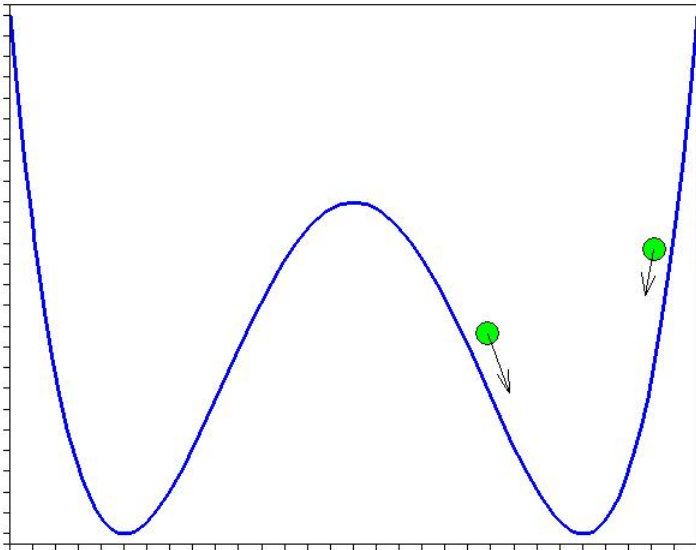


Slow-roll approximation

$$\cancel{\ddot{\phi}} = -3H\dot{\phi} - \lambda\phi(\phi^2 - v^2)$$

$$\dot{H} = -4\pi G\dot{\phi}^2$$

$$H^2 = \frac{4\pi G}{3} \left\{ \dot{\phi}^2 + \frac{1}{2}\lambda(\phi^2 - v^2)^2 \right\}$$



New variables

$$x = \frac{\kappa}{\sqrt{6}} \frac{\dot{\phi}}{H}, \quad y = \sqrt[4]{\frac{\lambda}{12}} \frac{\sqrt{\kappa}}{\sqrt{H}} \sqrt{|\phi^2 - v^2|}$$

$$z = \frac{\sqrt[4]{3\lambda}}{\sqrt{\kappa H}}, \quad u = \frac{\kappa v}{\sqrt{6}}$$

Mathematical aspects

System of differential equations

$$x' = 3x^3 - 3x - 2y^2z\sqrt{y^2 + u^2z^2}$$

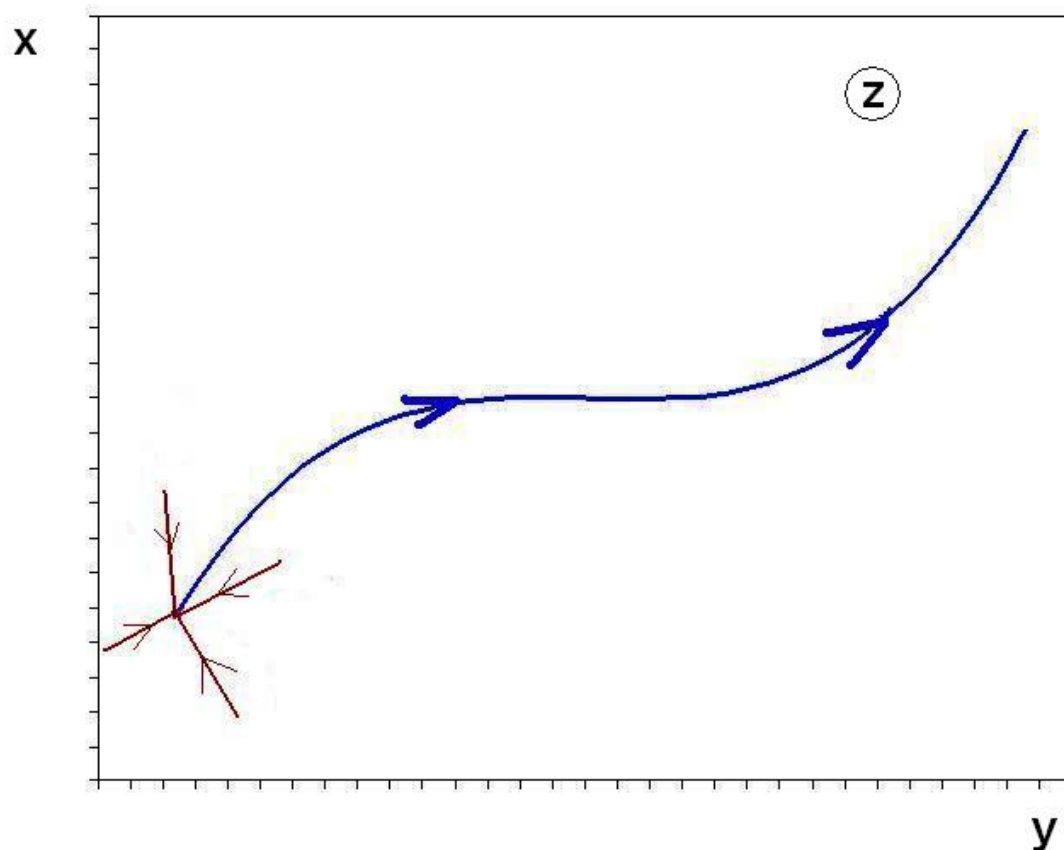
$$yy' = \frac{3}{2}x^2y^2 + xz\sqrt{y^2 + u^2z^2}$$

$$z' = \frac{3}{2}x^2z$$

Friedmann equation $x^2 + y^4 = 1$

System motion

1. the system “falls” to the quasiattractor
2. the critical point drifts during the evolution



Mathematical aspects

Critical points ($x' = 0, \quad y' = 0; \quad x \neq 0, \quad y \neq 0$)

$$\frac{3}{2}x_c\sqrt{1-x_c^2} + z\sqrt{\sqrt{1-x_c^2} + u^2z^2} = 0$$

$$\mathcal{B} = -3 + 6x_c^2 - \frac{2}{3}\frac{z^2}{y_c^2} < 0$$

$$\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = C \begin{pmatrix} 2y_c^3 \\ -x_c \end{pmatrix} e^{\mathcal{B}N}$$

The Universe inflation

$$\ddot{a} > 0$$

$$x_{\text{end}}^2 = \frac{1}{3}$$

$$y_{\text{end}}^4 = \frac{2}{3}$$

$$z_{\text{end}}^2 = \frac{\sqrt{3u^2 + 1} - 1}{u^2 \sqrt{6}}$$

Inhomogeneity:

- Scalar perturbations

$$P_S(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2 = \frac{\lambda}{8\pi^2} \frac{1}{x_c^2 z^4}$$

- Tensor perturbations

$$P_T(k) = 8\kappa^2 \left(\frac{H}{2\pi}\right)^2 = \frac{6\lambda}{\pi^2} \frac{1}{z^4}$$

Inhomogeneity:

- Tensor contribution $r \equiv \frac{P_T(k)}{P_S(k)} = 48 x_c^2$
- Spectral index $n_S - 1 \equiv \frac{d \ln P_S}{d \ln k} = \frac{4(9x_c^2 - z^2)}{3(3x_c^2 - 1)}$

$$N_{\text{total}} = \frac{2}{3} \int_{z_{\text{in}}}^{z_{\text{end}}} \frac{dz}{x_c^2 z} \approx \frac{3}{4} \left(\frac{1}{z_{\text{in}}^2} - u^2 \ln \frac{1 + u^2 z_{\text{in}}^2}{u^2 z_{\text{in}}^2} \right)$$

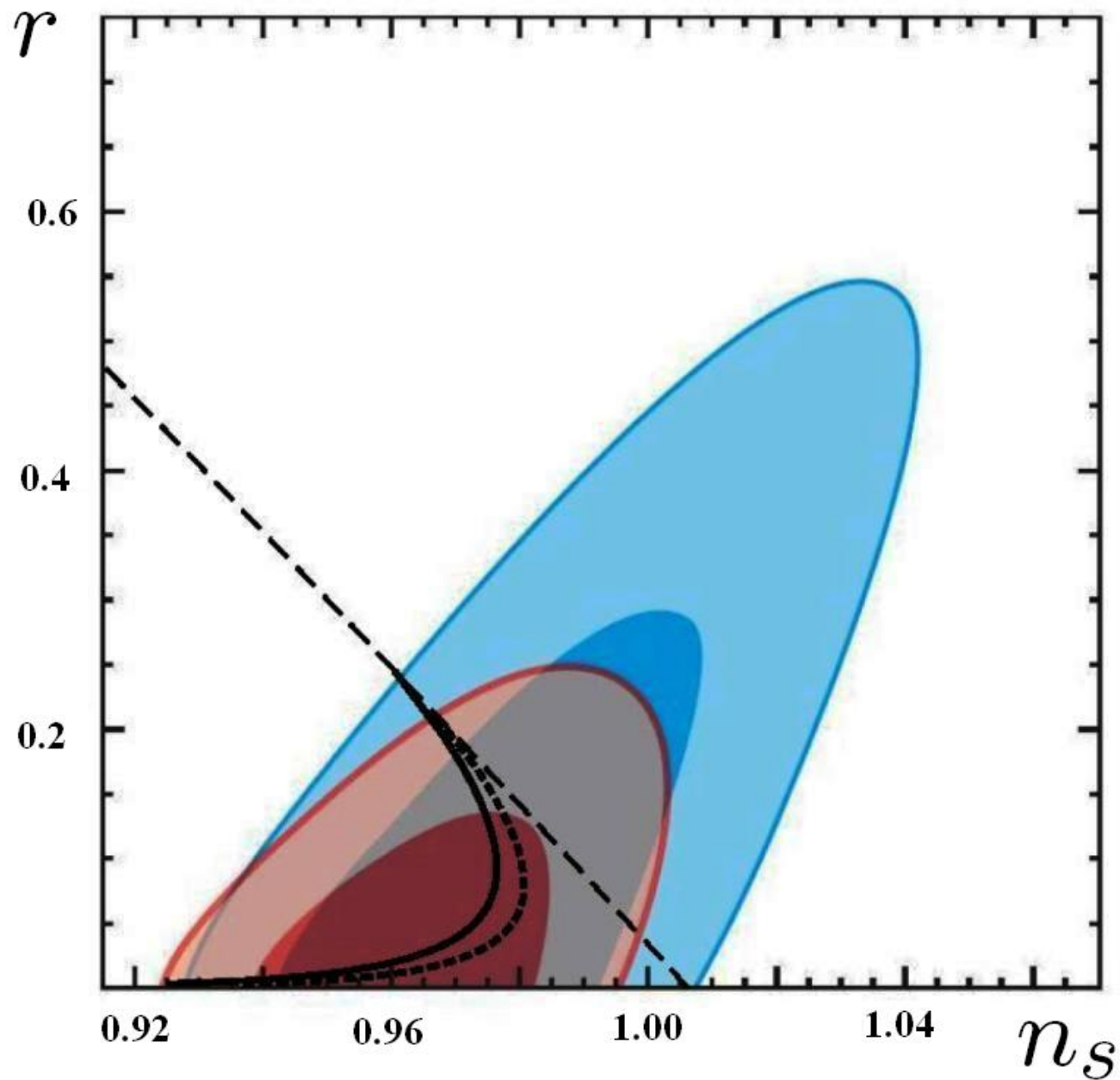
Limiting cases

- quartic term dominates $r = \frac{16}{N}$, $n_S - 1 = -\frac{3}{N}$
at $u^2 \rightarrow 0$

- quadratic term dominates $r = \frac{8}{N}$, $n_S - 1 = -\frac{2}{N}$
at $u^2 \rightarrow \infty$

Comparing data with experiment

WMAP data



Comparing data with experiment

Inflaton parameters:

- Mass

$$1 \cdot 10^{13} \text{ GeV} \leq m \leq 1.7 \cdot 10^{13} \text{ GeV}$$

- Constant of self-action

$$0 \leq \lambda \leq 9.0 \cdot 10^{-14}$$

- Expansion

$$N = 60_{-20}^{+40} \qquad 25 \leq u^2 \leq \infty$$

- We considered the inflation dynamics in the framework of the quasiattractor.
- The model is consistent with the observation data.
- We have precisely enough determined the inflaton mass.

Thank you for your attention!