"Imagery of Symmetry in Current Physics"

Dedicated to the memory
of Albert Tavkhelidze

Dmitry V. Shirkov
Bogoliubov Lab. JINR, Dubna

Aleko and Symmetry

Albert – man of Idea :

New Quantum Number for Quarks, 1965

Aleko and Passion

Albert – man of Action

New Quantum Number

for Quarks

ICTP lectures 1965

(C.N.Yang by F. Dyson)

vs.T.D. Leeand Wu

Symmetry in Life and Science

- Symmetry around us
  - Symmetry in Art
  - Distorted Symmetry
  - Symmetry and Beauty

Symmetry in Life and Science

- Symmetry around us
  - Symmetry in Art
  - Distorted Symmetry
  - Symmetry and Beauty
- Symmetry in Science :
  - Quantum Numbers
  - Continuous Symmetry
  - Brocken Symmetry

Phase transition and broken symmetry †

Connection btwn Phase transition and symmetry breaking was evident long ago before QM creation –> e.g., from physics of crystals.

Landau 1937 paper on phase transitions:

- starts by introducing symmetries,

Phase transition and broken symmetry †

Connection btwn Phase transition and symmetry breaking was evident long ago before QM creation –> e.g., from physics of crystals.

Landau 1937 paper on phase transitions:

- starts by introducing symmetries,
- but, only discrete symmetries :
- on SuperFluid – "He II is not a liquid crystal !"

Meanwhile, Landau's "Mechanics" (1937/40) is based upon continuous symmetries, invariance and conservation laws.

———————————————

† In Phys.Usp.52:549-557,2009, arXiv:0903.3194

Symmetries and groups

Symmetries and groups : discrete and continuous.

Continuous group –> Lie group of transformations.

Lagrangian –> Invariance –> Nöther theorem –> current –> conservation law.

Symmetries and groups

Symmetries and groups : discrete and continuous.

Continuous group –> Lie group of transformations.

Lagrangian –> Invariance –> Nöther theorem –> current –> conservation law.

Quantum Symmetries:

- Non-relativistic 2nd-quantized neutral field Phase transformation= $a \longrightarrow e^{-i\alpha} a, \; a^* \longrightarrow e^{i\alpha} a^*$ –> $N =$ const. Conserving Number of particles
- Charged (2-, 3-component) field; Gauge=phase transformation –> Current; Charge conservation

Quantum Symmetries

Qu-Symmetries: Phase, Gauge, Chiral, SuSy,

Qu-Symmetries are quite different from "Classical" ones, like spatial (boosts, rotations, Lorentz) and internal (isospin, flavor) ones.

For their formulation and understanding one has to use quantum notions :

* nonobservability of the ψ-function phase;
* spin, chirality ;
* distinction btwn Bose– and Fermi–statistics.

Bogoliubov model for SF He II

Bogoliubov 1946 microscopic theory starts with

$$H_{\mathrm{B-gas}} = \sum_{\vec{p}} \frac{p^2}{2m} a_p^+ a_p + \frac{1}{2V} \sum v(p_1 - p_2) a_{p_1}^+ a_{p_2}^+ a_{p_2} a_{p_1} \; ;$$

non-ideal Bose gas with weak repulsion $v(p) > 0\,.$

Bogoliubov model for SF He II

Bogoliubov 1946 microscopic theory starts with

$$H_{\mathrm{B-gas}} = \sum_{\vec{p}} \frac{p^2}{2m} a_p^+ a_p + \frac{1}{2V} \sum v(p_1 - p_2) a_{p_1}^+ a_{p_2}^+ a_{p_2} a_{p_1} \, ;$$

non-ideal Bose gas with weak repulsion $v(p) > 0$.

The Hamiltonian obeys $\boxed{a \to e^{-i\alpha} a, \; a^* \to e^{i\alpha} a^*} =$

phase symmetry –> No of particles conservation,

as $H_{\mathrm{B-gas}}$ commutes with $\mathbf{N} = \sum_{\vec{p}} a_p^+ a_p$

Bogoliubov model for SF He II

Bogoliubov 1946 microscopic theory starts with

$$H_{\mathrm{B-gas}} = \sum_{\vec{p}} \frac{p^2}{2m} a_p^+ a_p + \frac{1}{2V} \sum v(p_1 - p_2) a_{p_1}^+ a_{p_2}^+ a_{p_2} a_{p_1} \, ;$$

non-ideal Bose gas with weak repulsion $v(p) > 0$.

The Hamiltonian obeys $\boxed{a \to e^{-i\alpha} a, \; a^* \to e^{i\alpha} a^*}$ =

phase symmetry –> No of particles conservation,

as $H_{\mathrm{B-gas}}$ commutes with $\mathbf{N} = \sum_{\vec{p}} a_p^+ a_p$

Bogoliubov's physical hypothesis:

“macroscopic condensate” $\boxed{\mathbf{N_{p=0} = a_0^+ \, a_0} \sim \mathrm{N_A}}$

Corollary: condensate operators $a_0^+, a_0 \sim \sqrt{N_0}$ = c-numbers

Bogoliubov 1946 SuperFlu model

Shift $\boxed{\psi(\mathbf{x}) = \mathbf{\Psi_0} + \phi(\mathbf{x})}$ by "big" constant $\Psi_0 \sim \sqrt{N_0}$

results in bilinear approximate Hamiltonian

$$\mathbf{H_{Bog}} = \sum_{\mathbf{p}\neq\mathbf{0}} \left(\frac{\mathbf{p^2}}{\mathbf{2m}} + \frac{\mathbf{N_0}}{\mathbf{V}} \mathbf{v}(\mathbf{p}) \right) \mathbf{b}^{+}_{\mathbf{p}} \mathbf{b}_{\mathbf{p}}, + \frac{N_0}{2V} \sum_{p\neq 0} v(p) [b^{+}_{p} b^{+}_{-p} + b_p b_{-p}]$$

Bogoliubov 1946 SuperFlu model

Shift $\boxed{\psi(\mathbf{x}) = \mathbf{\Psi_0} + \phi(\mathbf{x})}$ by "big" constant $\Psi_0 \sim \sqrt{N_0}$
results in bilinear approximate Hamiltonian

$$\mathbf{H_{Bog}} = \sum_{\mathbf{p}\neq\mathbf{0}} \left(\frac{\mathbf{p^2}}{\mathbf{2m}} + \frac{\mathbf{N_0}}{\mathbf{V}} \mathbf{v(p)} \right) \mathbf{b_p^+ b_p}, + \frac{N_0}{2V} \sum_{p\neq 0} v(p) [b_p^+ b_{-p}^+ + b_p b_{-p}]$$

with b_p^+, b_p – "above-condensate" Bose-operators.
$\mathrm{H_{Bog}}$ describes creation of pairs of Helium atoms with opposite momenta from condensate and their "annihilation" into condensate.

Interaction between pairs is small $\sim N_0^{-1/2}$ and omitted.

Total number of the correlated pairs is not fixed.
No phase symmetry !

Bogoliubov-1946 SuperFlu, 3

In the leading order : $H_{B1} = E_0 + H_{Bog2}(b) + ...$

(E_0 =condensate energy), H_{Bog2} – bilinear operator form

$$H_{Bog2}(b) = \frac{N_0}{2V}\sum_{p\neq 0} v(p)\,[b_p^+ b_{-p}^+ + b_p b_{-p}] + \sum_{p\neq 0}\left(T(p) + \frac{N_0}{V} v(p)\right) b_p^+ b_p,$$

Bogoliubov-1946 SuperFlu, 3

In the leading order : $H_{B1} = E_0 + H_{Bog2}(b) + \ldots$
(E_0 =condensate energy), H_{Bog2} – bilinear operator form

$$H_{Bog2}(b) = \frac{N_0}{2V} \sum_{p\neq 0} v(p)\,[b^+_p b^+_{-p} + b_p b_{-p}] + \sum_{p\neq 0} \left(T(p) + \frac{N_0}{V} v(p)\right) b^+_p b_p,$$

diagonalized by Bogoliubov (u, v) transformation

$$b_p \longrightarrow \xi_p\,; \qquad \boxed{\xi_p = u_p b_p + v_p b^+_{-p}; \quad \xi^+_p = u_p b^+_p + v_p b_{-p}}$$

with real coefficients $u_p^2 - v_p^2 = 1;\ u_{-p} = u_p;\ v_{-p} = v_p\,.$
In 2nd quant. – by unitary Bog's transformation
$\xi_p = U^{-1}_\alpha b_p U_\alpha = u_p b_p + v_p b^+_{-p}$, with new ground state

$$\Phi_0 \longrightarrow \Psi_0 = U_\alpha \Phi_0\,; \qquad \boxed{U_\alpha = e^{\sum_p \alpha(p)\,[b^+_p b^+_{-p} - b_p b_{-p}]}\,.}$$

Bogoliubov spectrum instead of Landau *

The $b_p \to \xi_p$ transformation correlates pairs of He II atoms with opposite momenta. New Hamiltonian

$$H_{Bog2}(b) = H_{Bog3}(\xi)\,; \quad H_{Bog3} = \sum_{p\neq 0} E(p)\,\xi_p^+\,\xi_p\,,$$

$$E(p) = \sqrt{(T(p))^2 + T(p)\,v(p)}; \qquad T(p) = \frac{p^2}{2m}$$

describes new collective excitations [bogolons].

Bogoliubov spectrum instead of Landau *

The $b_p \longrightarrow \xi_p$ transformation correlates pairs of He II atoms with opposite momenta. New Hamiltonian

$$H_{Bog2}(b) = H_{Bog3}(\xi)\,; \quad H_{Bog3} = \sum_{p\neq 0} E(p)\,\xi_p^+\,\xi_p\,,$$

$$E(p) = \sqrt{(T(p))^2 + T(p)\,v(p)}; \qquad T(p) = \frac{p^2}{2m}$$

describes new collective excitations [bogolons].

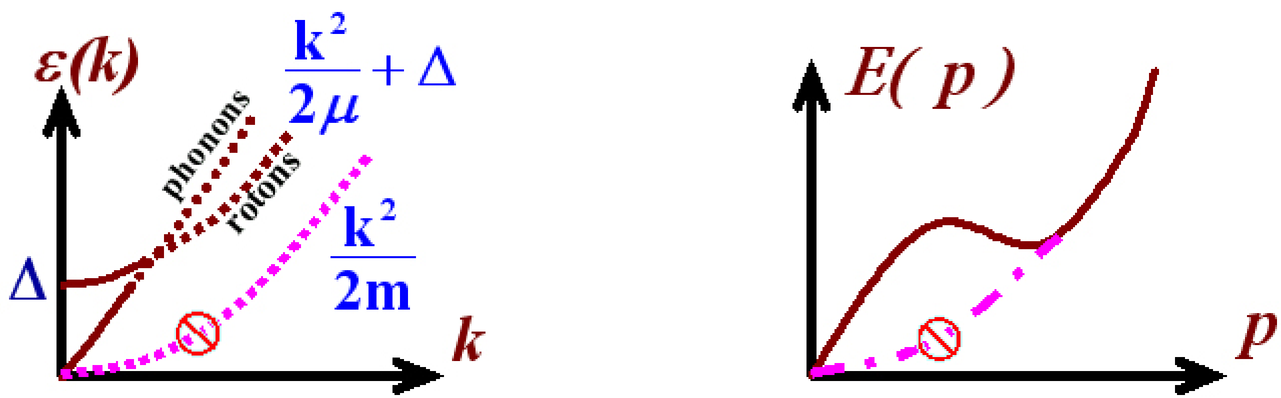
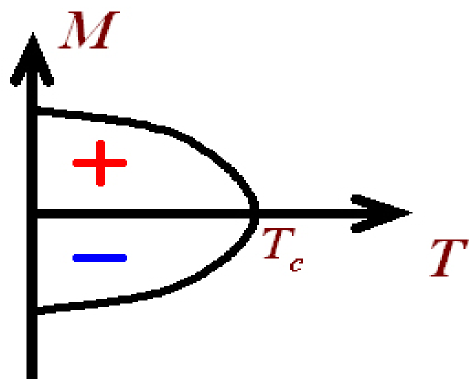


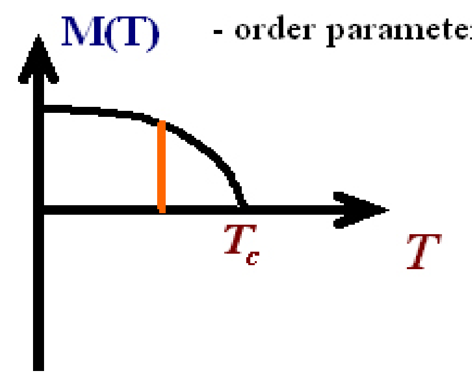
Figure 2: (a) Phonon + roton spectra – Landau phenomenology; (b) Bogoliubov spectrum from non-ideal Bose-gas microscopical model.

Phase transition with Symmetry Breaking *

Order parameter [Landau 1937] in magnetics,



- add a weak field δH_+
- tend $V \to \infty$
- put $\delta H_+ \to 0$



Ferromagnetism in a finite volume **V**. In the thermodynamic limit

Correlation function:

$$K_{\sigma\sigma}(\mathbf{r}) =< \sigma(\mathbf{0})\sigma(\mathbf{r}) > - < \sigma(\mathbf{0}) >< \sigma(\mathbf{r}) >$$

$$K_{\sigma\sigma}(\mathbf{r} \to \infty) = \begin{cases} 0, & T > T_c \\ M^2(T), & T < T_c \end{cases}$$

Ginzburg-Landau [1950] SuperConductivity

For description of system of electrons, G+L used

2-component order parameter $\Psi(r) = |\Psi(r)| e^{i\Phi(r)}$.

Ginzburg-Landau [1950] SuperConductivity

For description of system of electrons, G+L used 2-component order parameter $\Psi(r) = |\Psi(r)|e^{i\Phi(r)}$.
They defined "by hands" Free energy functional responsible for SuperConducting transition

$$F = F_n + \int \left(\frac{\hbar}{2m^*} |\vec{\nabla}\Psi(r)|^2 + a|\Psi(r)|^2 + b|\Psi(r)|^4 \right) dV$$

with $a \sim T - T_c$, $b \approx \text{const}$, m^* – effective mass

Ginzburg-Landau [1950] SuperConductivity

For description of system of electrons, G+L used

2-component order parameter $\Psi(r) = |\Psi(r)|e^{i\Phi(r)}$.

They defined "by hands" Free energy functional

responsible for SuperConducting transition

$$F = F_n + \int \left(\frac{\hbar}{2m^*} |\vec{\nabla}\Psi(r)|^2 + a|\Psi(r)|^2 + b|\Psi(r)|^4 \right) dV$$

with $a \sim T - T_c$, $b \approx \text{const}$, m^* – effective mass

SC current $j_\alpha = \frac{e^*\hbar}{m^*} |\Psi|^2 \nabla_\alpha \Phi$, $e*$ – effective charge.

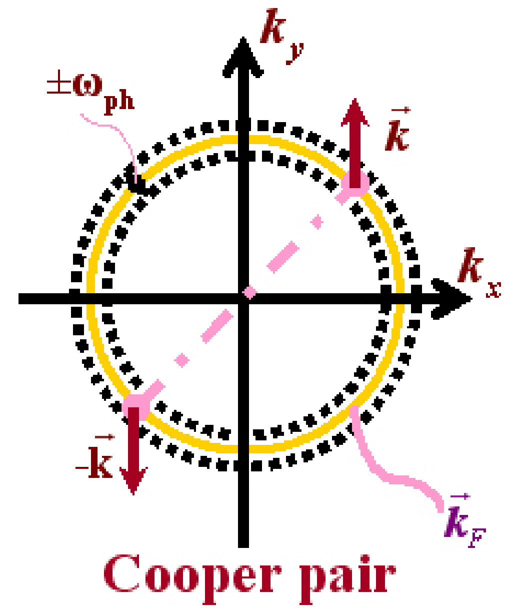
———————————————-

Gor'kov (1959): $m^* = 2m$, $e^* = 2e$, $|\Psi|^2 = n_s/2$.

BSC effective SuperConductivity

BCS model:

$$H = \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}} c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{k},\vec{k}'} V_{\vec{k},\vec{k}'} c^{+}_{\vec{k}\uparrow} c^{+}_{-\vec{k}\downarrow} c_{-\vec{k}'\downarrow} c_{\vec{k}'\uparrow} ,$$



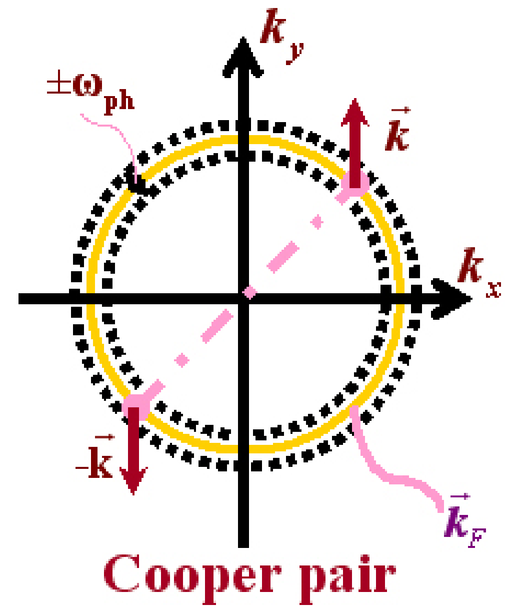
- eff. Cooper pairs (antipodes) attraction

$\varepsilon_{\vec{k}} = \frac{\vec{k}^2}{2m} - \varepsilon_F$ - electron energy above ε_F

BSC effective SuperConductivity

BCS model:

$$H = \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}} c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{k},\vec{k}'} V_{\vec{k},\vec{k}'} c^{+}_{\vec{k}\uparrow} c^{+}_{-\vec{k}\downarrow} c_{-\vec{k}'\downarrow} c_{\vec{k}'\uparrow} ,$$

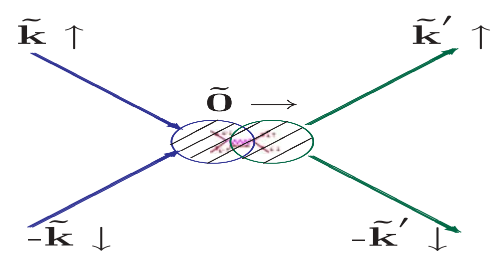


- eff. Cooper pairs (antipodes) attraction

$\varepsilon_{\vec{k}} = \frac{\vec{k}^2}{2m} - \varepsilon_F$ - electron energy above ε_F

Effect. electron-electron attraction only in vicinity of Fermi surface

$$V(\vec{k},\vec{k}') = \begin{cases} -V_C, & |\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}| < \omega_{ph} \\ 0, & |\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}| > \omega_{ph} \end{cases}$$



Semi-Phenomen. BSC theory, 2

Variational BCS wave function

$$|\Psi_{BCS}>= \prod_{\vec{k}}(u_{\vec{k}} + v_{\vec{k}} c^{+}_{\vec{k}\uparrow} c^{+}_{-\vec{k}\downarrow})|0>; \quad c_{\vec{k}\sigma}|0>= 0.$$

New SC ground state: $\boxed{c_{\vec{k}\sigma}|\Psi_{BCS}>\neq 0}$

Semi-Phenomen. BSC theory, 2

Variational BCS wave function

$$|\Psi_{BCS}>=\prod_{\vec{k}}(u_{\vec{k}}+v_{\vec{k}}c^{+}_{\vec{k}\uparrow}c^{+}_{-\vec{k}\downarrow})|0>; \quad c_{\vec{k}\sigma}|0>=0.$$

New SC ground state: $\boxed{c_{\vec{k}\sigma}|\Psi_{BCS}>\neq 0}$

- SC order parameter = Cooper pair condensate:
  $< c^{+}_{\vec{k}\uparrow}c^{+}_{-\vec{k}\downarrow} >= \Psi(\vec{k}) = |\Psi(\vec{k})|exp[i\Phi(\vec{k})]$
- Phase symm breaking: $\widetilde{c}^{+}_{\vec{k}\sigma} = e^{i\phi}c^{+}_{\vec{k}\sigma} \Rightarrow \boxed{\widetilde{\Psi}(\vec{k}) = e^{2i\phi}\Psi(\vec{k})}$
- Energy gap: $\Psi(\vec{k}) = \frac{\triangle_{\vec{k}}}{2E_{\vec{k}}}$ $\quad \boxed{\triangle_0 \approx exp\left(-\frac{1}{\lambda}\right);}$ $\quad \lambda = N_0 V_C$
- SC temperature $\quad T_c = 1.14\,\omega_{ph}\,exp\left(-\frac{1}{\lambda}\right); \quad 2\triangle_0 = 3.52T_c$

Bogoliubov SuperCond micro-theory

with Fröhlich electron-phonon interaction: $H_{Fr} =$

$$= \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}} c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{q}} \omega_{\vec{q}} b^{+}_{\vec{q}} b_{\vec{q}} + g_{Fr} \sum_{\vec{k},\vec{k}',\sigma} \sqrt{\frac{\omega(\vec{q})}{2V}} c^{+}_{\vec{k}\sigma} c_{\vec{k}'\sigma} (b^{+}_{\vec{q}} + b_{-\vec{q}})$$

Bogoliubov (u,v) transformation:

$$\alpha_{\vec{k}\uparrow} = u_{\vec{k}} c_{\vec{k}\uparrow} - v_{\vec{k}} c^{+}_{-\vec{k}\downarrow}; \quad \alpha^{+}_{\vec{k}\uparrow} = u_{\vec{k}} c^{+}_{\vec{k}\uparrow} + v_{\vec{k}} c_{-\vec{k}\downarrow}$$

$$u^2_{\vec{k}} = 1 - v^2_{\vec{k}} = \frac{1}{2}\left(1 + \frac{\varepsilon_{\vec{k}}}{E_{\vec{k}}}\right)$$

Bogoliubov SuperCond micro-theory

with Fröhlich electron-phonon interaction: $H_{Fr} =$

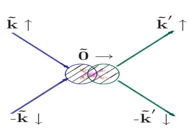
$$= \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}} c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{q}} \omega_{\vec{q}} b^{+}_{\vec{q}} b_{\vec{q}} + g_{Fr} \sum_{\vec{k},\vec{k}',\sigma} \sqrt{\frac{\omega(\vec{q})}{2V}} c^{+}_{\vec{k}\sigma} c_{\vec{k}'\sigma} (b^{+}_{\vec{q}} + b_{-\vec{q}})$$

Bogoliubov (u,v) transformation:

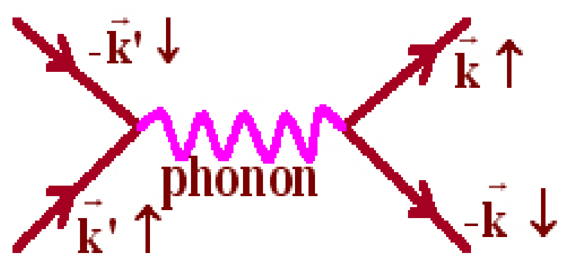
$$\alpha_{\vec{k}\uparrow} = u_{\vec{k}} c_{\vec{k}\uparrow} - v_{\vec{k}} c^{+}_{-\vec{k}\downarrow}; \quad \alpha^{+}_{\vec{k}\uparrow} = u_{\vec{k}} c^{+}_{\vec{k}\uparrow} + v_{\vec{k}} c_{-\vec{k}\downarrow}$$

$u^2_{\vec{k}} = 1 - v^2_{\vec{k}} = \frac{1}{2}\left(1 + \frac{\varepsilon_{\vec{k}}}{E_{\vec{k}}}\right)$ Gap solution : $\boxed{\triangle_B = \widetilde{\omega}\, exp\left(-\frac{1}{\rho_B}\right)}$

BCS phenom $\lambda = V_{\mathrm{C}} N_0$ *vs* Microscopical $\rho_B = g^2_{Fr} N_0$



$\Rightarrow$



Message to XXI

"Phase transition in Quantum system, as a rule, is accompanied by Spontaneous Symmetry Breaking".

XXth Century Folklore

Message to XXI

"Phase transition in Quantum system, as a rule, is accompanied by Spontaneous Symmetry Breaking".

XXth Century Folklore

1. Microscopic BCS-Bogoliubov SuperConductivity was shown to be SuperFluidity of Cooper pairs (Bogoliubov, 1958)

2. The SuperFlu and SuperCond transitions proceed escorted by Spont Symm Breaking of phase ("gauge") symmetry, related to No of particles conservation.

Message to XXI

"Phase transition in Quantum system, as a rule, is accompanied by Spontaneous Symmetry Breaking".

XXth Century Folklore

1. Microscopic BCS-Bogoliubov SuperConductivity was shown to be SuperFluidity of Cooper pairs (Bogoliubov, 1958)

2. The SuperFlu and SuperCond transitions proceed escorted by Spont Symm Breaking of phase ("gauge") symmetry, related to No of particles conservation.

3. QFT Higgs = formal replica of Ginzburg-Landau SuperCond phenomenology. Yet no physics under it.

Tavkhelidze at Dubna

SSB Transition to QFT; Early 60s

Spontaneous Breaking of Chiral (γ_5) Invariance

2-dim models with cutoff Λ

- Vaks + Larkin I,II [Aug 1960]
- Tavkhelidze [Aug 1960] {ref: Bogoliubov, Sept '60}
- Nambu [? 1960 Purdue Conf] {ref: Nobel Comm '08 doc }
- Nambu, Jona-Lasinio I [Oct 1960]

2-dim, + cutoff Λ

- Nambu, Jona-Lasinio II [May 1961]

2-dim without cutoff

- Arbuzov, Tavkhelidze, Faustov [Nov 1961]

Implication to QFT; Higgs field

Lagrangian for normal quantum scalar field with quartic self-interaction and stable ground state

$$L(\varphi, g) = \frac{1}{2}\,(\partial_\mu \varphi)^2 - V(\varphi), \qquad \boxed{V(\varphi) = \frac{m^2}{2}\,\varphi^2 + g\,\varphi^4;\ \ g > 0}$$

Since 60s, in QFT play with toy models *à la Ginzburg-Landau* with phantom scalar field $\mathbf{\Phi}(x)$, like the Higgs (1964) one

$$V_{\rm Higgs}(\Phi^2) = \lambda\left(\mathbf{\Phi}(x)^2 - \Phi_0^2\right)^2\ ; \quad \mathbf{\Phi}^2 = \Phi_1^2 + \Phi_2^2\,; \quad \Phi_0^2 = \text{const.}$$

with imaginary initial mass $\mu_{\rm H}^2 = -4\lambda\,\Phi_0^2$ and the final one $m_{\rm Higgs} = 2\sqrt{2\lambda}\,\Phi_0$ obtained after shift of field operator by constant

$$\Phi(x) \to \varphi(x) = \Phi(x) - \Phi_0\,,$$

like in Bogoliubov's SuperFluidity.

Bogoliubov with Tavkhelidze

Different Symmetries in Macro- and Micro-

Thus, the Broken Symmetry of micro- theory of SuperConductivity (like in Bogoliubov's SuperFluidity) – is the phase Symmetry.

Nonconservation of the No of Cooper pairs or He II atoms relevant for the phase transition.

Different Symmetries in Macro- and Micro-

Thus, the Broken Symmetry of micro- theory of SuperConductivity (like in Bogoliubov's SuperFluidity) – is the phase Symmetry.

Nonconservation of the No of Cooper pairs or He II atoms relevant for the phase transition.

Cf. with Champagne-bottle-Symmetry of macro- phenomenological Ginzburg-Landau theory.

Micro and Macro Symmetries for the same object are different; Essentially different !

Symmetries: exact and approximate

What is Symmetry (broken) of physical problem ?

Symmetries: exact and approximate

What is Symmetry (broken) of physical problem ?

* In theory we deal with –

- Symmetry (the S.Lie one) of a classical Eq. and of some Approximation to it (including symmetry of effective macroscopic models.

Symmetries: exact and approximate

What is Symmetry (broken) of physical problem ?

* In theory we deal with –

- Symmetry (the S.Lie one) of a classical Eq. and of some Approximation to it (including symmetry of effective macroscopic models.
- Symmetry of Quantum Problem (operator eq., eigenvectors and matrix elements); of Approximation to Qu-Problem.

Symmetries: exact and approximate

What is Symmetry (broken) of physical problem ?

* In theory we deal with –

- Symmetry (the S.Lie one) of a classical Eq. and of some Approximation to it (including symmetry of effective macroscopic models.
- Symmetry of Quantum Problem (operator eq., eigenvectors and matrix elements); of Approximation to Qu-Problem.

* In what sense Do they relate to Symmetry of a given physical system ?

Levels of Symmetry

At least 3 levels of Symmetries:

- Classical (obvious) Symmetries: in crystals;

Levels of Symmetry

At least 3 levels of Symmetries:

- Classical (obvious) Symmetries: in crystals;
- "Intermediate" Symmetries

  – artificial but scenically transparent –

  {in order-parameter phenomenology macro-models} , Champagne-bottle = Mexican hat;

Levels of Symmetry

At least 3 levels of Symmetries:

- Classical (obvious) Symmetries: in crystals;
- "Intermediate" Symmetries – artificial but scenically transparent – {in order-parameter phenomenology macro-models} , Champagne-bottle = Mexican hat;
- Quantum Symmetries (hidden), like phase=gauge, Chiral and Super- Symmetry.

Levels of Symmetry

At least 3 levels of Symmetries:

- Classical (obvious) Symmetries: in crystals;
- "Intermediate" Symmetries

  – artificial but scenically transparent –

  {in order-parameter phenomenology macro-models} , Champagne-bottle = Mexican hat;
- Quantum Symmetries (hidden), like phase=gauge, Chiral and Super- Symmetry.
- Among Qu-Sym, approximate (in pQCD)

Modern Pilatus vs Critical phenomena

– What is the Symmetry of a physical system ?

(Symmetry involved in phase transition)

– In classical case, it is evident (bottle, crystals

– Can it be formulated independently of models ?

Modern Pilatus vs Critical phenomena

– What is the Symmetry of a physical system ?

(Symmetry involved in phase transition)

– In classical case, it is evident (bottle, crystals

– Can it be formulated independently of models ?

Heretical form : – Does the symmetry exist only in the conscience of theoreticians ?

Modern Pilatus vs Critical phenomena

– What is the Symmetry of a physical system ?

(Symmetry involved in phase transition)

– In classical case, it is evident (bottle, crystals

– Can it be formulated independently of models ?

Heretical form : – Does the symmetry exist only in the conscience of theoreticians ?

"What is the Verity ?" = that's the old question (by Pilatus to Jesus). The modern analog:

What is the Symmetry ?

Pilatus in XXI

"Quid est symmetria ?"

Reduction of Dimensions

Dimensional reduction, the transition 4D —> 2D, was used in 90s in HE Regge scattering (Aref'eva, Lipatov). In XXI, it got impetus in quantum gravity opening the way to (super)renormalizability.

We study the coupling behavior in $g\,\varphi^4$ model defined in both the 4D, 2D domains; the $\bar{g}(Q^2)$ evolutions being duly conjugated at a reduction scale $Q \sim M$.

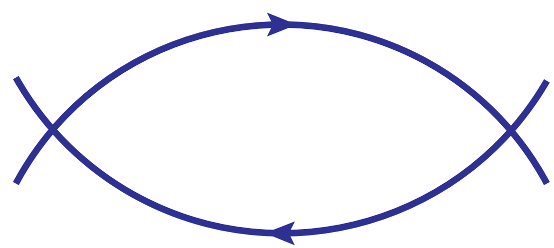
[Sh., arXiv: hep-th 1004.1510]

The $g\,\varphi^4$ model in 4D and 2D

Consider

$$L = T - V; \quad V(m, g; \varphi) = \frac{m^2}{2}\,\varphi^2 + \frac{4\pi^{d/2}\,M^{d-4}}{9}\,g\,\varphi^4; \qquad g > 0$$

in parallel in 4D (d=4) and 2D (d=2).
Limit ourselves to 1-loop leading level for $\bar{g}$ corresponding to only diagram, the 1st correction to 4-vertex function,.



Its contribution I enters into running coupling as follows:

$$\bar{g}(q^2) = \frac{g_i}{1 - g_i\,I\,(q^2; m^2, m_i^2)}\,.$$

Smooth DR in the mom picture

DR in Feynman integral by modifying metric

$$dk = d^4\,k \to d_M k \;=\; \frac{d^4\,k}{1 + k^2/M^2}\,; \qquad k^2 = \mathbf{k}^2 - k_0^2\,.$$

$$I\left(\frac{q^2}{m^2}\right) = \frac{i}{\pi^2}\int \frac{dk}{(m^2+k^2)[m^2+(k+q)^2]} \;\to$$

$$\to \frac{i\,M^2}{\pi^2}\int \frac{dk}{(m^2+k^2)[m^2+(k+q)^2]\,[M^2+k^2]} = J(\kappa;\mu)\,,$$

with $\kappa = q^2/4\,m^2$, $\mu = M^2/m^2$, $q^2 = \mathbf{q}^2 - q_0^2$. Explicitely;

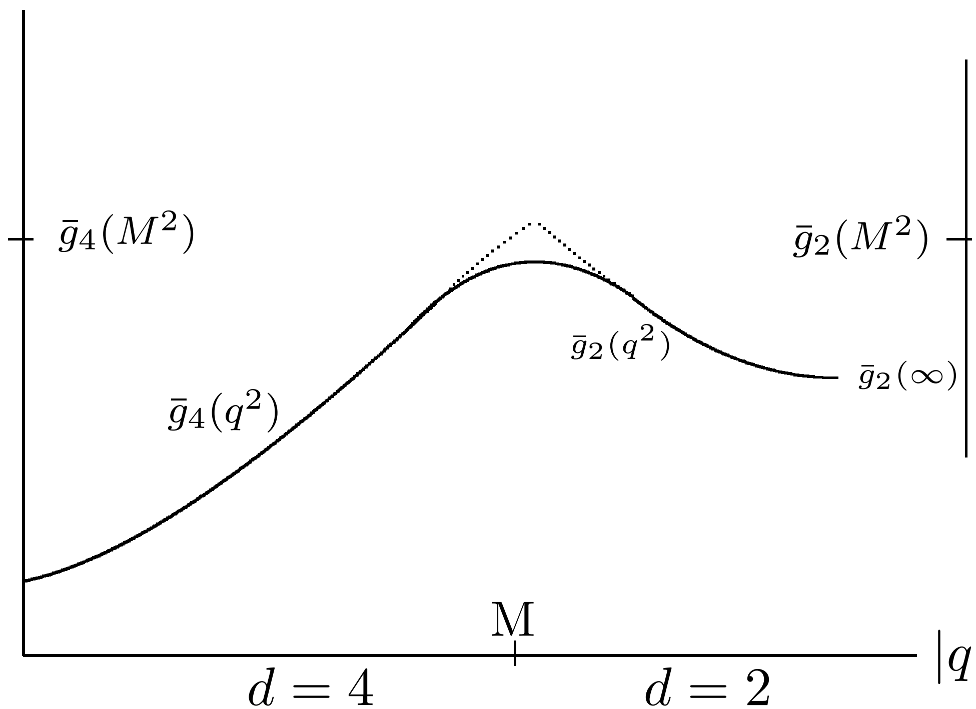
$$J_i^{[4]}(\kappa;\mu)\;\sim \ln\left(\frac{q^2}{m_i^2}\right)\;; \qquad J_i^{[2]}(\kappa;\mu)\;\sim \ln\left(\frac{4\,M^2}{m_i^2}\right) + \frac{M^2}{q^2}\ln\frac{q^2}{M^2}$$

at $m^2 \ll q^2 \ll M^2$ and at $M^2 \ll q^2$ $\sim q^2 \gg M^2$. 1st intermediate asymptote is rising; 2nd, final - decreasing.

The $\bar{g}$ UV fixed point by DR

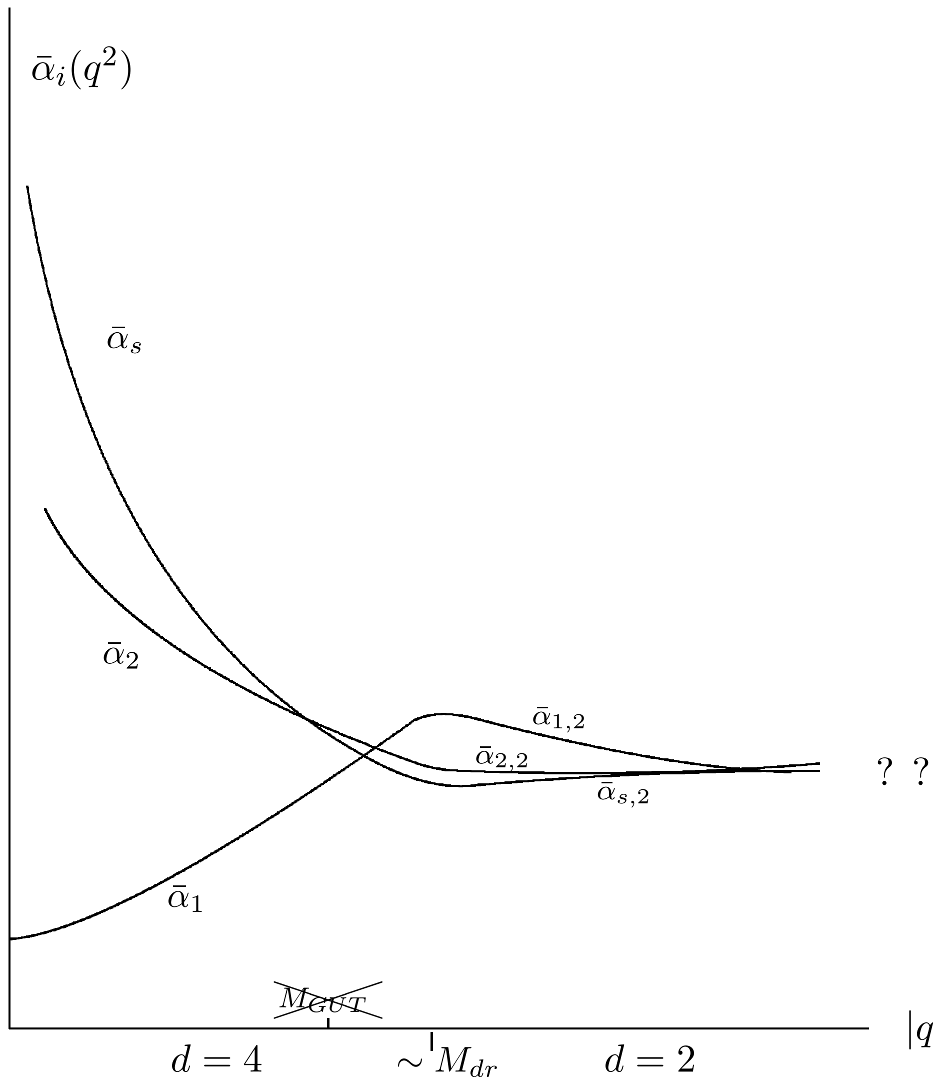
Coupling evolution changes drastically. The $\bar{g}(q^2)$ diminishes beyond DR scale tending to finite value

$$\bar{g}_2(\infty) = \frac{g_M}{1 + g_M\, I_2(M^2/m^2)} \;<\; g_M \;:$$



The dotted lines corresponds to hard conjunction at DR scale.

Great Unification via DR Looking-Glass ?



New brave Great Unification by DR instead of leptoquarks.

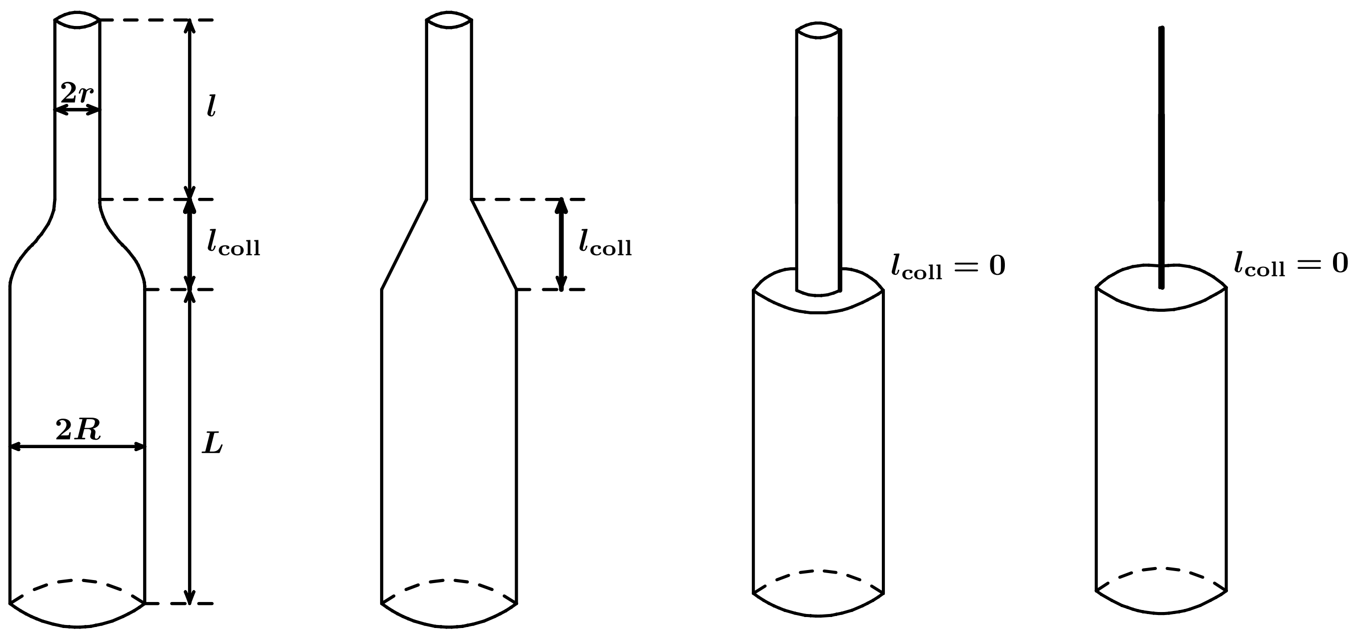
Resume of the DR hypothesis

The notable observation is that change of geometry could yield the same final result as an explicit change of dynamics (adding leptoquarks ...).

[arXiv: hep-th 1004.1510]

Toy models for the DR Looking-Glass

for studying (classic/quantum) problems on variable geometry manifold, like surface of "bottles" :



(a) (b) (c) (d)

– to learn on possible physical signal "from/through looking-glass at scale M"

Resume of the DR hypothesis, 2

The possibility of detecting some signal "through the looking-glass at DR scale" that would provide us with direct evidence on the existence of dimension reduction of any kind.

In micro -

Resume of the DR hypothesis, 2

The possibility of detecting some signal "through the looking-glass at DR scale" that would provide us with direct evidence on the existence of dimension reduction of any kind.

In micro - and/or in macro-world ?