QUARKS-2010
16th International Seminar on High Energy Physics
Kolomna, Russia, June 6 – 12, 2010.

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Quantum corrections in supersymmetric theories with the higher covariant derivative regularization

Regularization for the supersymmetric theories

Quantum corrections in supersymmetric theories are investigated for a long time, for example in the papers

L.V.Avdeev, O.V.Tarasov, Phys.Lett. 112B, (1982),356;
A.Parkes, P.West, Phys.Lett. 138B, (1983), 99;
I.Jack, D.R.T.Jones, C.G.North, Phys.Lett B386, (1996), 138;
Nucl.Phys. B486 (1997), 479;
I.Jack, D.R.T.Jones, A.Pickering, Phys.Lett. B435, (1998), 61.

Most calculations were made with the dimensional reduction

W.Siegel, Phys.Lett. 84B, (1979), 193; 94B, (1980), 37.

(The dimensional regularization breaks the supersymmetry.) With the dimensional reduction the β-function was calculated even in the four-loop approximation. However, the dimensional reduction is inconsistent from the mathematical poit of view and can lead to some problems in higher loops.

The higher covariant derivatives

A consistent regularization, which does not break the supersummetry is the higher covariant derivative regularization, proposed by A.A.Slavnov:

A.A.Slavnov, Nucl.Phys., **B31**, (1971), 301; Theor.Math.Phys. **13**, (1972), 1064.

It was generalized to the supersymmetric case in the papers

V.K.Krivoshchekov, Theor.Math.Phys. **36**, (1978), 745; P.West, Nucl.Phys. B268, (1986), 113.

The first (one-loop) calculation with the higher derivative regularization was made for the (nonsupersymmetric) Yang–Mills theory in

C.Martin, F.Ruiz Ruiz, Nucl.Phys. **B 436**, (1995), 645.

Taking into account correction, made in

M.Asorey, F.Falceto, Phys.Rev **D 54**, (1996), 5290; T.Bakeyev, A.Slavnov, Mod.Phys.Lett. **A11**, (1996), 1539.

the result coincided with the usual β-function of the Yang–Mills theory.

$N = 1$ supersymmetric theories

N=1 supersymmetric Yang-Mills theory with matter in the massless case is described by the action

$$S = \frac{1}{2e^2}\mathrm{Re}\,\mathrm{tr}\int d^4x\,d^2\theta\,W_a C^{ab} W_b + \frac{1}{4}\int d^4x\,d^4\theta\,(\phi^*)^i (e^{2V})_i{}^j \phi_j +$$
$$+\Big(\frac{1}{6}\int d^4x\,d^2\theta\,\lambda^{ijk}\phi_i\phi_j\phi_k + \mathrm{h.c.}\Big),$$

where ϕ_i are chiral scalar matter superfields, V is a real scalar gauge superfield, and the supersymmetric gauge field stress tensor is given by

$$W_a = \frac{1}{8}\bar{D}^2\Big[e^{-2V} D_a e^{2V}\Big].$$

The action is invariant under the gauge transformations

$$e^{2V} \to e^{i\Lambda^+} e^{2V} e^{-i\Lambda}; \qquad \phi \to e^{i\Lambda}\phi$$

if $(T^A)_m{}^i \lambda^{mjk} + (T^A)_m{}^j \lambda^{imk} + (T^A)_m{}^k \lambda^{ijm} = 0.$

Background field method

We use the background field method: $e^{2V} \to e^{2V'} \equiv e^{\mathbf{\Omega}^+} e^{2V} e^{\mathbf{\Omega}}$, where $\mathbf{\Omega}$ is a background field. Background covariant derivatives are given by

$$\boldsymbol{D} \equiv e^{-\mathbf{\Omega}^+} \frac{1}{2}(1+\gamma_5) D e^{\mathbf{\Omega}^+}; \qquad \bar{\boldsymbol{D}} \equiv e^{\mathbf{\Omega}} \frac{1}{2}(1-\gamma_5) D e^{-\mathbf{\Omega}};$$

$$\boldsymbol{D}_\mu \equiv -\frac{i}{4}(C\gamma^\mu)^{ab} \Big\{ \boldsymbol{D}_a, \bar{\boldsymbol{D}}_b \Big\}.$$

The background gauge invariance

$$\phi \to e^{i\Lambda}\phi; \quad V \to e^{iK} V e^{-iK}; \quad e^{\mathbf{\Omega}} \to e^{iK} e^{\mathbf{\Omega}} e^{-i\Lambda}; \quad e^{\mathbf{\Omega}^+} \to e^{i\Lambda^+} e^{\mathbf{\Omega}^+} e^{-iK},$$

where K is an arbitrary real superfield, and Λ is a background-chiral superfield.

This invariance allows to choose $\mathbf{\Omega} = \mathbf{\Omega}^+ = \mathbf{V}$.

It is desirable to fix a gauge and to introduce a regularization in such a way, that the background gauge invariance will be unbroken.

Quantization

The gauge is fixed by adding the following term:

$$S_{gf} = -\frac{1}{32e^2}\,\mathrm{tr}\int d^4x\,d^4\theta\left(V\boldsymbol{D}^2\bar{\boldsymbol{D}}^2V + V\bar{\boldsymbol{D}}^2\boldsymbol{D}^2V\right).$$

(Then the terms, quadratic in the quantum field, have the simplest form.)

The corresponding ghost Lagrangian is

$$S_c = i\,\mathrm{tr}\int d^4x\,d^4\theta\left\{(\bar{c}+\bar{c}^+)V\left[(c+c^+)+\mathrm{cth}\,V(c-c^+)\right]\right\}.$$

Also it is necessary to add the Nielsen-Kallosh ghosts

$$S_B = \frac{1}{4e^2}\mathrm{tr}\int d^4x\,d^4\theta\,B^+e^{\boldsymbol{\Omega}^+}e^{\boldsymbol{\Omega}}\,B.$$

Higher derivative regularization

To regularize the theory we use the higher covariant derivative regularization.

For a theory with the nontrivial cubic superpotential it is also necessary to introduce the higher covariant derivative term for the matter superfields. We add to the action the term

$$S_\Lambda = \frac{1}{2e^2}\mathrm{tr}\,\mathrm{Re}\int d^4x\,d^4\theta\,V\frac{(\boldsymbol{D}_\mu^2)^{n+1}}{\Lambda^{2n}}V + \frac{1}{4}\int d^4x\,d^4\theta\,\Big((\phi^*)^i \times$$

$$\times\Big[e^{\boldsymbol{\Omega}^+}\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}e^{\boldsymbol{\Omega}}\Big]_i{}^j\phi_j\Big).$$

Presence of the higher derivatives in the matter kinetic terms makes the calculations much more complicated.

After adding of the term with the higher derivatives divergences remain only in the one-loop approximation.

Higher derivative regularization

In order to regularize the remaining one-loop divergences, it is necessary to introduce Pauli-Villars determinants into the generating functional

L.D.Faddeev, A.A.Slavnov, Gauge fields, introduction to quantum theory, Benjamin, Reading, 1990.

$$Z[J,\mathbf{\Omega}] = \int D\mu \prod_I \Big(\det PV(V,\mathbf{V},M_I)\Big)^{c_I} \times$$

$$\times \exp\Big\{iS + iS_\Lambda + iS_{gf} + iS_B + iS_{gh} + \mbox{Sources}\Big\},$$

where the coefficients satisfy the conditions $\sum\limits_I c_I = 1; \qquad \sum\limits_I c_I M_I^2 = 0.$

It is convenient to write the Pauli–Villars determinants as

$$\det PV(V,\mathbf{V},M) = \Big(\int D\Phi^* D\Phi e^{iS_{PV}}\Big)^{-1}.$$

In order to cancel the remaining one-loop divergences of the theory with the higher derivative term the Pauli–Villars action S_{PV} should contain the higher derivatives.

Pauli–Villars fields

We considered the following form of the Pauli–Villars action:

$$S_{PV} = \frac{1}{4}\int d^4x\, d^4\theta\, (\Phi^*)^i \Big[e^{\boldsymbol{\Omega}^+}\Big(1+\frac{(\boldsymbol{D}_\alpha^2)^m}{\Lambda^{2m}}\Big)e^{\boldsymbol{\Omega}}\Big]_i{}^j\,\Phi_j$$
$$+\Big(\frac{1}{4}\int d^4x\, d^2\theta\, M^{ij}\Phi_i\Phi_j + \text{h.c.}\Big).$$

(A regularized one-loop diagram with cubic matter vertex is finite.)

In order to obtain the gauge invariance the mass should satisfy

$$(T^A)_k{}^i M^{kj} + (T^A)_k{}^j M^{ki} = 0.$$

Also we assume that

$$M^{ij}M^*_{jk} = M^2\delta^i_k \qquad M^{ij} = a^{ij}\Lambda,$$

where a^{ij} are constants. (There is the only dimensionful parameter Λ.)

Two-loop β-function for $N=1$ supersymmetric Yang-Mills theory

Two-loop calculation gives the following result:

$$\beta(\alpha) = -\frac{3\alpha^2}{2\pi}C_2 + \alpha^2 T(R) I_0 + \alpha^3 C_2^2 I_1 + \frac{\alpha^3}{r} C(R)_i{}^j C(R)_j{}^i I_2 +$$
$$+\alpha^3 T(R) C_2 I_3 + \alpha^2 C(R)_i{}^j \frac{\lambda^*_{jkl}\lambda^{ikl}}{4\pi r} I_4 + \ldots,$$

where we do not write the integral for the one-loop ghost contribution and the integrals I_0–I_4 are given below, and the following notation is used:

$$\text{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$
$$f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \qquad r \equiv \delta_{AA}.$$

Taking into account Pauli–Villars contributions,

$$I_i = I_i(0) - \sum_I I_i(M_I), \qquad i = 0, 2, 3$$

where I_i are given by

Factorization of integrands into total derivatives

$$I_0(M) = 8\pi \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{q^2} \frac{d}{dq^2} \left\{ \frac{1}{2} \ln\left(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2\right) + \right.$$

$$\left. + \frac{M^2}{2(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)} - \frac{mq^{2m}/\Lambda^{2m} q^2 (1+q^{2m}/\Lambda^{2m})}{q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2} \right\};$$

$$I_1 = 96\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{k^2} \frac{d}{dk^2} \left\{ \frac{1}{q^2(q+k)^2(1+q^{2n}/\Lambda^{2n})} \times \right.$$

$$\left. \times \frac{1}{(1+(q+k)^{2n}/\Lambda^{2n})} \left(\frac{n+1}{(1+k^{2n}/\Lambda^{2n})} - \frac{n}{(1+k^{2n}/\Lambda^{2n})^2} \right) \right\};$$

$$I_2(M) = -64\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{1}{q^2} \frac{d}{dq^2} \left\{ \frac{q^2}{k^2(1+k^{2n}/\Lambda^{2n})} \times \right.$$

$$\times \frac{(1+(q+k)^{2m}/\Lambda^{2m})}{((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m}) + M^2)} \left[\frac{q^2(1+q^{2m}/\Lambda^{2m})^3}{(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)^2} + \right.$$

$$\left.\left. + \frac{mq^{2m}/\Lambda^{2m}}{q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2} - \frac{2mq^{2m}/\Lambda^{2m} M^2}{(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2)^2} \right] \right\};$$

Factorization of integrands into total derivatives

$$I_3(M) = 16\pi^2 \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d}{d\ln\Lambda}\Bigg\{\frac{\partial}{\partial q_\alpha}\Bigg[\frac{k_\alpha(1+q^{2m}/\Lambda^{2m})}{(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)}\times$$

$$\times\frac{1}{(k+q)^2(1+(q+k)^{2n}/\Lambda^{2n})}\Bigg(-\frac{(1+k^{2m}/\Lambda^{2m})^3}{(k^2(1+k^{2m}/\Lambda^{2m})^2+M^2)^2}+$$

$$+\frac{mk^{2m}/\Lambda^{2m}}{k^2(1+k^{2m}/\Lambda^{2m})^2+M^2}-\frac{2mk^{2m}/\Lambda^{2m}M^2}{(k^2(1+k^{2m}/\Lambda^{2m})^2+M^2)^2}\Bigg)\Bigg]-$$

$$-\frac{1}{k^2}\frac{d}{dk^2}\Bigg[\frac{2(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}{(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)\,((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2+M^2)}\times$$

$$\times\Bigg(\frac{1}{(1+k^{2n}/\Lambda^{2n})}+\frac{nk^{2n}/\Lambda^{2n}}{(1+k^{2n}/\Lambda^{2n})^2}\Bigg)\Bigg]\Bigg\};$$

$$I_4 = 64\pi^2\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d}{d\ln\Lambda}\frac{1}{q^2}\frac{d}{dq^2}\Bigg[\frac{1}{k^2(q+k)^2(1+k^{2m}/\Lambda^{2m})}\times$$

$$\times\frac{1}{(1+(q+k)^{2m}/\Lambda^{2m})}\Bigg(\frac{1}{(1+q^{2m}/\Lambda^{2m})}+\frac{mq^{2m}/\Lambda^{2m}}{(1+q^{2m}/\Lambda^{2m})^2}\Bigg)\Bigg].$$

Two-loop β-function for $N=1$ supersymmetric Yang-Mills theory

The integrals can be calculated using the identity

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{d}{dk^2} f(k^2) = \frac{1}{16\pi^2}\Big(f(k^2=\infty) - f(k^2=0)\Big).$$

(This is a total derivative in the four-dimensional spherical coordinates.)

The result for the two-loop β-function is given by

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\Big(3C_2 - T(R)\Big) + \frac{\alpha^3}{(2\pi)^2}\Big(-3C_2^2 + T(R)C_2 +$$

$$+\frac{2}{r}C(R)_i{}^j C(R)_j{}^i\Big) - \frac{\alpha^2 C(R)_i{}^j \lambda^*_{jkl}\lambda^{ikl}}{8\pi^3 r} + \ldots.$$

Two-loop β-function for $N = 1$ supersymmetric Yang-Mills theory

Comparing the result with the one-loop anomalous dimension

$$\gamma_i{}^j(\alpha) = -\frac{\alpha C(R)_i{}^j}{\pi} + \frac{\lambda^*_{ikl}\lambda^{jkl}}{4\pi^2} + \ldots,$$

gives the exact NSVZ β-function in the considered approximation.

$$\beta(\alpha) = -\frac{\alpha^2\Big[3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha)/r\Big)\Big]}{2\pi(1 - C_2\alpha/2\pi)}.$$

V.Novikov, M.A.Shifman, A.Vainstein, V.Zakharov, Nucl.Phys. B 229, (1983), 381; Phys.Lett. 166B, (1985), 329; M.Shifman, A.Vainshtein, Nucl.Phys. B 277, (1986), 456.

(The result also agrees with the DRED calculations.)

Three-loop calculation for SQED

The notation is

$$\Gamma^{(2)} = \int \frac{d^4p}{(2\pi)^4}\, d^4\theta \Big(-\frac{1}{16\pi} \mathbf{V}(-p)\, \partial^2\Pi_{1/2} \mathbf{V}(p)\, d^{-1}(\alpha,\mu/p) +$$

$$+\frac{1}{4}(\phi^*)^i(-p,\theta)\,\phi_j(p,\theta)\,(ZG)_i{}^j(\alpha,\mu/p)\Big).$$

The main result: (It was obtained as the equality of some well defined integrals)

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\Lambda/p)-\alpha_0^{-1}\Big)\Big|_{p=0} = -\frac{d}{d\ln\Lambda}\alpha_0^{-1}(\alpha,\mu/\Lambda) =$$

$$= \frac{1}{\pi}\Big(1-\frac{d}{d\ln\Lambda}\ln G(\alpha_0,\Lambda/q)\Big|_{q=0}\Big) = \frac{1}{\pi} + \frac{1}{\pi}\frac{d}{d\ln\Lambda}\Big(\ln ZG(\alpha,\mu/q) -$$

$$-\ln Z(\alpha,\Lambda/\mu)\Big)\Big|_{q=0} = \frac{1}{\pi}\Big(1-\gamma\Big(\alpha_0(\alpha,\Lambda/\mu)\Big)\Big).$$

The reason is that the integrands are again total derivatives.

Factorization into total derivatives in $N=1$ SUSY QED for some classes of diagrams.

Let us consider the Abelian case (SUSY QED) formally (without contributions of the Pauli–Villars fields).

$$S = \frac{1}{4e^2}\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab} f(\partial^2/\Lambda^2) W_b + \frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi\Big)$$

where $f(\partial^2/\Lambda^2)$ is a regulator, for example

$$f = 1 + \frac{\partial^{2n}}{\Lambda^{2n}}.$$

It seems that the corresponding proof for contributions of the Pauli–Villars fields can be made similarly.

1. One-loop result

We can use results of explicit calculation, presented above. Here the contribution of the Pauli–Villars field are important.

2. Some technical simplifications

a. (This step can be omitted, but it simplifies the calculations) Due to the Ward identity the two-point Green function is proportional to

$$\int d^4\theta \mathbf{V}\partial^2\Pi_{1/2}\mathbf{V} \times \text{(Momentum integral)},$$

where $\partial^2\Pi_{1/2} \sim D^a\bar{D}^2D_a$. Therefore, in order to find β-function it is possible to substitute

$$\mathbf{V} \to \bar{\theta}^a\bar{\theta}_a\theta^b\theta_b$$

(Note that we consider the limit $p \to 0$)

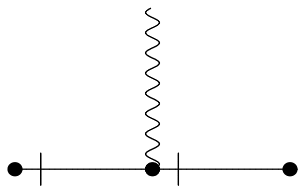
b. Integral of a total derivative in the coordinate representation is given by

$$\text{Tr}\Big([x^\mu, \text{Something}]\Big) = 0.$$

We will try to reduce the sum of diagrams to such commutators.

3. Summation of subdiagrams

Let us consider the following sum of subdiagrams, substituting $\mathbf{V} \to \bar\theta^a\bar\theta_a\theta^b\theta_b$:

 $+$ 

$$= -\theta^a\theta_a\bar\theta^b\frac{\bar D_b D^2}{4\partial^2} + \theta^a\theta_a\frac{D^2}{4\partial^2}$$

$$-i\bar\theta^b(\gamma^\mu)_b{}^a\theta_a\frac{\bar D^2 D^2\partial_\mu}{\partial^4} + i\theta^a(\gamma^\mu)_a{}^b\frac{\bar D_b D^2\partial_\mu}{4\partial^4} + \frac{\bar D^2 D^2}{16\partial^4}$$

Only the terms, written by the blue color, give nontrivial contributions to the two-point function of the gauge superfield, because finally it is necessary to obtain

$$\int d^4\theta\,\theta^a\theta_a\bar\theta^b\bar\theta_b,$$

and calculating the θ-part of the graph can not produce powers of θ or $\bar\theta$.

For investigation of the Pauli–Villars contribution it is necessary to consider the massive case. This is made similarly, but all expressions become more complicated.

4. External $\mathbf{V}$-lines are attached to different loops of the matter superfields

Let us denote by $*$ a chain of propagators, connecting vertexes with quantum gauge field. Then each loop will be proportional to

$$\mathrm{Tr}\Big(- i\bar\theta^c(\gamma^\nu)_c{}^d\theta_d \frac{\bar D^2 D^2\partial_\nu}{8\partial^4} * -\theta^c\theta_c\bar\theta^d \frac{\bar D_d D^2}{4\partial^2} * \Big)$$

$$= \mathrm{Tr}\Big(- \theta^c\theta_c\bar\theta^d * \frac{\bar D_d D^2}{4\partial^2} * -\bar\theta^d\theta^c * \frac{\bar D^2 D_c}{4\partial^2} * \frac{\bar D_d D^2}{4\partial^2} *$$

$$-i\bar\theta^c(\gamma^\nu)_c{}^d\theta_d * \frac{\bar D^2 D^2\partial_\nu}{8\partial^4} * +\theta^2, \bar\theta^1, \theta^1, \theta^0 \text{ terms}\Big)$$

After some simple algebra the first three terms can be written as

$$\mathrm{Tr}\Big(- 2\theta^c\theta_c\bar\theta^d[\bar\theta_d, *] - i\bar\theta^c(\gamma^\nu)_c{}^d\theta_d[y^*_\nu, *] + \ldots\Big) = 0 + \ldots$$

where $y^*_\nu = x_\nu - i\bar\theta^a(\gamma_\nu)_a{}^b\theta_b/2 + i\theta^a(\gamma_\nu)_a{}^b\bar\theta_b/2$.

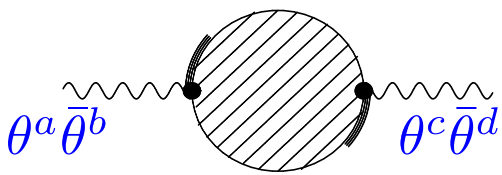
Multiplication of the other terms gives 0 after the integration over $d^4\theta$.

Therefore, the sum of all such diagrams is given by an integral of a total derivative and is equal to 0.

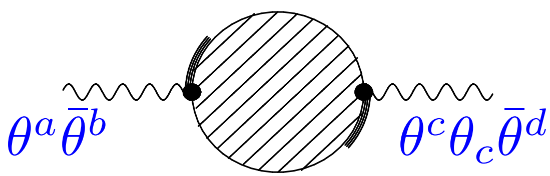
5. External $\mathbf{V}$-lines are attached to a single loop of the matter superfields

a. Calculation of Feynman diagrams

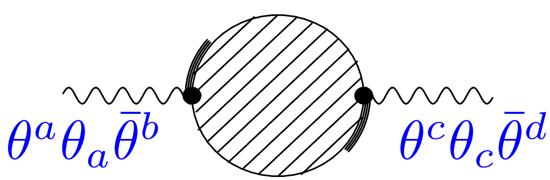
Since calculating θ-integrals can not increase a degree of θ, it is possible to shift θ-s to an arbitrary point of the loop, commuting them with matter propagators. This gives



$$\sim -\frac{1}{128}\mathrm{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b\frac{\bar D^2 D^2\partial^\mu}{\partial^4} * \frac{\bar D^2 D^2\partial_\mu}{\partial^4} * \Big)$$



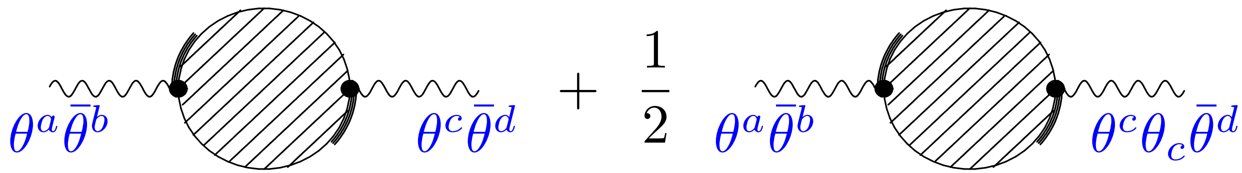
$$\sim -i(\gamma^\mu)_c{}^d\mathrm{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b\frac{\bar D^2 D^2\partial_\mu}{16\partial^4} * \frac{\bar D^2 D_c}{16\partial^2} * \frac{\bar D^d D^2}{\partial^2} * + \frac{\bar D^2 D_c\partial_\mu}{16\partial^4} * \frac{\bar D^d D^2}{\partial^2} * \Big)$$



$$\sim \mathrm{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b\frac{\bar D_d D^2}{4\partial^2} * \frac{\bar D^2 D^c}{8\partial^2} * \frac{\bar D^2 D_c}{8\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * - \frac{\bar D_d D^2}{4\partial^2} * \frac{\bar D^2}{4\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * - \frac{\bar D_d}{2\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * + \frac{\bar D_d D^c}{2\partial^2} * \frac{\bar D^2 D_c}{8\partial^2} * \frac{\bar D^d D^2}{4\partial^2} * \Big)$$

b. Extracting NSVZ β-function

First we calculate



Using

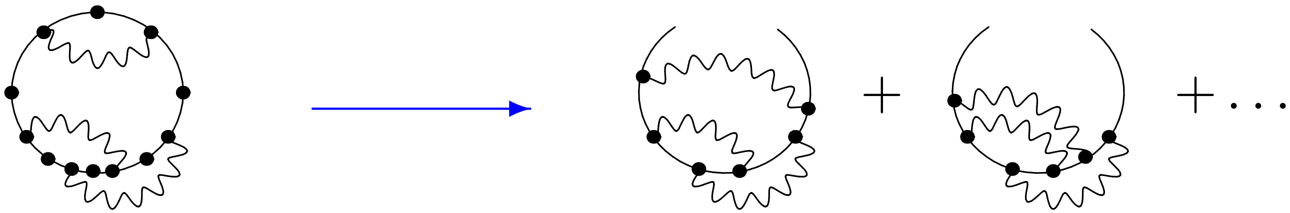
$$[x^\mu, \frac{\partial_\mu}{\partial^4}] = [i\frac{\partial}{\partial p_\mu}, -\frac{ip^\mu}{p^4}] = 2\pi^2\delta^4(p)$$

after some algebra we obtain

$$\mathrm{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b\Big(\frac{\pi^2}{128} * \bar D^2 D^2 \delta^4(\partial_\alpha) + \Big[y^*_\mu, \frac{\bar D^2 D^2 \partial^\mu}{16^2\partial^4} *\Big]$$
$$+ i(\gamma^\mu)_c{}^d\Big\{\theta^c, *\frac{\bar D_d D^2}{8\partial^2} * \frac{\bar D^2 D^2\partial_\mu}{16\partial^4}\Big\} + i(\gamma^\mu)_c{}^d\Big\{\bar\theta^d, *\frac{\bar D^2 D_c\partial_\mu}{8\partial^4}\Big\}\Big)\Big)$$
$$= \mathrm{Tr}\Big(\frac{\pi^2}{128}\theta^a\theta_a\bar\theta^b\bar\theta_b * \bar D^2 D^2\delta^4(\partial_\alpha)\Big)$$

δ-function allow to perform integration over the momentum, corresponding to the considered matter loop. This corresponds to cutting the diagram, which gives diagrams for the two-point Green function of the matter superfield.

A. Smilga, A. Vainstein, Nucl. Phys. B 704, (2005), 445.



Comparing the coefficients it is possible to obtain

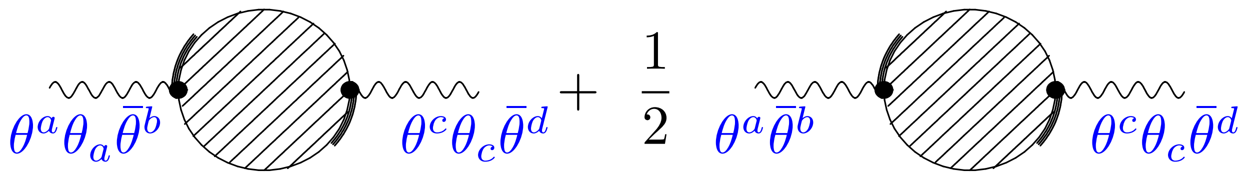
$$\beta(\alpha) \leftarrow -\frac{\alpha^2}{\pi}\gamma(\alpha).$$

Therefore, the sum of all such diagrams is an integral of a total derivative and gives the exact NSVZ β-function.

Alternatively, this result can be obtained substituting solution of the Ward identity into one of the effective diagrams in the Schwinger–Dyson equation.

c. Remaining diagrams

Now let us calculate



In this case the method, based on the substituting solution of the Ward identity into Schwinger–Dyson equation does not work. (Here contributions of the transversal parts of the Green functions are essential.)

The considered sum is given by

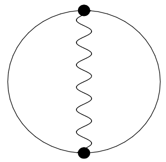
$$\mathrm{Tr}\Bigg(-\frac{i}{32}(\gamma^\mu)_d{}^c\Big(\frac{\bar D^2 D^2\partial_\mu}{\partial^4}*\frac{\bar D^2 D_c}{16\partial^2}*\frac{\bar D^d D^2}{\partial^2}*+\frac{\bar D^2 D_c\partial_\mu}{\partial^4}*\frac{\bar D^d D^2}{\partial^2}*\Big)$$

$$-\frac{\bar D_d}{2\partial^2}*\frac{\bar D^d D^2}{4\partial^2}*-\frac{\bar D_d D^2}{4\partial^2}*\frac{\bar D^2}{4\partial^2}*\frac{\bar D^d D^2}{4\partial^2}*+\frac{\bar D_d D^c}{2\partial^2}*\frac{\bar D^2 D_c}{8\partial^2}*\frac{\bar D^d D^2}{4\partial^2}*$$

$$+\frac{\bar D_d D^2}{4\partial^2}*\frac{\bar D^2 D^c}{8\partial^2}*\frac{\bar D^2 D_c}{8\partial^2}*\frac{\bar D^d D^2}{4\partial^2}*\Bigg)\theta^a\theta_a\bar\theta^b\bar\theta_b$$

This expression can not be factorized into a total derivative by the method, described above. However, for planar diagrams with a single loop of the matter superfield the factorization can be proven in all orders:

Let us assume that the diagram contain a line, which cut it:



Then there are the following possibilities:

I. There is $*\dfrac{\bar{D}^a D^2}{8\partial^2}* = [*, \bar\theta^a] = 0$ or $*\dfrac{D^2 D^a}{8\partial^2}* = [*, \theta^a] = 0$ on the left (right) side, because $\theta_1 \delta^4(\theta_1 - \theta_2) = \theta_2 \delta^4(\theta_1 - \theta_2)$.

II. There is $*\dfrac{\bar{D}^a D^2}{16\partial^2} * \dfrac{\bar{D}_a D^2}{16\partial^2}*$ on the left (right) side. Then on the other side we have

$$\sim \{\theta^b[\theta_b, *]\} = 0$$

III. There is $*\dfrac{\bar{D}^a D^2}{16\partial^2} * \dfrac{\bar{D}^2 D_b}{16\partial^2}*$ on the left (right) side. Then on the other side we have

$$\sim (\gamma^\mu)_a{}^b [y^*_\mu, *]\theta^c\theta_c\bar\theta^d\bar\theta_d$$

IV. All modified propagators are on the same side of the diagram.

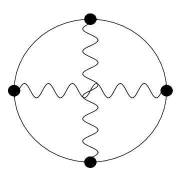
The considered diagrams can be cut into parts with no more than two matter propagators. Then

$$\text{Tr}\Big(-\frac{i}{32}(\gamma^\mu)_d{}^c\frac{\bar{D}^2 D_c\partial_\mu}{\partial^4}\cdot\frac{\bar{D}^d D^2}{\partial^2}-\frac{\bar{D}_d}{2\partial^2}\cdot\frac{\bar{D}^d D^2}{4\partial^2}\Big)\theta^a\theta_a\bar{\theta}^b\bar{\theta}_b$$
$$\sim(\gamma^\mu)_c{}^d\,\text{Tr}\Big(\Big[y_\mu^*,\frac{\bar{D}^2 D_d}{\partial^2}\cdot\frac{\bar{D}^c D^2}{\partial^2}\Big]\theta^a\theta_a\bar{\theta}^b\bar{\theta}_b\Big)=0$$

Note that all expressions become well defined after adding contributions of the Pauli–Villars fields.

Therefore, planar diagrams with a single loop of matter superfields are given by an integral of a total derivative and are equal to 0.

Using this method it is possible to prove factorization for some non-planar diagrams, for example, for



In particular this explains the result of the three loop calculation in $N=1$ SUSY QED.

Conclusion

- ✓ Possibly, with the higher derivative regularization integrals, defining the β-function, are the integrals of total derivatives. This allows to calculate at least one of the integrals analytically.
- ✓ For $N = 1$ supersymmetric electrodynamics, regularized by higher derivatives, it is possible to prove factorization of integrals into total derivative for some classes of diagrams, for example, for planar diagrams with a single matter loop. (Here we present the proof without PV contributions. It seems that they can be analyzed in a similar manner.)
- ✓ So far it is not clear, if it is possible to prove the factorization for all Feynman diagrams.
- ✓ Factorization of integrands into total derivatives is related with the NSVZ exact β-function.

Thank you for the attention!