Recent progress in f(R) models of inflation and dark energy

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QUARKS-2010, Kolomna, Russia, 10.06.2010

Definition of DE

DE – unclustered component, seen by gravitational interaction only. Quantitative definition - through gravitational field equations conventionally written in the Einstein form:

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu R\right) = -\left(T_{\mu\,(vis)}^\nu + T_{\mu\,(DM)}^\nu + T_{\mu\,(DE)}^\nu\right) \,,$$

$G = G_0 = const$ - the Newton gravitational constant measured in laboratory.

In the absence of direct interaction between DM and DE:

$$T_{\mu\,(DE);\nu}^\nu = 0 \,.$$

Possible forms of DE

- Physical DE
  New non-gravitational field of matter. DE proper place – in the rhs of gravity equations.
- Geometrical DE
  Modified gravity. DE proper place – in the lhs of gravity equations.
- Λ - intermediate case.

$f(R)$ gravity

$$S = \frac{1}{8\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R^\mu_\mu \ .$$

The effective energy-momentum tensor of DE in $f(R)$ gravity:

$$8\pi G T^\nu_{\mu\,(DE)} = F'(R)\, R^\nu_\mu - \frac{1}{2}\, F(R)\delta^\nu_\mu + \left(\nabla_\mu\nabla^\nu - \delta^\nu_\mu\nabla_\gamma\nabla^\gamma\right) F'(R)\,.$$

De Sitter solutions in the absence of matter: roots $R = R_{DS}$ of the algebraic equation

$$Rf'(R_) = 2f(R)\ .$$

Conditions for viable $f(R)$ models

I. Conditions of classical and quantum stability:

$f'(R) > 0\ , \quad f''(R) > 0\ .$

Even the saturation of these inequalities should be avoided:

1. $f'(R_0) = 0$: a generic anisotropic space-like curvature singularity forms.

2. $f''(R_0) = 0$: a weak singularity forms, loss of predictability of the Cauchy evolution.

$$a(t) = a_0 + a_1(t - t_s) + a_2(t - t_s)^2 + a_3|t - t_s|^{5/2} + ...\ .$$

The metric in C^2, but not C^3, continuous across this singularity, and there is no unambiguous relation between the coefficients a_3 for $t < t_s$ and $t > t_s$.

II. Conditions for the existence of the Newtonian limit:
$|F| \ll R, \quad |F'| \ll 1, \quad RF'' \ll 1$ for $R \gg R_{now}$ and up to some very large R.
The same conditions for smallness of deviations from GR.
III. Laboratory and Solar system tests.
No deviation from the Newton law up to 50 μ.
No deviation from the Einstein values of the post-Newtonian coefficients β and γ up to 10^{-4} in the Solar system.
IV. Existence of a future stable (or at least metastable) de Sitter asymptote:

$$f'(R_{DS})/f''(R_{DS}) \geq R_{DS} \ .$$

Required since observed properties of DE are close to that of a cosmological constant.

V. Cosmological tests:
among them the anomalous growth of matter perturbations for recent redshifts

$$\left(\frac{\delta\rho}{\rho}\right)_m \propto t^{\frac{\sqrt{33}-1}{6}}$$

at the matter-dominated stage for $k \gg M(R)a$, where $M^2(R) = 1/3F"(R)$.
VI. $f(R)$ cosmology should not destroy previous successes of present and early Universe cosmology in the scope of GR, including the existence of the matter-dominated stage driven by non-relativistic matter preceded by the radiation-dominated stage with the correct BBN and, finally, inflation.

An example of a viable DE model

A. A. Starobinsky, JETP Lett. **86**, 157 (2007)

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1 + \frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

with $n \geq 2$. Similar models in:
1. W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).
2. A. Appleby and R. Battye, Phys. Lett. B **654**, 7 (2007).
Generic feature: phantom behaviour for $z > 1$,
crossing of the phantom boundary $w_{DE} = -1$ for $z < 1$.

Still not the end of the story!

Three new problems

In the early Universe:

- Unlimited growth of $M(R)$ for $t \to 0$: when $M(R)$ exceeds M_{Pl}, quantum-gravitational loop corrections invalidate the use of an effective quasi-classical $f(R)$ gravity.
- Unlimited growth of the amplitude of δR oscillations for $t \to 0$ (the "scalaron overproduction" problem).
- "Big Boost" singularity before the Big Bang: $a(t) = a_0 + a_1(t - t_0) + a_2|t - t_0|^k + \ldots$, $1 < k = \frac{2n+1}{n+1} < 2$, if $F(R) \propto R^{-2n}$ for $R \to \infty$.

Curing all three problems

S. A. Appleby, R. A. Battye and A. A. Starobinsky, JCAP **1006**, 005 (2010).

Add $\frac{R^2}{6M^2}$ to $f(R)$ with M not less than the scale of inflation. Then the first and and third problems go away. The second problem still remains, but (any) inflation can solve it. However, in all known inflationary models R may be negative during reheating after inflation (e.g. when $V(\phi) = 0$). Necessity of an extension of $f(R)$ to $R < 0$ keeping $f''(R) > 0$. As a result, a non-zero g-factor $(0 < g < 1/2)$ arises:

$$g = \frac{f'(R) - f'(-R)}{2f'(R)} , \quad R_0 \ll R \ll M^2 .$$

An example: g-extended R^2-corrected AB model

$$f(R) = (1-g)R + g\epsilon \log\left[\frac{\cosh\left(R/\epsilon - b\right)}{\cosh b}\right] + \frac{R^2}{6M^2} \ .$$

Combined inflationary–DE models

If $M \approx 3 \times 10^{-6} M_{Pl}$, the scalaron can play the role of an inflaton, too. Then the inflationary predictions are formally the same as for the pure $R + R^2/6M^2$ inflationary model (A. A. Starobinsky, 1980) which does not describe the present DE:

$$n_s = 1 - \frac{2}{N}\ , \quad r = \frac{12}{N^2}.$$

However, N is different, $N \sim 70$ for the unified model (versus $N \sim 55$ for the purely inflationary one) because the stage if reheating after inflation becomes completely different: it consists of unequal periods with $a \approx const$ and $a \propto t^{1/2}$, so $a(t) \propto t^{1/3}$ on average for a long time after the end of inflation.

Observable prediction which is, however, degenerate with other inflationary models in $f(R)$ gravity.

Reheating – due to gravitational particle creation which occurs mainly at the end of inflation. Less efficient than in the pure $f(R) = R + R^2/6M^2$ inflationary model,

$$t = t_{\rm reh} \sim M^{-4} M_{\rm Pl}^3 \sim 10^{-18}\ {\rm s}\ .$$

Conclusions about $f(R)$ models

- Though a very narrow class among all $f(R)$ models of present DE still remains viable, it is not empty: it is possible to construct models satisfying all existing cosmological, Solar system and laboratory data, and distinguishable from ΛCDM.
- To achieve this, previously constructed viable $f(R)$ DE models should be extended to large R with the $\sim R^2$ asymptotic behaviour and to negative R keeping $f'(R) > 0,\ f''(R) > 0$ at least up to the scale of inflation.
- Unified description of primordial DE producing inflation and present DE in the scope of $f(R)$ gravity is possible for the specific choice of M: $M \approx 3 \times 10^{-6} M_{Pl}$.

- Combined inflationary – DE $f(R)$ models have a significantly different reheating stage after inflation as compared to pure inflationary $f(R)$ models, with strongly non-linear oscillations of the scale factor $a(t)$.
- The most critical test for all $f(R)$ models of present dark energy: anomalous growth of density perturbations in the matter component at recent redshifts $z \sim 1 - 3$.