Non-relativistic approach to quantum gravity

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Quarks 2010

Attempt to quantize gravity as a (weakly coupled) field theory

advantage: explicit

drawback: requires rejection of Lorentz invariance

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approach 1.5 year old:
still in its infancy

... despite hundreds of papers ...

Plan

- Gravity as quantum field theory: problems of relativistic formulation
- Quantum gravity with anisotropic scaling
- Covariant form of non-relativistic gravity. Relation with Einstein-aether model
- Coupling to matter. Phenomenological constraints
- Outlook

Problem of quantum gravity:
Einstein-Hilbert action is non-renormalizable

$$M_P^2 \int R\sqrt{-g}\, d^4x \quad \Rightarrow \quad M_P^2 \int \left((\partial h)^2 + h(\partial h)^2 + \ldots \right) d^4x$$

Quadratic part is invariant under the scaling:

$$\mathbf{x} \mapsto b^{-1}\mathbf{x} \ , \ t \mapsto b^{-1} t \ ,$$

$h \mapsto b\, h$ $\Rightarrow$ scaling dimension of h is 1

$$\int h(\partial h)^2\, d^4x \ \mapsto \ b \underbrace{\int h(\partial h)^2\, d^4x}_{\text{irrelevant interaction}}$$

We need to reduce *dim h* to 0

Can be achieved by including terms with higher derivatives: $R^2, \quad R_{\mu\nu}R^{\mu\nu}, \quad R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$

$$\int (R^2 + M_P^2 R)\sqrt{-g}\, d^4x \quad \Rightarrow$$

$$\int \left((\Box h)^2 + h(\Box h)^2 + M_P^2(\partial h)^2 + M_P^2\, h(\partial h)^2 + \dots \right) d^4x$$

$(\Box h)^2$: sets *dim h* in UV

$h(\Box h)^2$: marginal interaction

$M_P^2(\partial h)^2 + M_P^2\, h(\partial h)^2$: relevant deformations

Scaling: $\mathbf{x} \mapsto b^{-1}\mathbf{x}\ ,\ t \mapsto b^{-1}t\ ,\ h \mapsto h$

$$\mathit{dim}\ h = 0$$

IR dynamics is determined by terms with 2 derivatives

But higher time derivatives ⇒ ghosts

⇒ loss of unitarity

Stelle (1978)

Gravity with anisotropic scaling I

Horava (2009)

Split coordinates in space and time:

ADM decomposition of the metric (in GR -- a gauge choice)

$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dt dx^i - \gamma_{ij} dx^i dx^j$$

Think of the splitting as physical

$\Leftrightarrow$ equip space-time with foliation by spacelike surfaces

4d Diffs are broken down to foliation preserving subgroup (FDiffs)

$$\mathbf{x} \mapsto \tilde{\mathbf{x}}(\mathbf{x}, t) \ , \qquad t \mapsto \tilde{t}(t)$$

Gravity with anisotropic scaling II

$$S = \frac{M_P^2}{2} \int d^3x dt \sqrt{\gamma} N \Big(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V} \Big)$$

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

$$\mathcal{V}_I = -R \quad + \quad M_*^{-2} \big(A_1 \Delta R + A_2 R_{ij} R^{ij} + \dots \big)$$
$$+ \quad M_*^{-4} \big(B_1 \Delta^2 R + B_2 R_{ij} R^{jk} R^i_k + \dots \big)$$

difference from GR

R_{ij} -- 3d Ricci tensor

Variations

- projectable: $N = N(t)$ (compatible with FDiffs)
- non-projectable: $N = N(t, \mathbf{x})$
- with/without detailed balance

- collection of marginal and relevant operators under scaling:

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\ , \quad t \mapsto b^{-3}t$$

$$N,\ \gamma_{ij} \mapsto N,\ \gamma_{ij}$$

$$N_i \mapsto b^2 N_i$$

➡ Theory is power-counting renormalizable

- higher-derivative terms are unimportant in IR

➡ recovery of GR provided λ flows to 1

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Too quick !!

To make long story short ...

- explicit breaking of Diffs (gauge group of GR) down to FDiffs

  extra light degree of freedom -- "scalar graviton"

- ill-behaved in both models explicitly proposed by Horava (ghost / gradient instability / strong coupling)

Charmousis, Niz, Padilla, Saffin (2009)
Blas, Pujolas, S.S. (2009)

A failure of the program ?

or of the specific realizations ?

- Foliation is physical $\Rightarrow$ extra scalar is unavoidable
- Can we make it well-behaved by adjusting the action ?

The third attempt

A healthy model

Blas, Pujolas, S.S. (2009)

is obtained by a straightforward (and natural) generalization of the non-projectable case

$a_i \equiv N^{-1}\partial_i N$ -- covariant under FDiffs

$dim\ a_i = 1$ ⇒

$$\mathcal{V}_{II} = \mathcal{V}_I - \alpha a_i a^i$$
$$+ M_*^{-2}\left(C_1 a_i \Delta a^i + C_2 (a_i a^i)^2 + C_3 a_i a_j R^{ij} + \ldots\right)$$
$$+ M_*^{-4}\left(D_1 a_i \Delta^2 a^1 + D_2 (a_i a^i)^3 + D_3 a_i a^i a_j a_k R^{jk} + \ldots\right)$$

Scalar mode dispersion relation:

$$\omega^2 = \frac{\lambda - 1}{2(3\lambda - 1)} \frac{P[-p^2/M_*^2]}{Q[-p^2/M_*^2]} p^2$$

$$\begin{aligned} P[x] =& (g_2^2 - g_1 g_3)x^4 - (g_1 f_3 + g_3 f_1 - 2 g_2 f_2)x^3 \\ & + (f_2^2 - 4g_2 - f_1 f_3 - 2g_3 - g_1\alpha)x^2 \\ & - (2f_3 + f_1\alpha + 4f_2)x + (4 - 2\alpha) \\ Q[x] =& g_3 x^2 + f_3 x + \alpha \end{aligned}$$

- stable throughout the momentum range
- right scaling in IR: $\omega^2 \propto p^2$
- and in UV: $\omega^2 \propto p^6$

TOWARDS PHENOMENOLOGY

Stueckelberg formalism I

To identify the effect of the new d.o.f.: restore gauge invariance by introducing Stueckelberg field

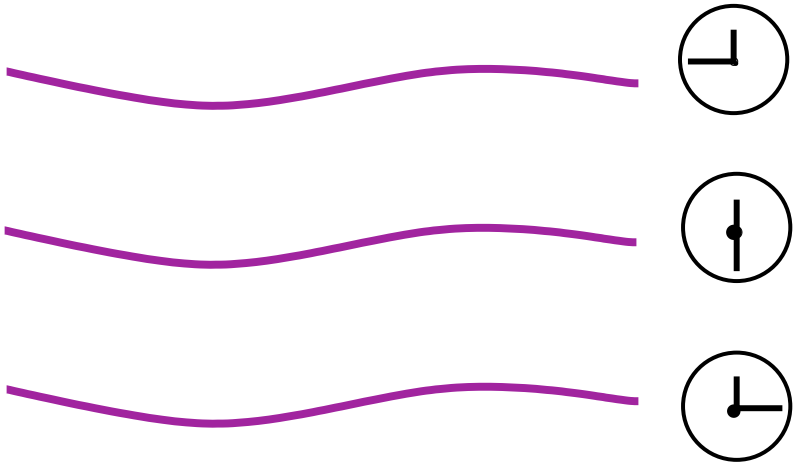
In case of gravity equivalent to covariantization

- parametrize foliation surfaces with scalar field:

$$\sigma(x) = const$$

ADM frame = gauge fixing $t = \sigma$

σ sets global time



CHRONON

Stuecke[illegible]rmalism I

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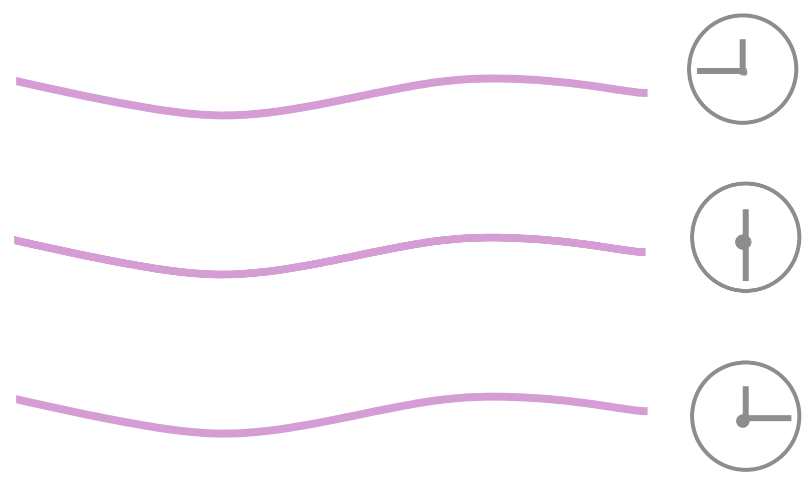

CHRONON

CHRONO-METRIC THEORY

Stuecke … ormalism I

To identify th … e new d.o.f.: resto …
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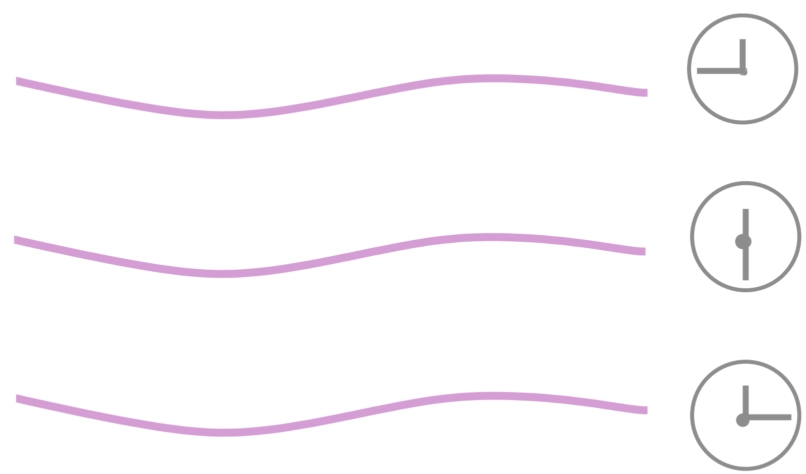

Stueckelberg formalism II

- Time reparameterizations in ADM frame

➡ symmetry $\sigma \mapsto \tilde{\sigma} = f(\sigma)$

Invariant object -- unit normal to the foliation surfaces:

$$u_\mu = \frac{\partial_\mu \sigma}{\sqrt{(\partial\sigma)^2}}$$

- identify covariant geometric structures in ADM frame
- obtain the covariant (low-energy) action:

$$S = -\frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big\{{}^{(4)}R + (\lambda-1)(\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u^\rho \nabla_\nu u_\rho\Big\}$$

compare with Einstein-aether model

Jacobson, Mattingly (2001)

N.B. In our case there are no transverse vector modes

Chronon dynamics: low-energy perspective

expand $\sigma = t + \chi$

$$S_\chi = \int d^4x \left[\frac{M_\alpha^2}{2}(\partial_i \dot{\chi})^2 - \frac{M_\lambda^2}{2}(\Delta\chi)^2 - M_\lambda^2\, \dot{\chi}(\Delta\chi)^2 + M_\alpha^2 \left(\dot{\chi}\partial_i\ddot{\chi}\partial_i\chi - \partial_i\dot{\chi}\partial_j\chi\partial_i\partial_j\chi\right) + \ldots \right]$$

$$M_\alpha \equiv \sqrt{\alpha} M_P$$
$$M_\lambda \equiv \sqrt{\lambda - 1}\, M_P$$

- linear order $\Delta(M_\alpha^2 \ddot{\chi} - M_\lambda^2 \Delta\chi) = 0$
- derivative self-interaction

  for $M_\alpha \sim M_\lambda$ would-be strong coupling at $\Lambda \sim M_\alpha$

  resolved by higher derivatives

$$M_* \lesssim M_\alpha, M_\lambda$$

N.B. Λ goes down in case of hierarchy between M_α and M_λ

Coupling to matter I

SM fields couple to u_μ

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- with additional derivatives

$$a_\mu \bar{\psi} \gamma^\mu \psi \qquad K^{\mu\nu} \bar{\psi} \gamma_\mu \partial_\nu \psi$$

derivative interaction via χ

suppressed by M_*

Coupling to matter I

SM fields couple to u_μ

- with additional derivatives

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derivative interaction via χ

suppressed by M_*

- without derivatives

$$u_\mu \bar{\psi} \gamma^\mu \psi \qquad u^\mu u^\nu \bar{\psi} \gamma_\mu \partial_\nu \psi \qquad u^\mu u^\nu \bar{\psi} \partial_\mu \partial_\nu \psi$$

lead to violation of Lorentz symmetry within the SM

Coupling to matter II

operators of dim > 4 ($u^\mu u^\nu \bar{\psi} \partial_\mu \partial_\nu \psi$)

UV modification of dispersion relations

$$E^2 = m^2 + p^2 + \frac{p^4}{\left(M_*^{(mat)}\right)^2} + \dots$$

timing of AGN's and GRB's

MAGIC (2008)
Fermi GMB/LAT (2009)

$$M_*^{(mat)} \gtrsim 10^{10} \div 10^{11} \mathrm{GeV}$$

N.B. $M_*^{(mat)}$ may be different from M_*

Coupling to matter III

operators of dim ≤ 4 $\quad (u_\mu \bar{\psi} \gamma^\mu \psi, \;\; u^\mu u^\nu \bar{\psi} \gamma_\mu \partial_\nu \psi)$

tightly constrained

e.g. dim 4 correct "speed of light" for different species

$$E^2 = m^2 + c^2 p^2$$

experimental bound:

Lamoreaux et al. (1986)
Coleman, Glashow (1999)

$$|c_\gamma - c_{p,e}| \leq 6 \times 10^{-22} \quad !$$

➡ A mechanism for suppression of Lorentz breaking at dim up to 4 is required

Universal coupling

Minimal coupling to effective metric

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \beta u_\mu u_\nu$$

- trade $g_{\mu\nu}$ for $\tilde{g}_{\mu\nu}$

$$S = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} \Big\{ {}^{(4)}R - \beta \nabla_\mu u_\nu \nabla^\nu u^\mu$$

$$+ \lambda' (\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u^\rho \nabla_\nu u_\rho \Big\}$$

$$\lambda - 1 + \beta$$

- exploit connection to Einstein-aether

PPN parameters I

Spherically symmetric solutions the same as in Einstein-aether

→ all PPN parameters the same as in GR except α_1^{PPN} , α_2^{PPN}

measure preferred frame effects

PPN parameters I

Spherically symmetric solutions the same as in Einstein-aether

$\Rightarrow$ all PPN parameters the same as in GR except α_1^{PPN} , α_2^{PPN}

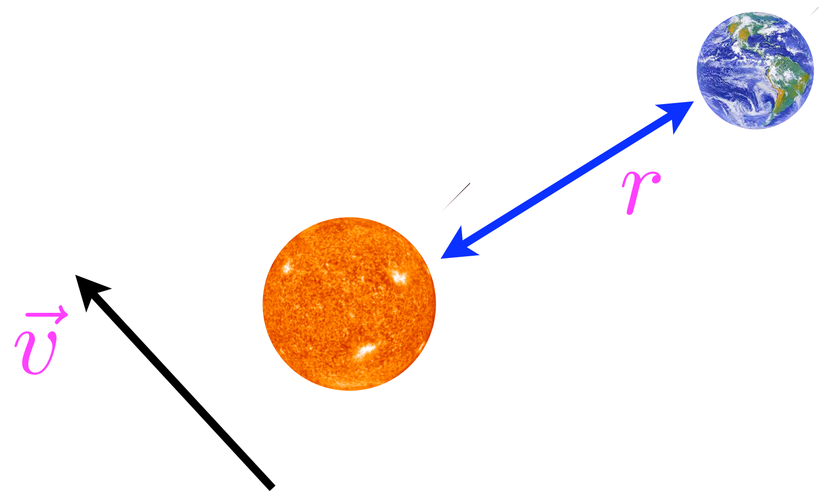
$\underbrace{\alpha_1^{PPN}\,,\ \alpha_2^{PPN}}$ measure preferred frame effects

Solar system bounds:

$$|\alpha_1^{PPN}| \lesssim 10^{-4}\,, \qquad |\alpha_2^{PPN}| \lesssim 10^{-7}$$

PPN parameters II

Solar system bounds



$$h_{00} = -2G_N \frac{m}{r}\left(1 - \frac{\alpha_2^{PPN}}{2}\frac{(x^i v^i)^2}{r^2}\right)$$

$$h_{0i} = \frac{\alpha_1^{PPN}}{2} G_N \frac{m}{r} v^i$$

PPN parameters III

$$\alpha_1^{PPN} = -4(\alpha + 2\beta)$$

$$\alpha_2^{PPN} = \frac{(\alpha + 2\beta)(\alpha - \lambda' + 3\beta)}{2(\lambda' - \beta)}$$

- vanish if $\alpha + 2\beta = 0$
- α_2^{PPN} vanishes when $\beta = 0$, $\lambda' = \alpha$ ($c_\chi = 1$)
- barring cancellations

$$\alpha \, , \, \beta \, , \, \lambda' \lesssim 10^{-7} \div 10^{-6}$$

+ Absence of strong coupling ➡ upper bound on the scale of quantum gravity

$$M_* \lesssim 10^{15} \div 10^{16} \mathrm{GeV}$$

To get back LI in IR
use the supertool

SUPERSYMMETRY !!

Lorentz invariance from supersymmetry

Nibbelink, Pospelov (2004)
Bolokhov, Nibbelink, Pospelov (2005)

Given SUSY, Lorentz invariance emerges as accidental symmetry at low energies

It is impossible to write any LV operator in MSSM of dim < 5

Dim 5 operators are CPT odd → may be forbidden
→ LV starts from dim 6

SUSY breaking generates dim 4 LV operators suppressed by

$$\left(m_{soft}/M_*\right)^2$$

Conclusions and Outlook

- A consistent power counting renormalizable model of gravity with anisotropic scaling exists
- It does not reduce to GR in the infrared: light scalar mode, violation of LI
- Compatible with experimental data for the scale of LV between 10^{10} and 10^{16} GeV
- Open issues: proof of renormalizability, UV completeness, singularities, cosmology, black holes, emergence of LI, instantaneous interaction, binary pulsars, ...

  Calcagni (2009), Kiritsis & Kofinas(2009), Brandenberger (2009), Kiritsis (2009), Kobayashi et al. (2010), Armendariz-Picon et al. (2010), ...

- Beyond 4d: higher and lower dims, lattice models, condensed matter, causal dynamical triangulation, ...

  Iengo & Serone (2010), Horava (2009), Xu & Horava (2010), Ambjorn et al. (2010), ...

Thanks to the OK (Organizing ~~C~~K Committee) !